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Compositional Semantics
Heinrich Heine University
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Model Answers to Problem Set 2

Reading: Dowty, Wall and Peters (1981), pp. 14–35, 44–47

Question 1. Verify that $[K(d,j) \wedge M(d)]$ is a well-formed sentence of L_0 given the formation rules in (2-1) and (2-2).

(2-1)

Category	*Basic expressions*
Names	d, n, j, m
One-place predicates	M, B
Two-place predicates	K, L

(2-2)
1. If δ is a one place predicate and α is a name, then $\delta(\alpha)$ is a sentence.
2. If γ is a two place predicate and α and β are names, then $\gamma(\alpha, \beta)$ is a sentence.
3. If ϕ is a sentence, then $\neg\phi$ is a sentence.
4. If ϕ and ψ are sentences, then $[\phi \wedge \psi]$ is a sentence.
5. If ϕ and ψ are sentences, then $[\phi \vee \psi]$ is a sentence.
6. If ϕ and ψ are sentences, then $[\phi \rightarrow \psi]$ is a sentence.
7. If ϕ and ψ are sentences, then $[\phi \leftrightarrow \psi]$ is a sentence.

Solution: Since d and j are names and K is a two-place predicate according to (2-1), $K(d,j)$ is a sentence according to (2-2)-2. And since d is a name and M is a one place predicate according to (2-1), $M(d)$ is a sentence according to (2-2)-1. Two sentences joined by $\wedge$ form a sentence according to rule (2-2)-4, so the entire expression is a sentence.

Question 2. What sorts of semantic values do one-place predicates have in L_0?

Answer: Sets of individuals.

Question 3. If M is a one-place predicate and j denotes an individual, then how do we determine the truth value of $M(j)$ in L_0?

Answer: $[\![M(j)]\!]$ = 1 iff $[\![j]\!] \in [\![M]\!]$, because M is a one-place predicate, and j is a name. According to Rule (2-8)B1 (p. 21): "If δ is a one-place predicate and α is a name, then $\delta(\alpha)$ is true iff $[\![\alpha]\!] \in [\![\delta]\!]$."

Question 4. Give an example of a two-place relation K such that $\langle a, c\rangle \in K$.

Example answers:

- a stands for "apples" and c stands for "carrots" and K is the "is sweeter than" relation. [Defining K by description]
- $K = \{\langle a, a\rangle, \langle a, b\rangle, \langle a, c\rangle\}$ [Defining K by listing its members]

Question 5. Give interpretations like the ones in (2-7) for the predicates K and M and the constants d and j that would make sentence 1 of example (2-4) true, keeping the semantic rules in (2-8).

Sentence 1 of example (2-4), for reference: $K(d, j) \wedge M(d)$

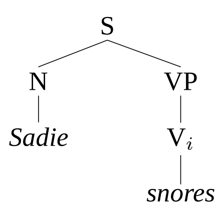
Example solution:

$[\![K]\!]$ = the set of all pairs of people such that the first one killed the second one
$[\![M]\!]$ = the set of all living people who are professional killers
$[\![d]\!]$ = Lee Harvey Oswald
$[\![j]\!]$ = John F. Kennedy

(thanks to Sebastian Klinge)

Question 6. Construct a phrase structure tree for one of the sentences in (2-10).

Solution (based on the phrase-structure rules given in (2-9)):



Alternate notation for this tree: $[_{\text{S}}\ [_{\text{N}}\ \textit{Sadie}\ ]\ [_{\text{VP}}\ [_{\text{V}_i}\ \textit{snores}\ ]\ ]\ ]$

Question 7. Let the set of individuals A be $\{a, b, c, d, e, f, g\}$. What is the characteristic function of the set $\{a, b, c\}$?

Answer:

$$\begin{bmatrix} a & \rightarrow & 1 \\ b & \rightarrow & 1 \\ c & \rightarrow & 1 \\ d & \rightarrow & 0 \\ e & \rightarrow & 0 \\ f & \rightarrow & 0 \end{bmatrix}$$

Alternate notation for the same answer: $\{\langle a,1\rangle,\langle b,1\rangle,\langle c,1\rangle,\langle d,0\rangle,\langle e,0\rangle,\langle f,0\rangle\}$

Alternate answer: The function f, defined such that $f(x)=1$ if $x\in\{a,b,c\}$ and 0 otherwise.

Question 8. (i) Are the semantic values of intransitive verbs in L_{0E} sets of individuals or characteristic functions of sets of individuals? (ii) What about L_0? (iii) Is there any reason to choose one over the other (p. 28)?

Answers:

(i) The semantic values of intransitive verbs in L_{0E} are characteristic functions.
(ii) In L_0 they are sets (as stated in the answer to question #2).
(iii) There is no *crucial* (i.e. empirical) reason, since "sets and characteristic functions are essentially two ways of looking at what amounts to the same thing." However, "[i]t may be more elegant to formalize semantic values as characteristic functions rather than sets in that the semantic rules which produce a truth value as output are assimilated to other rules which work by applying a function to an argument."

Question 9. Do problem (2-6), p. 29.

PROBLEM (2-6). Determine by means of the semantic rules just given the semantic values of the phrase structure trees of *Hank sleeps* and *Liz is-boring*.

The semantic rules that had just been given:

(2-19) If α is $[_\gamma\ \beta]$,[1] where γ is any lexical category and β is any lexical item, and $\gamma\to\beta$ is a syntactic rule, then $[\![\alpha]\!]=[\![\beta]\!]$

(2-20) If α is $[_{\text{VP}}\ \beta\ ]$ and β is V_i, then $[\![\alpha]\!]=[\![\beta]\!]$.

(2-21) If α is N and β is VP, and if γ is $[_{\text{S}}\ \alpha\ \beta]$, then $[\![\gamma]\!]=[\![\beta]\!]([\![\alpha]\!])$

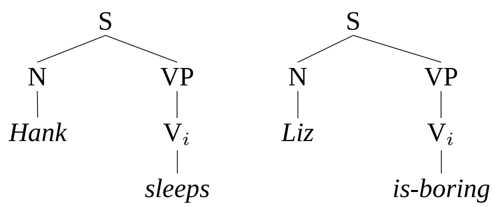
We will also need lexical entries for *Liz, Hank, sleeps,* and *is-boring*:

$$[\![\textit{sleeps}]\!] = \begin{bmatrix} \text{Anwar Sadat} & \to & 1 \\ \text{Queen Elizabeth} & \to & 0 \\ \text{Henry Kissinger} & \to & 0 \end{bmatrix}$$

$$[\![\textit{is-boring}]\!] = \begin{bmatrix} \text{Anwar Sadat} & \to & 1 \\ \text{Queen Elizabeth} & \to & 1 \\ \text{Henry Kissinger} & \to & 1 \end{bmatrix}$$

Phrase structure trees:

[1] See question 6 regarding this way of notating trees.



Computation of semantic value for *Hank sleeps*.

$$
\begin{array}{lll}
 & [\![\, [_{S} \, [_{N} \textit{ Hank} \,] \, [_{VP} \, [_{V_i} \textit{ sleeps} \,] \,] \,] \,]\!] & \\
= & [\![\, [_{VP} \, [_{V_i} \textit{ sleeps} \,] \,] \,]\!] (\, [\![\, [_{N} \textit{ Hank} \,] \,]\!] \,) & \text{(2-21)} \\
= & [\![\, [_{V_i} \textit{ sleeps} \,] \,]\!] (\, [\![\, [_{N} \textit{ Hank} \,] \,]\!] \,) & \text{(2-20)} \\
= & [\![\textit{sleeps}]\!] (\, [\![\textit{Hank}]\!] \,) & \text{(2-19)} \\
= & [\![\textit{sleeps}]\!] (\text{Henry Kissinger}) & \text{(lex. entry for } \textit{Hank}) \\
= & 0 & \text{(lex. entry for } \textit{sleeps})
\end{array}
$$

The computation for *Liz is-boring* is analogous, yielding a value of 1 in this case.

Question 10. What is the truth value of *Henry Kissinger sleeps* in L_{0E}? (p. 30)

It is not a sentence of L_{OE}, because it is not generated by the phrase structure rules, so it has no truth value!

Question 11. How do Dowty, Wall and Peters reconcile the following two facts: 1) VPs have as their semantic values functions from individuals to truth values; 2) Transitive verbs seem to express binary relations between individuals? (pp. 30–31)

By taking advantage of the isomorphism between relations and functions that allows the former to be rewritten as functions which yield other functions as its outputs.

Question 12. Do problem (2-8).

PROBLEM (2-8). Determine the truth value assigned to each of the phrase structure trees constructed in Problem (2-3), under the assumed assignments of semantic values to terminal symbols of L_{OE}.

PROBLEM (2-3). Construct all the phrase structure trees associated with the sentences in (2-10). The fourth sentence should have two trees. Set your results aside for Problem (2-8).

(2-10)
1. *Sadie snores.*
2. *Liz sleeps.*
3. *It-is-not-the-case-that Hank snores.*
4. *Sadie sleeps or Liz is-boring and Hank snores.*
5. *It-is-not-the-case-that it-is-not-the-case-that Sadie sleeps.*

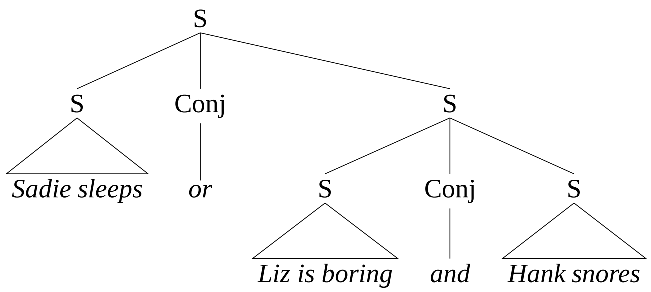
Phrase structure trees:

1. $[_{S}[_{N}\textit{Sadie}][_{VP}[_{V_i}\textit{snores}]]]$

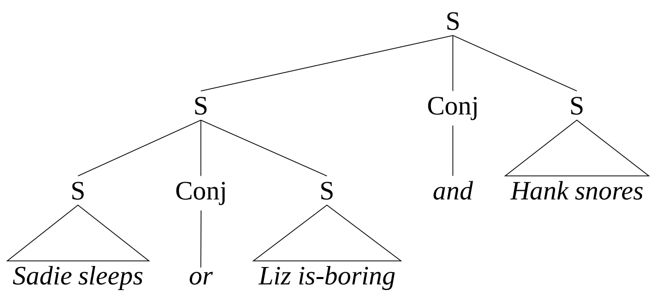
2. $[_{S}[_{N}\textit{Liz}][_{VP}[_{V_i}\textit{sleeps}]]]$

3. $[_{S}\textit{It-is-not-the-case-that}[_{S}[_{N}\textit{Hank}][_{VP}[_{V_i}\textit{snores}]]]]$

4a.



4b.



5. $[_{S}\textit{It-is-not-the-case-that}[_{S}\textit{it-is-not-the-case-that}[_{S}[_{N}\textit{Sadie}][_{VP}[_{V_i}\textit{sleeps}]]]]]$

Semantic values for terminal symbols:

$[\![\textit{Sadie}]\!]$	$=$	Anwar Sadat	(2-12)
$[\![\textit{Liz}]\!]$	$=$	Queen Elizabeth II	(2-12)
$[\![\textit{Hank}]\!]$	$=$	Henry Kissinger	(2-12)
$[\![\textit{snores}]\!]$	$=$	$\begin{bmatrix} \text{Anwar Sadat} & \rightarrow & 1 \\ \text{Queen Elizabeth} & \rightarrow & 1 \\ \text{Henry Kissinger} & \rightarrow & 0 \end{bmatrix}$	(2-13)
$[\![\textit{sleeps}]\!]$	$=$	$\begin{bmatrix} \text{Anwar Sadat} & \rightarrow & 1 \\ \text{Queen Elizabeth} & \rightarrow & 0 \\ \text{Henry Kissinger} & \rightarrow & 0 \end{bmatrix}$	(2-14)
$[\![\textit{is-boring}]\!]$	$=$	$\begin{bmatrix} \text{Anwar Sadat} & \rightarrow & 1 \\ \text{Queen Elizabeth} & \rightarrow & 1 \\ \text{Henry Kissinger} & \rightarrow & 1 \end{bmatrix}$	(2-15)
$[\![\textit{it-is-not-the-case-that}]\!]$	$=$	$\begin{bmatrix} 1 & \rightarrow & 0 \\ 0 & \rightarrow & 1 \end{bmatrix}$	(2-31)

$$[\![\mathit{and}]\!] = \begin{bmatrix} \langle 1,1\rangle & \rightarrow & 1 \\ \langle 1,0\rangle & \rightarrow & 0 \\ \langle 0,1\rangle & \rightarrow & 0 \\ \langle 0,0\rangle & \rightarrow & 0 \end{bmatrix} \quad \text{(2-33)}$$

$$[\![\mathit{or}]\!] = \begin{bmatrix} \langle 1,1\rangle & \rightarrow & 1 \\ \langle 1,0\rangle & \rightarrow & 1 \\ \langle 0,1\rangle & \rightarrow & 1 \\ \langle 0,0\rangle & \rightarrow & 0 \end{bmatrix} \quad \text{(2-34)}$$

The semantic composition rules that we need are, in addition to (2-19), (2-20), and (2-21), the following two:

(2-32) If α is Neg and ϕ is S, and if ψ is $[_{\mathrm{S}}\ \alpha\ \phi]$, then $[\![\psi]\!] = [\![\alpha]\!]([\![\psi]\!])$

(2-35) If α is Conj, ϕ is S, and ψ is S, and if ω is $[_{\mathrm{S}}\ \phi\ \alpha\ \psi]$, then $[\![\omega]\!] = [\![\alpha]\!](\langle[\![\phi]\!], [\![\psi]\!]\rangle)$

Solutions.

1. *Sadie snores*.

$$\begin{array}{lll} & [\![[_{\mathrm{S}}[_{\mathrm{N}}\mathit{Sadie}][_{\mathrm{VP}}[_{\mathrm{V}_i}\mathit{snores}]]]]\!] & \\ = & [\![[_{\mathrm{VP}}[_{\mathrm{V}_i}\mathit{snores}]]]\!]([\![[_{\mathrm{N}}\mathit{Sadie}]]\!]) & \text{(2-21)} \\ = & [\![[_{\mathrm{V}_i}\mathit{snores}]]\!]([\![[_{\mathrm{N}}\mathit{Sadie}]]\!]) & \text{(2-20)} \\ = & [\![\mathit{snores}]\!]([\![\mathit{Sadie}]\!]) & \text{(2-19)} \\ = & [\![\mathit{snores}]\!](\text{Anwar Sadat}) & \text{(lex. entry for } \mathit{Sadie}) \\ = & 1 & \text{(lex. entry for } \mathit{snores}) \end{array}$$

2. *Liz sleeps*.

$$\begin{array}{lll} & [\![[_{\mathrm{S}}[_{\mathrm{N}}\mathit{Liz}][_{\mathrm{VP}}[_{\mathrm{V}_i}\mathit{sleeps}]]]]\!] & \\ = & [\![[_{\mathrm{VP}}[_{\mathrm{V}_i}\mathit{sleeps}]]]\!]([\![[_{\mathrm{N}}\mathit{Liz}]]\!]) & \text{(2-21)} \\ = & [\![[_{\mathrm{V}_i}\mathit{sleeps}]]\!]([\![[_{\mathrm{N}}\mathit{Liz}]]\!]) & \text{(2-20)} \\ = & [\![\mathit{sleeps}]\!]([\![\mathit{Liz}]\!]) & \text{(2-19)} \\ = & [\![\mathit{sleeps}]\!](\text{Queen Elizabeth II}) & \text{(lex. entry for } \mathit{Liz}) \\ = & 0 & \text{(lex. entry for } \mathit{sleeps}) \end{array}$$

3. *It-is-not-the-case-that Hank snores*.

$$\begin{array}{lll} & [\![[_{\mathrm{S}}[_{\mathrm{Neg}}\mathit{It\text{-}is\text{-}not\text{-}the\text{-}case\text{-}that}][_{\mathrm{S}}[_{\mathrm{N}}\mathit{Hank}][_{\mathrm{VP}}[_{\mathrm{V}_i}\mathit{snores}]]]]]\!] & \\ = & [\![[_{\mathrm{Neg}}\mathit{It\text{-}is\text{-}not\text{-}the\text{-}case\text{-}that}]]\!]([\![[_{\mathrm{S}}[_{\mathrm{N}}\mathit{Hank}][_{\mathrm{VP}}[_{\mathrm{V}_i}\mathit{snores}]]]]\!]) & \text{(2-32)} \\ = & [\![[_{\mathrm{Neg}}\mathit{It\text{-}is\text{-}not\text{-}the\text{-}case\text{-}that}]]\!](0) & \text{(analogous to \#1 and \#2)} \\ = & [\![\mathit{It\text{-}is\text{-}not\text{-}the\text{-}case\text{-}that}]\!](0) & \text{(2-19)} \\ = & 1 & \text{(lex. entry 2-31)} \end{array}$$

4a. *[Sadie sleeps] or [Liz is-boring and Hank snores]*

First let us compute the semantic values for the sentences. Let us use the name SS for the phrase structure tree corresponding to *Sadie sleeps*, LB for the tree for *Liz is-boring*, and HS for the tree for *Hank snores*.

$$\begin{array}{lcl} [\![\text{SS}]\!] & = & 1 \\ [\![\text{LB}]\!] & = & 1 \\ [\![\text{HS}]\!] & = & 0 \end{array}$$

So what we want to compute is: $[\![\, [_{\text{S}}\ \text{SS}\ [_{\text{Conj}}\ \textit{or}\ ]\ [_{\text{S}}\ \text{LB}\ [_{\text{Conj}}\ \textit{and}\ ]\ \text{HS}\ ]\ ]\ ]\ ]\!]$.

$$\begin{array}{lll} & [\![\, [_{\text{S}}\ \text{SS}\ [_{\text{Conj}}\ \textit{or}\ ]\ [_{\text{S}}\ \text{LB}\ [_{\text{Conj}}\ \textit{and}\ ]\ \text{HS}\ ]\ ]\ ]\ ]\!] & \\ = & [\![\, [_{\text{Conj}}\ \textit{or}\ ]\ ]\!](\langle\ [\![\text{SS}]\!], [\![\ [_{\text{S}}\ \text{LB}\ [_{\text{Conj}}\ \textit{and}\ ]\ \text{HS}]\ ]\!]\ \rangle) & \text{(2-35)} \\ = & [\![\textit{or}]\!](\langle\ [\![\text{SS}]\!], [\![[_{\text{S}}\ \text{LB}\ [_{\text{Conj}}\ \textit{and}\ ]\ \text{HS}]]\!]\rangle) & \text{(2-19)} \\ = & [\![\textit{or}]\!](\langle\ [\![\text{SS}]\!], [\![\textit{and}]\!](\langle\ [\![\text{LB}]\!]\ ,\ [\![\text{HS}]\!]\ \rangle)) & \text{(2-19, 2-33)} \\ = & [\![\textit{or}]\!](\langle\ [\![\text{SS}]\!]\ ,\ 0\ \rangle) & \text{(lex. entry for } \textit{and}\text{)} \\ = & [\![\textit{or}]\!](\langle\ 1, 0\rangle) & \text{(computed above)} \\ = & 1 & \text{(lex. entry for } \textit{or}\text{)} \end{array}$$

4b. *[Sadie sleeps or Liz is-boring] and Hank snores*

What we want to compute is $[\![\ [_{\text{S}}\ [_{\text{S}}\text{SS}\ [_{\text{Conj}}\ \textit{or}\ ]\ \text{LB}\ ][_{\text{Conj}}\ \textit{and}\ ]\ \text{HS}\ ]\ ]\!]$.

This will end up as:

$$\begin{array}{ll} & [\![and]\!]([\![or]\!](\langle[\![\text{SS}]\!], [\![\text{LB}]\!]\rangle), [\![\text{HS}]\!]\rangle) \\ = & [\![and]\!](\langle[\![or]\!](\langle 1, 1\rangle), 0\rangle) \\ = & [\![and]\!](\langle 1, 0\rangle) \\ = & 0 \end{array}$$

5. *It-is-not-the-case-that it-is-not-the-case-that Sadie sleeps*.

After two applications of the negation rule (2-32), we get:

$$\begin{array}{ll} & [\![\textit{it-is-not-the-case-that}]\!]([\![\textit{it-is-not-the-case-that}]\!](SS)) \\ = & [\![\textit{it-is-not-the-case-that}]\!]([\![\textit{it-is-not-the-case-that}]\!](1)) \\ = & [\![\textit{it-is-not-the-case-that}]\!](0) \\ = & 1 \end{array}$$

The fourth sentence in (2-10) would have posed a problem if we had attempted to assign semantic values to terminal strings rather than trees or labelled bracketings. Why? Why could be assign semantic values directly to terminal strings if we were dealing with a syntactically ambiguous language.

Answer: We want semantic values to be assigned by a *function*, and a function gives a single output for every input.

Question 13. It is important to recognize that a sentence can be true with respect to one model but false with respect to another. Dowty, Wall and Peters illustrate this by giving three models that yield different truth values for the sentence $M(d)$. Give another sentence ϕ of L_0 such that $[\![\phi]\!]^{M_1} = 1$ and $[\![\phi]\!]^{M_2} = 0$ (where M_1 and M_2 are defined as on pp. 46–7), and explain why.

The constants in the two models are interpreted as follows:

	F_1	F_2
M	{Maine, New Hampshire, Vermont, Massachusetts, Connecticut, Rhode Island}	the set of all odd integers
B	states that have Pacific coasts	perfect squares
K	states such that some part of the first lies west of some part of the second	pairs of integers such that the first is greater than the second
L	pairs of states such that the first is larger than the second	pairs of integers such that the first is the square of the second
m	Michigan	2
j	California	0
d	Alaska	9
n	Rhode Island	-1

One possible answer: $L(j, n)$. California is bigger than Rhode Island, but 0 is not the square of -1.

Histogram of scores