

Homework 9: Presuppositions of Quantified Sentences (or: *Every nation cherishes its king*)

Read: Heim (1983), *On the Projection Problem for Presuppositions*, §1.3, 3-4.

1. Why is *Every nation cherishes its king* a problem for Gazdar's theory?
2. Extra credit: Give another example of a sentence containing a part that has a presupposition containing a free variable (not involving possessives).
3. On p. 402, Heim suggests treating contexts as sets of pairs $\langle g, w \rangle$, where g is an assignment function (a sequence of individuals, or a function from integers to individuals) and w is a world.¹ For example, suppose:

$$c_1 = \left\{ \begin{array}{l} \langle \langle \text{Sweden, Sweden, Sweden} \rangle, w_1 \rangle \\ \langle \langle \text{Sweden, Sweden, Sweden} \rangle, w_2 \rangle \\ \langle \langle \text{Sweden, Sweden, Sweden} \rangle, w_3 \rangle \\ \langle \langle \text{Sweden, Sweden, America} \rangle, w_1 \rangle \\ \langle \langle \text{Sweden, Sweden, America} \rangle, w_2 \rangle \\ \langle \langle \text{Sweden, Sweden, America} \rangle, w_3 \rangle \\ \langle \langle \text{America, Sweden, America} \rangle, w_1 \rangle \\ \langle \langle \text{America, Sweden, America} \rangle, w_2 \rangle \\ \langle \langle \text{America, Sweden, America} \rangle, w_3 \rangle \\ \langle \langle \text{America, Sweden, Sweden} \rangle, w_1 \rangle \\ \langle \langle \text{America, Sweden, Sweden} \rangle, w_2 \rangle \\ \langle \langle \text{America, Sweden, Sweden} \rangle, w_3 \rangle \end{array} \right\}$$

And assume the following distribution of facts in worlds (which we can refer to as a 'model'):

- Sweden is a nation: $\{w_1, w_2, w_3\}$
- America is a nation: $\{w_1, w_2, w_3\}$
- Sweden has a king: $\{w_1, w_2, w_3\}$
- America has a king: $\{w_3\}$
- Sweden cherishes its king: $\{w_1, w_3\}$
- America cherishes its king: $\{w_3\}$

With this way of representing contexts, we can update contexts with sentences containing free variables. Updating a context c with (9) ' x_i has a king' gives: $c_1 \cup \{\langle g, w \rangle : g(i) \text{ has a king in } w\}$. Let $c_2 = 'c_1 + 'x_3 \text{ has a king}'$. What sequence-world pairs does c_2 contain?

4. The CCP that Heim gives for *every* is in (21), repeated here:

¹I will represent assignment functions as ordered sequences like $\langle \text{America, Sweden, America} \rangle$ but also use function notation, so if $g = \langle \text{America, Sweden, America} \rangle$ then $g(1) = \text{America}$, $g(2) = \text{Sweden}$, $g(3) = \text{America}$.

$c + \text{Every } x_i \text{ A, B} = \{ \langle g, w \rangle \in c : \text{for every } a, \text{ if } \langle g^{i/a}, w \rangle \in c + \text{A}, \text{ then } \langle g^{i/a}, w \rangle \in c + \text{A} + \text{B} \}$

where:

- $g^{i/a}$ is the assignment that is the same as g except that the individual a is assigned to the number i , and
- x_i must be a “new” variable in the sense of Heim’s (22): x_i is a **new variable** with respect to context c iff: For any two sequences g and g' that differ at most in their i -th member, and for any world w : $\langle g, w \rangle \in c$ iff $\langle g', w \rangle \in c$.²

Let us consider an example to see how this works. Suppose that $A = x_3$ is a nation, and $B = x_3$ cherishes x_3 ’s king. We want to compute $c +$ ‘Every x_3 , x_3 is a nation, x_3 cherishes x_3 ’s king’ for an appropriate c . Here is a guide:

- (a) Is x_3 “new” in c_1 in the sense made explicit in (22) assuming that the only two elements of the domain are Sweden and America? Explain.
- (b) Is $c_1 + A$ defined? Why or why not? If so, what is it? (Remember that an update is defined if the context entails all of its presuppositions. You can assume that ‘ x_3 is a nation’ has no presuppositions.)
- (c) Is $(c_1 + A) + B$ defined? Why or why not? If so, what is it? Remember that in order to admit (satisfy-the-presuppositions-of) ‘ x_i cherishes x_i ’s king’, a context must entail ‘ x_i has a king’. By this Heim means that “it has to be a context c such that, for every $\langle g, w \rangle \in c$, $g(i)$ has a king in w ’.”
- (d) Suppose that we change the model so that America has a king in w_1 , w_2 and w_3 . Is $(c_1 + A) + B$ defined? If so, what is it?
- (e) Is $c_1 +$ ‘Every x_3 , x_3 is a nation, x_3 cherishes x_3 ’s king’ defined? If so, explain why and compute its value and show how you arrived at your answer. If not, why not?
- (f) If we change the model so that America has a king in every world, is $c +$ ‘Every x_3 , x_3 is a nation, x_3 cherishes x_3 ’s king’ defined? If so, why? If not, why not?
- (g) Using the model where both Sweden and America have a king in every world, maintaining the assumption that Sweden cherishes its king in w_1 and w_3 , what is the resulting set of sequence-world pairs? I.e., what is the value of:

$$c_1 + \text{‘Every } x_3, x_3 \text{ is a nation, } x_3 \text{ cherishes } x_3 \text{’s king’?}$$

5. For any context c , it turns out that $c +$ ‘Every x_3 , x_3 is a nation, x_3 cherishes x_3 ’s king’ is defined whenever the following is true:

For every $\langle g, w \rangle \in c \cup \{ \langle g, w \rangle : g(3) \text{ is a nation in } w, g(3) \text{ has a king in } w. \}$

Combined with the fact that x_3 is required to be a “new” variable, this means that *Every nation cherishes its king* is predicted by Heim’s system to presuppose that every nation has a king. Explain how this universal presupposition arises.

6. Extra credit: Heim’s system predicts that (25) (*A fat man was pushing his bicycle*) presupposes that every fat man has a bicycle. Explain how this prediction comes about in your own words.

²In other words, for every pair $\langle g, w \rangle$ that you have in the context, for every x distinct from $g(i)$, there has to be another pair $\langle g', w \rangle$ where $g'(i) = x$. The value of i cannot be restricted to certain individuals.