

Model Answers to Problem Set 1, Part B

Reading: Heim and Kratzer (1998), chapter 1.

1. Let $A = \{a, b, c\}$ and $A' = \{A\}$.
 - (i) Is A a subset of A' ? [Recall that X is a subset of Y if all of the members of X are members of Y .]
No; the elements of A are $a, b,$ and $c,$ and none of these are members of $A',$ since the only member of A' is $A.$
 - (ii) Is $\{A\}$ a subset of A' ?
Yes, $\{A\} = A'$ and every set is a subset of itself.
 - (iii) Is A a member of A' ?
Yes.
 - (iv) Is a a member of A' ?
No; the only member of A' is $A.$
2. List all of the subsets of A . Don't forget the empty set!
 $\{a, b, c\}, \{a, b\}, \{a, c\}, \{b, c\}, \{a\}, \{b\}, \{c\}, \emptyset$
3. Let $A = \{a, b, c\}$ and $B = \{a, d, e\}$.
 - (i) What is the intersection of A and B ?
 $\{a\}$
 - (ii) What is the union of A and B ?
 $\{a, b, c, d, e\}$
 - (iii) What is the complement of A in B ?
 $\{d, e\}$
4. True or false: $\langle a, b \rangle = \langle b, a \rangle$. Why?
False [extra credit: unless $a=b!$], because the order matters in an ordered pair.
5. True or false: $\{a, b\} = \{b, a\}$. Why?
True, because sets that have the same elements are equal.
6. Under what circumstances is a relation a function?
When there is one and only one "output" for each "input"; for every object x that serves as the first element of an ordered pair in the relation $R,$ there is one and only one object y such that $R(x, y).$

7. Which of the following relations is a function:

$$(a) \begin{bmatrix} a \rightarrow b \\ c \rightarrow b \\ d \rightarrow c \end{bmatrix} \quad (b) \begin{bmatrix} a \rightarrow a \\ c \rightarrow b \\ d \rightarrow c \end{bmatrix}$$

Both! Trick question!

8. (i) What is the domain of the relation (a)?

$$\{a, c, d\}$$

(ii) What is its range?

$$\{b, c\} \text{ [Note: Technically this is equal to } \{b, b, c\} \text{ but I did not count the latter as a correct answer.]}$$

9. Give a function f such that $f(0) = 1, f(1) = 0, f(2) = 1, f(3) = 0, f(4) = 1,$
and $f(5) = 0$.

$$f(i) = 1 \text{ if } i \text{ is even; otherwise } f(i) = 0.$$