

Problem Set 5: Partial functions

Read:

- Heim and Kratzer (1998), chapter 4, pp. 73–83

Exercises:

1. What is the lexical entry that Heim and Kratzer give for *the* (p. 75)?
2. Heim and Kratzer say that $\llbracket \mathbf{the} \rrbracket$ applied to $\llbracket \mathbf{opera\ by\ Beethoven} \rrbracket$ yields the value *Fidelio*. Give a denotation for *opera by Beethoven* that would give this result when Heim and Kratzer’s lexical entry for *the* applies to it via Functional Application. Your answer should have the following form:

$\llbracket \mathbf{opera\ by\ Beethoven} \rrbracket = \underline{\hspace{2cm}}$

3. Suppose that $\llbracket \mathbf{president\ of\ the\ USA} \rrbracket = \lambda x.x \in \{\text{Barack Obama}\}$. What is $\llbracket \mathbf{the} \rrbracket(\llbracket \mathbf{president\ of\ the\ USA} \rrbracket)$? Explain how you arrived at your answer.
4. Regarding *the negative square root of 4*, Frege says, “We have here a case in which out of a concept-expression [i.e., an expression whose meaning is of type $\langle e, t \rangle$] a compound proper name is formed [that is to say, the entire expression is of type e] with the help of the definite article in the singular, which is at any rate permissible when one and only one object falls under the concept.” To flesh out Frege’s analysis of this example, Heim and Kratzer suggest that *square root* is a “transitive noun”, with a meaning of type $\langle e, \langle e, t \rangle \rangle$, and that “*of* is vacuous, $\llbracket \mathbf{square\ root} \rrbracket$ applies to 4 via Functional Application, and the result of that composes with $\llbracket \mathbf{negative} \rrbracket$ under predicate modification.” Using these ideas, along with Heim and Kratzer’s lexical entry for *the*, give:
 - (a) Lexical entries for *negative*, *square root*, *of*, and 4
 - (b) A compositional analysis of *the negative square root of 4*. (That is, give a syntactic analysis and then say what the denotation is at each node of the syntactic tree, and what semantic rule is used to compute the value of the non-terminal nodes from the values of their daughters.)
5. Heim and Kratzer inform us that South College has no escalator. We can represent this as follows:

$$\llbracket \text{escalator in South College} \rrbracket = \lambda x.x \in \emptyset$$

That is, $\llbracket \text{escalator in South College} \rrbracket$ is the characteristic function of the empty set, so given any input, it always returns false.

What is $\llbracket \text{the} \rrbracket(\llbracket \text{escalator in South College} \rrbracket)$? Explain how you arrived at your answer.

6. Heim and Kratzer also inform us that there is more than one elevator in South College. Let us label these Elevator₁... Elevator_n, where n is the total number of elevators in South College. We can then define the semantic value for *elevator in South College* as follows:

$$\llbracket \text{elevator in South College} \rrbracket = \lambda x.x \in \{\text{Elevator}_1 \dots \text{Elevator}_n\}$$

What is $\llbracket \text{the} \rrbracket(\llbracket \text{elevator in South College} \rrbracket)$? Explain how you arrived at your answer.

7. What is the domain of $\llbracket \text{the} \rrbracket$?
8. Is $\llbracket \text{the} \rrbracket$ a function from $D_{\langle e,t \rangle}$ to D_e ? Why?
9. Is $\llbracket \text{the} \rrbracket$ is a partial function from $D_{\langle e,t \rangle}$ to D_e ? Why?
10. Assuming that $D_{\langle \sigma, \tau \rangle}$ (for any types σ, τ) is the set of all partial functions from D_σ to D_τ , what is the semantic type of $\llbracket \text{the} \rrbracket$ (give the label)?
11. Assume as in the previous assignment that *is* is semantically vacuous and that $\llbracket \text{broken} \rrbracket = \lambda x.x$ is broken. Show why *the elevator in South College is broken* has no semantic value. (Hint: It has to do with the Functional Application rule on p. 76.)
12. Heim and Kratzer suggest (p. 78) that “ ϕ having no value represents the case that ϕ has a false presupposition.” For example, consider the case where $\phi = \textit{The elevator in South College is broken}$. What would the false presupposition be in this case?