Elizabeth Coppock
coppock@phil.hhu.de
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Compositional Semantics
Heinrich Heine University
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Models and Interpretation[1]

1 Review: Predicate calculus

Some formulas of Predicate Logic (or Predicate Calculus):

$\textsc{love}(\textsc{John}, \textsc{Mary})$ — 'John loves Mary'

$\forall x[\textsc{love}(\textsc{Mary}, x) \rightarrow \textsc{happy}(x)]$ — 'Everyone whom Mary loves is happy'

$\exists x[\textsc{even}(x) \wedge x > 1]$ — 'There are even numbers greater than 1'

Recall that predicate calculus is a formal language; a logic. Logics have a syntax and a semantics.

Syntax: specifies which expressions of the logic are well-formed, and what their syntactic categories are.

Semantics: specifies which objects the expression correspond to, and what their semantic categories are.

Syntactic categories. Formulas are built up from the following expressions of the syntactic categories including: **individual constants**, **variables**, **predicate constants**, **logical connectives**, and **quantifiers**.

Semantic types. Each expression belongs to a certain semantic type. The types of PC are: **individuals**, **relations**, and **truth values** (True or False).

For our examples we have:

[1] Based closely on lecture notes by V. Borschev and B. Partee

Expressions	Syntactic categories	Semantic Type
$\textsc{John}$,$\textsc{Mary}$	(individual) constant	individual
x	variable	individual
$\textsc{happy}$, $\textsc{even}$	unary predicate constant	unary relation
$\textsc{love}$, $>$	binary predicate constant	binary relation
$\textsc{love}(\textsc{John}, \textsc{Mary})$	(atomic) formula	truth value
$\textsc{happy}(x)$	(atomic) formula	truth value
$x > 1$	(atomic) formula	truth value
$\forall x[\textsc{love}(\textsc{Mary}, x) \rightarrow \textsc{happy}(x)]$	formula	truth value
$\exists x[\textsc{even}(x) \wedge x > 1]$	formula	truth value

Note: **Prefix notation** puts the relation first; **infix** notation puts the relation in the middle. Prefix notation is proper but we can use infix notation for readability.

2 Model theory

Model-theoretic semantics. The meaning of a formula (a sentence) is its truth-conditions: to know the meaning of a formula is to know what the world (the model) must be like if the formula is true. Knowing the meaning of a sentence does not require knowing whether the sentence is in fact true; it only requires being able to discriminate between situations in which the sentence is true and situations in which the sentence is false. Semantics which is based on truth-conditions is called *model-theoretic*.

Interpretation with respect to a model. Expressions of predicate calculus are *interpreted* in *models*. Models consist of a domain of individuals D and an interpretation function I which assigns values to all the constants:

$$\mathbf{M} = \langle D, I \rangle$$

An interpretation function $[\![\,]\!]^M$, built up recursively on the basis of the basic interpreation function I, assigns to every expression α of the language (not just the constants) a **semantic value** $[\![\alpha]\!]^M$.

Here are two models, M_r and M_f (r for "real", and f for "fantasy"/"fiction"/"fake"):

$$M_r = \langle D, I_r \rangle$$

$$M_f = \langle D, I_f \rangle$$

They share the same domain:

$$D = \{m, j, k\}$$

where m, j, and k denote actual individuals whose names are Mary, John and Kim.

In M_r, j is happy, but m and k are not:

$$I_r(\text{HAPPY}) = [\![\text{HAPPY}]\!]^{M_r} = \{j\}$$

In M_f, everybody is happy:

$$I_f(\text{HAPPY}) = [\![\text{HAPPY}]\!]^{M_f} = \{m, j, k\}$$

The interpretation of a constant such as MARY is always m:

$$I_r(\text{MARY}) = [\![\text{MARY}]\!]^{M_r} = m$$

$$I_f(\text{MARY}) = [\![\text{MARY}]\!]^{M_f} = m$$

What is the interpretation of HAPPY(MARY)? It should come out as false in M_r, and true in M_f. So what we want to get is:

$$[\![\text{HAPPY}(\text{MARY})]\!]^{M_r} = 0$$

$$[\![\text{HAPPY}(\text{MARY})]\!]^{M_f} = 1$$

What tells us this?

Principle of Compositionality: The semantics of formulas – their interpretation in every given model – is defined by semantic rules, which correspond in a direct way to the syntactic rules. The semantics of the whole is based on the semantics of parts by means of this pairing of semantic interpretation rules with syntactic formation rules.

Syntactic categories. Let us use the following:

Terms: (i) individual variables; (ii) individual constants.

Pred-1: RUN, WALK, HAPPY, CALM, ...EVEN, ODD, ...

Pred-2: LOVE, KISS, LIKE, SEE, ...

Composition rules. How to build formulas:

- If P ∈ Pred-1 and T ∈ Term, then P(T) ∈ Form.
- If R ∈ Pred-2 and T_1, T_2 ∈ Term, then R(T_1,T_2) ∈ Form.
- If ϕ ∈ Form , then $\neg\phi$ ∈ Form.
- If ϕ ∈ Form and ψ ∈ Form, then $[\phi \wedge \psi]$ ∈ Form.
- If ϕ ∈ Form and ψ ∈ Form, then $[\phi \vee \psi]$ ∈ Form.
- If ϕ ∈ Form and ψ ∈ Form, then $[\phi \rightarrow \psi]$ ∈ Form.
- If ϕ ∈ Form and ψ ∈ Form, then $[\phi \leftrightarrow \psi]$ ∈ Form.

Semantics. Domain D of entities (individuals); set of truth values $\{1, 0\}$, I: Interpretation function which assigns semantic values to all constants.

$$\mathbf{M} = \langle D, I \rangle$$

Semantic types assigned to semantic categories:

- Term ⇝ entities/individuals
- Pred-1 ⇝ sets (of entities). Semantic values of this type are members of $\wp(D)$, the powerset of of D. [What is the powerset of $\{m, j, k\}$?]
- Pred-2 ⇝ relations between entities (sets of pairs). Semantic values of this type are members of $\wp(D \times D)$, the powerset of the cross-product of D with itself. [What is the cross-product of $\{m, j, k\}$ with itself?]
- Pred-n ⇝ n-place relations; sets of n-tuples of entites. Values: members of $\wp(D \times ... \times D)$.
- Form ⇝ Truth values. Values are members of $\{0, 1\}$.

Notation: $[\![\phi]\!]^M$ is the value of ϕ with respect to model M.

Basic expressions:

- If α is a constant, then $[\![\alpha]\!]^M$ = I(α).

- If $P \in$ Pred-1 and $T \in$ Term, then $[[P(T)]]^M = 1$ iff $[[T]]^M \in [[P]]^M$.

  More general rule: If $R \in$ Pred-n and $T_1, ...T_n \in$ Term, then $[[R(T_1, ..., T_n)]]^M =$ 1 iff
  $$\langle [[T_1]]^M, ..., [[T_n]]^M \rangle \in [[R]]^M$$
- $[[\neg\phi]]^M = 1$ if $[[\phi]]^M = 0$; otherwise $[[\neg\phi]]^M = 0$.
- $[[\phi \wedge \psi]]^M = 1$ if $[[\phi]]^M = 1$ and $[[\psi]]^M = 1$; 0 otherise.
- Similarly for $[[\phi \vee \psi]]^M$, $[[\phi \to \psi]]^M$, and $[[\phi \leftrightarrow \psi]]^M$.

Variable assignments (teaser). $\forall x[\text{LOVE}(\text{MARY}, x) \to \text{HAPPY}(x)]$ is true in **M** if and only iff for every object d in the domain: $d \in [[\text{HAPPY}]]^M$ if $\langle [[\text{MARY}]]^M, d\rangle \in [[\text{LOVE}]]^M$. **More on this later!**

Example. Two models:
$$M_r = \langle D, I_r \rangle$$
$$M_f = \langle D, I_f \rangle$$
Domain for both models:
$$D = \{j, m, k\}$$
Interpretation functions:

$I_r(\text{JOHN}) = j$	$I_f(\text{JOHN}) = j$
$I_r(\text{MARY}) = m$	$I_f\text{MARY}) = m$
$I_r(\text{LOVE}) = \{\langle j,j\rangle, \langle j,m\rangle, \langle m,m\rangle, \langle m,j\rangle\}$	$I_f(\text{LOVE}) = \{\langle j,j\rangle, \langle m,j\rangle\}$
$I_r(\text{HAPPY}) = \{j\}$	$I_f(\text{HAPPY}) = \{m, j, k\}$

Building up formulae:

Because HAPPY $\in$ Pred-1 and MARY $\in$ Term, $[[\text{HAPPY}(\text{MARY})]]^{M_f} = 1$ iff:
$$[[\text{MARY}]]^{M_f} \in [[\text{HAPPY}]]^{M_f}$$
$$\iff m \in \{m, j, k\}$$
$[[\text{HAPPY}(\text{MARY})]]^{M_r} = 1$ iff:
$$[[\text{MARY}]]^{M_r} \in [[\text{HAPPY}]]^{M_r}$$
$$\iff m \in \{j\}$$

Homework: Dowty, Wall and Peters (1981), pp. 14–35, 44–47

1. Verify that $[K(d,j) \wedge M(d)]$ is a well-formed sentence of L_0 given the formation rules in (2-1) and (2-2).
2. What sorts of semantic values do one-place predicates have in L_0?
3. If M is a one-place predicate and j denotes an individual, then how do we determine the truth value of $M(j)$ in L_0?
4. Give an example of a two-place relation K such that $\langle a, c\rangle \in K$.
5. Give interpretations like the ones in (2-7) for the predicates K and M and the constants d and j that would make sentence 1 of example (2-4) true, keeping the semantic rules in (2-8).
6. Construct a phrase structure tree for one of the sentences in (2-10).
7. Let the set of individuals A be $\{a, b, c, d, e, f, g\}$. What is the characteristic function of the set $\{a, b, c\}$?
8. (i) Are the semantic values of intransitive verbs in L_{0E} sets of individuals or characteristic functions of sets of individuals? (ii) What about L_0? (iii) Is there any reason to choose one over the other (p. 28)?
9. Do problem (2-6), p. 29.
10. What is the truth value of *Henry Kissinger sleeps* in L_{0E}? (p. 30)
11. How do Dowty, Wall and Peters reconcile the following two facts: 1) VPs have as their semantic values functions from individuals to truth values; 2) Transitive verbs seem to express binary relations between individuals? (pp. 30–31)
12. Do problem (2-8).
13. It is important to recognize that a sentence can be true with respect to one model but false with respect to another. Dowty, Wall and Peters illustrate this by giving three models that yield different truth values for the sentence $M(d)$. Give another sentence ϕ of L_0 such that $[[\phi]]^{M_1} = 1$ and $[[\phi]]^{M_2} = 0$ (where M_1 and M_2 are defined as on pp. 46–7), and explain why.