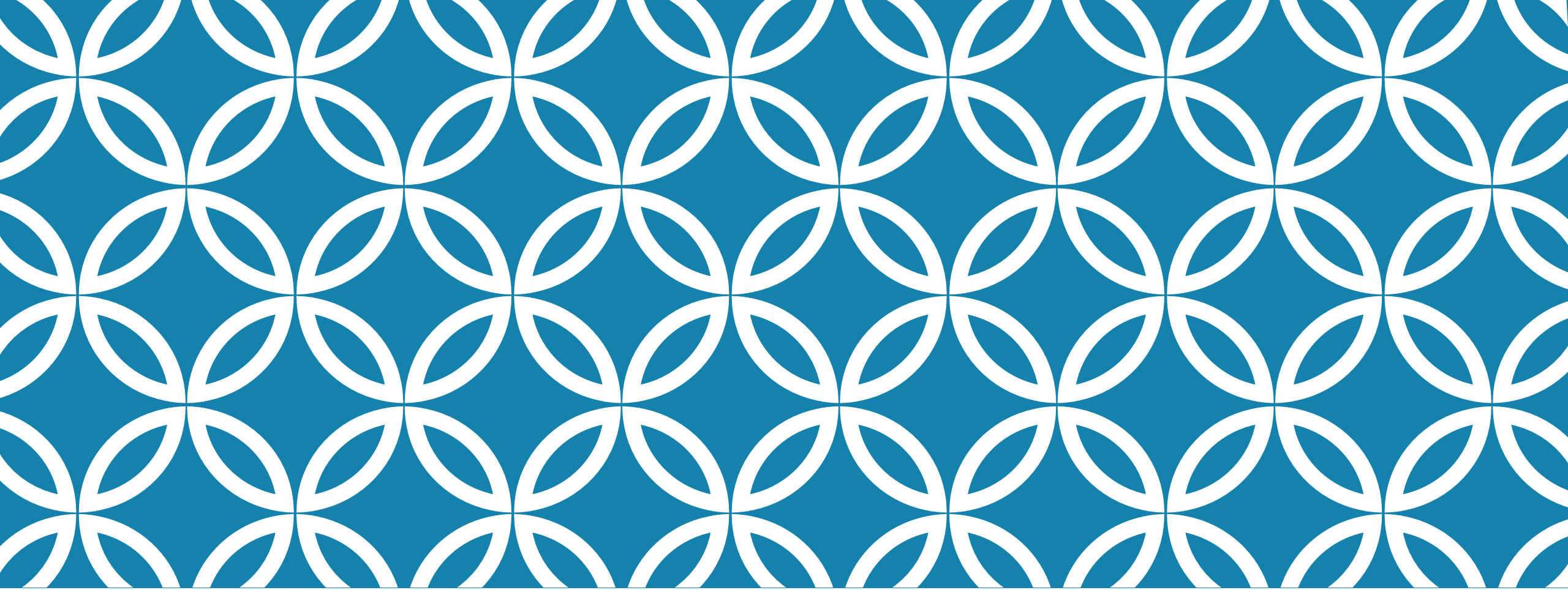
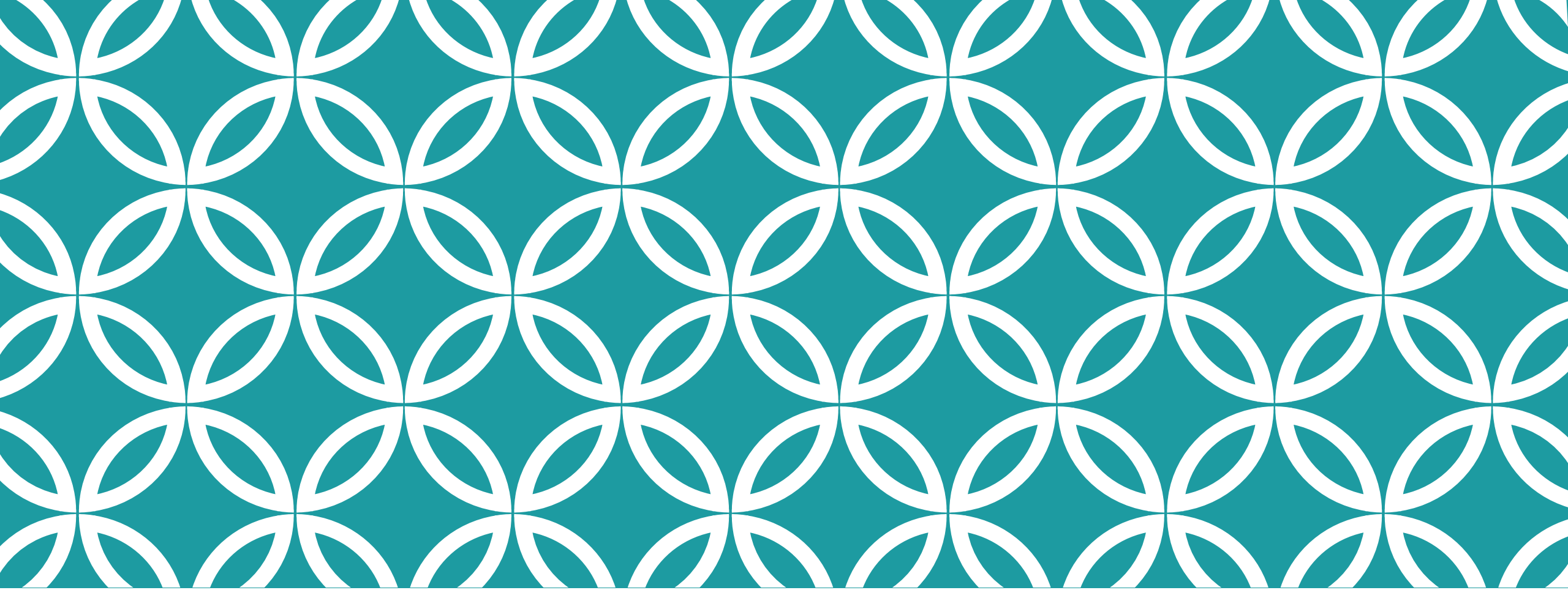


LECTURE 2 | Propositional logic

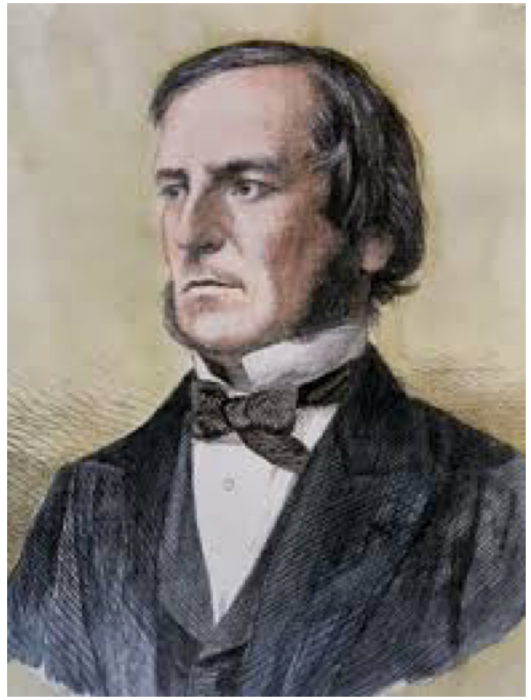
EGG 2019 | Introduction to Semantics | Elizabeth Coppock



1 Boolean connectives

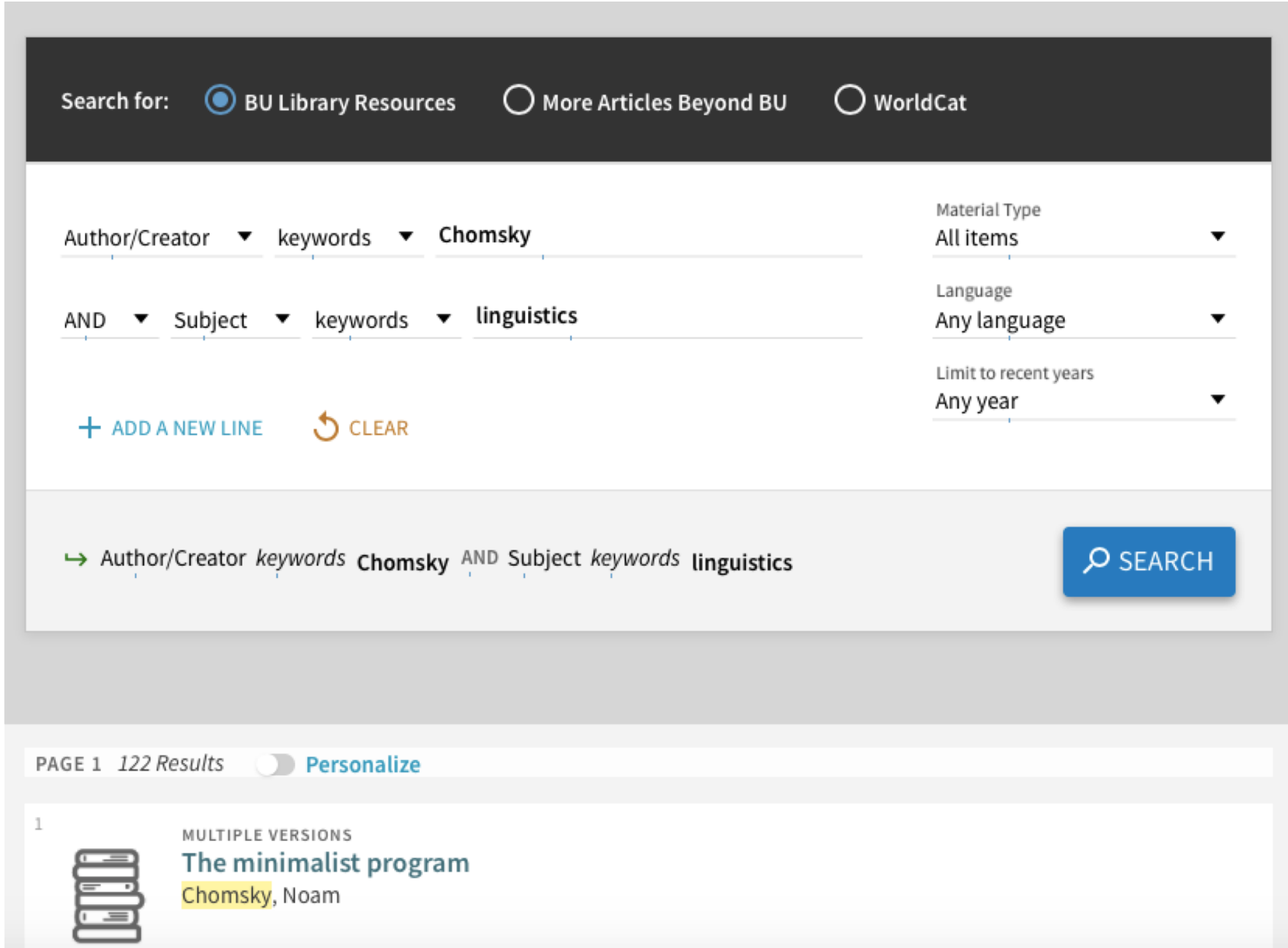
BOOLEAN CONNECTIVES

- and - $\wedge$
- or - $\vee$
- not - $\neg$



George Boole
(1815-1864)

BOOLEAN SEARCH



DISJUNCTION

An “or” statement is called a disjunction.

The statements that are disjoined are called the disjuncts.

CONJUNCTION

An “and” statement is called a conjunction.

The statements that are conjoined are called the conjuncts.

NEGATION

A “not” statement is called a negation.

Not is a unary connective, because it only applies to a single statement.

Conjunction and disjunction are binary connectives.

SCOPE AMBIGUITY WITH NEGATION & CONJUNCTION

Antonio didn't take Phonology and Syntax.

SCOPE AMBIGUITY WITH NEGATION & CONJUNCTION

Antonio didn't take Phonology and Syntax.

p = Antonio took Phonology *q = Antonio took Syntax*

SCOPE AMBIGUITY WITH NEGATION & CONJUNCTION

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Reading 1: Conjunction of negations

$[\neg p \ \& \ \neg q]$

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Antonio didn't take Phonology and Syntax.

p = *Antonio took Phonology* *q* = *Antonio took Syntax*

Reading 1: Conjunction of negations

$[\neg p \ \& \ \neg q]$

Reading 2: Negation of a conjunction

$\neg[p \ \& \ q]$

Suppose he took both

	Phonology (week 1)	Syntax (week 1)	Semantics (week 1)
10.00-11.15		**Hedde Zeijlstra** – Introduction to Syntax (2 weeks) ✓	**Serge Minor** – The Semantics of Grammatical Aspect in Slavic (and Beyond) (2 weeks)
11.30-12.45	**Shanti Ulfsbjorninn** – A typology of morphological segment-zero alternations	**Thomas McFadden** – Clause-Peripheral Agreements – Allocutivity, Complementizer Agreement and the Theory of Agree	**Elizabeth Coppock** – Introduction to Formal Semantics (2 weeks) ✓
14.15-15.30	**David Odden** – Introduction of Phonological Computation and Representation ✓	**Thomas McFadden, Sandhya Sundaresan & Hedde Zeijlstra** – Structure Building, Selection & Selective Opacity (2 weeks)	**Simon Charlow** – Alternatives in Semantics (2 weeks)
15.45-17.00	**Christian Uffmann** – Distinctive Feature Theory	**Karlos Arregi & Emily Hanink** – The syntax of switch reference	**Deniz Rudin** – The Semantics-Discourse-Pragmatics Interface: Theory and Applications (2 weeks) ✓
17.15-18.30 Open podium			

$[\neg p \ \& \ \neg q]$

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$[\neg p \ \& \ \neg q]$ - false

$\neg[p \ \& \ q]$ - false

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17.15-18.30 Open podium			

$[\neg p \ \& \ \neg q]$ - true

$\neg[p \ \& \ q]$ - true

Suppose he took only one

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$\neg[p \ \& \ q]$

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$[\neg p \,\&\, \neg q]$ - false

$\neg[p \,\&\, q]$ - true

SCOPE AMBIGUITY WITH NEGATION & CONJUNCTION

Antonio didn't take Phonology and Syntax.

p = *Antonio took Phonology* *q* = *Antonio took Syntax*

Reading 1: Conjunction of negations

$[\neg p \ \& \ \neg q]$

Reading 2: Negation of a conjunction

$\neg[p \ \& \ q]$

SEMANTICS FOR CONNECTIVES

truth-functional: truth value of whole depends entirely on truth value of parts

SEMANTIC RULE FOR NEGATION

- If p is true, then $\neg p$ is false.
- Likewise, if p is false, then $\neg p$ is true.

SEMANTIC RULE FOR NEGATION

- If p is true, then $\neg p$ is false.
- Likewise, if p is false, then $\neg p$ is true.

In truth table format:

p	$\neg p$
T	F
F	T

TRUTH TABLE FOR ∧ (“AND”)

p	q	$p \wedge q$
T	T	
T	F	
F	T	
F	F	

TRUTH TABLE FOR ∧ (“AND”)

p	q	$p \wedge q$
T	T	T
T	F	
F	T	
F	F	

TRUTH TABLE FOR ∧ (“AND”)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	
F	F	

TRUTH TABLE FOR ∧ (“AND”)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	

TRUTH TABLE FOR ∧ (“AND”)

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

TRUTH TABLES

p	q	$\neg p$	$\neg q$	$\neg p \ \& \ \neg q$
T	T	F	F	F
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

TRUTH TABLES

p	q	$\neg p$	$\neg q$	$\neg p \ \& \ \neg q$	$p \ \& \ q$	$\neg[p \ \& \ q]$
T	T	F	F	F	T	F
T	F	F	T	F	F	T
F	T	T	F	F	F	T
F	F	T	T	T	F	T

TRUTH TABLE FOR ∨ (“OR”)

p	q	$p \vee q$
T	T	
T	F	
F	T	
F	F	F

TRUTH TABLE FOR ∨ (“OR”)

p	q	$p \vee q$
T	T	
T	F	T
F	T	T
F	F	F

"OR": INCLUSIVE OR EXCLUSIVE?

I can meet you today or tomorrow.

→ not both days. [exclusive]

Your standard deduction is higher if you are 65 years or older or blind.

→ Still higher if you are both. [inclusive]

TRUTH TABLE FOR ∨ (INCLUSIVE "OR")

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

TRUTH TABLE FOR XOR (EXCLUSIVE "OR")

p	*q*	*p* XOR *q*
T	T	F
T	F	T
F	T	T
F	F	F

TWO HYPOTHESES

Hypothesis A: English “or” is ambiguous between an inclusive and an exclusive interpretation.

Hypothesis B: English “or” has only one interpretation, namely the inclusive one.

NEGATED INCLUSIVE DISJUNCTION

p	q	$p \vee q$	$\neg[p \vee q]$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

NEGATED EXCLUSIVE DISJUNCTION

p	q	p XOR q	$\neg[p$ XOR $q]$
T	T	F	T
T	F	T	F
F	T	T	F
F	F	F	T

WHICH PREDICTION IS BORNE OUT?

I can't meet you today or tomorrow

WHICH PREDICTION IS BORNE OUT?

I can't meet you today or tomorrow

means I can't meet you today AND I can't meet you tomorrow.

VALID VS. SOUND ARGUMENTS

An argument is valid if and only if the conclusion is entailed by the collection of the premises.

VALID VS. SOUND ARGUMENTS

An argument is valid if and only if the conclusion is entailed by the collection of the premises.

An argument is sound if and only if it is valid, and moreover, the premises are all true.

FALLACIES

An argument that is invalid can be called a fallacy.

THE CHOCOLATE LOVER'S ARGUMENT

- I can't stop eating these chocolates.
- I really love chocolate, or I seriously lack willpower.
- I know I really love chocolate.

Therefore:

I must not lack willpower.

AFFIRMING A DISJUNCT

p or q

p

Therefore: *not q*

PROVING VALIDITY USING TRUTH TABLES

- An argument is valid if the conclusion is entailed by the collection of the premises.

- To check whether an argument is valid:
  - ✓ Identify the situations where *all* of the premises are true.
  - ✓ Check whether the conclusion is true in all those situations. (If so, yes.)

POSSIBLE SITUATIONS

I ♥ chocolate	I lack willpower
TRUE	TRUE
TRUE	FALSE
FALSE	TRUE
FALSE	FALSE

ADDING COLUMNS FOR PREMISES AND CONCLUSION

Premise 2		Premise 1	Conclusion
I ♥ chocolate	I lack willpower	I ♥ chocolate or I lack willpower	I don't lack willpower
TRUE	TRUE	TRUE	FALSE
TRUE	FALSE	TRUE	TRUE
FALSE	TRUE	TRUE	FALSE
FALSE	FALSE	FALSE	TRUE

WHEN PREMISES BOTH ARE TRUE, CONCLUSION MAY BE FALSE

Premise 2		Premise 1	Conclusion
I ♥ chocolate	I lack willpower	I ♥ chocolate or I lack willpower	I don't lack willpower
TRUE	TRUE	TRUE	FALSE
TRUE	FALSE	TRUE	TRUE
FALSE	TRUE	TRUE	FALSE
FALSE	FALSE	FALSE	TRUE

WITH EXCLUSIVE "OR", CONCLUSION FOLLOWS

Premise 2		Premise 1	Conclusion
I ♥ chocolate	I lack willpower	I ♥ chocolate XOR I lack willpower	I don't lack willpower
TRUE	TRUE	FALSE	FALSE
TRUE	FALSE	TRUE	TRUE
FALSE	TRUE	TRUE	FALSE
FALSE	FALSE	FALSE	TRUE

AFFIRMING A DISJUNCT

p or q

p

Therefore: *not q*

Is valid only if *or* is treated as *exclusive* disjunction.

SCOPE AMBIGUITY WITH NEGATION & CONJUNCTION

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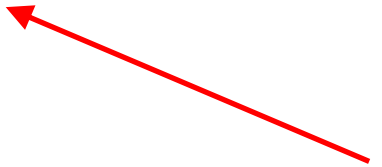

Paraphrasable as

Antonio didn't take Phonology or Syntax.

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Reading 2: Negation of a conjunction

$\neg[p \ \& \ q]$

Paraphrasable as

$\neg[p \ V \ q]$

DEMORGAN'S LAWS

$\neg p \wedge \neg q$ is equivalent to $\neg[p \vee q]$

$\neg p \vee \neg q$ is equivalent to $\neg[p \wedge q]$

PROOF

What is the relationship between $\neg p \wedge \neg q$ and $\neg[p \vee q]$?

p	q	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$p \vee q$	$\neg[p \vee q]$
T	T					
T	F					
F	T					
F	F					

PROOF

What is the relationship between $\neg p \wedge \neg q$ and $\neg[p \vee q]$?

p	q	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$p \vee q$	$\neg[p \vee q]$
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T	F					
F	T					
F	F					

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T	T	F				
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F	T	T				
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T	T	F				
T	F	F				
F	T	T				
F	F	T				

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T	T	F	F			
T	F	F	T			
F	T	T	F			
F	F	T	T			

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What is the relationship between $\neg p \wedge \neg q$ and $\neg[p \vee q]$?

p	q	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$p \vee q$	$\neg[p \vee q]$
T	T	F	F			
T	F	F	T			
F	T	T	F			
F	F	T	T			

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p	q	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$p \vee q$	$\neg[p \vee q]$
T	T	F	F	F		
T	F	F	T	F		
F	T	T	F	F		
F	F	T	T	T		

PROOF

What is the relationship between $\neg p \wedge \neg q$ and $\neg[p \vee q]$?

p	q	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$p \vee q$	$\neg[p \vee q]$
T	T	F	F	F		
T	F	F	T	F		
F	T	T	F	F		
F	F	T	T	T		

PROOF

What is the relationship between $\neg p \wedge \neg q$ and $\neg[p \vee q]$?

p	q	$\neg p$	$\neg q$	$\neg p \wedge \neg q$	$p \vee q$	$\neg[p \vee q]$
T	T	F	F	F	T	
T	F	F	T	F	T	
F	T	T	F	F	T	
F	F	T	T	T	F	

PROOF

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T	F	F	T	F	T	
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PROOF

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T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	T	F	T

PROOF

What is the relationship between $\neg p \wedge \neg q$ and $\neg(p \vee q)$?

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F	T	T	F	F	T	F
F	F	T	T	T	F	T

PROOF

What is the relationship between $\neg p \wedge \neg q$ and $\neg[p \vee q]$?

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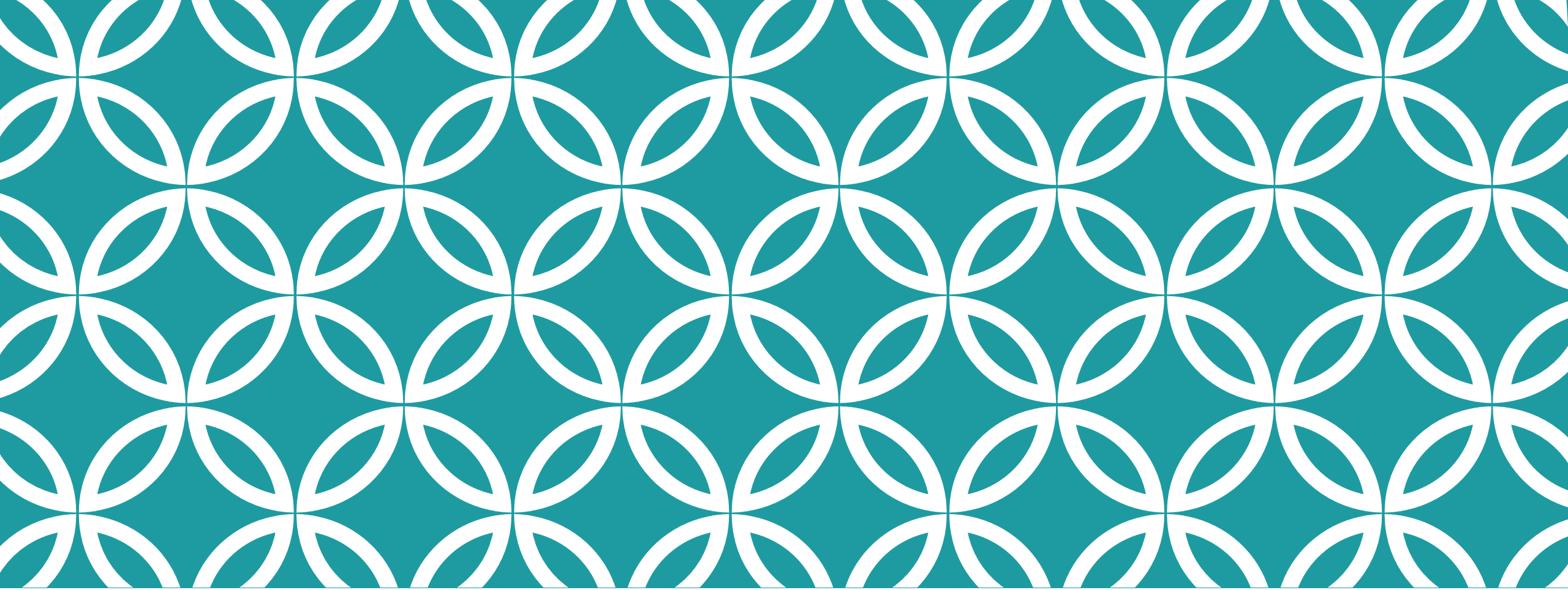
T	T	F	F	F	T	F
T	F	F	T	F	T	F
F	T	T	F	F	T	F
F	F	T	T	T	F	T

They are equivalent!

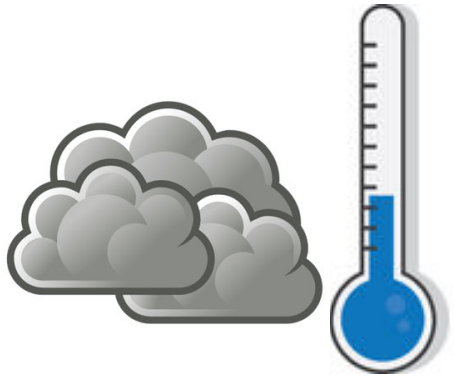


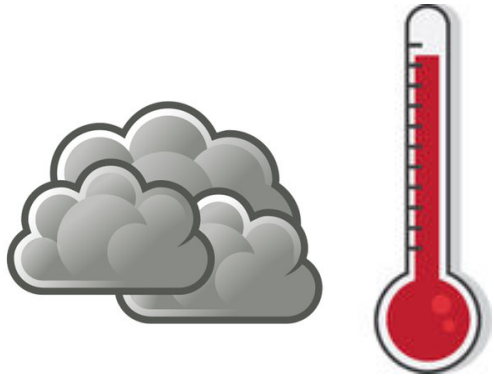
11 | Material conditional

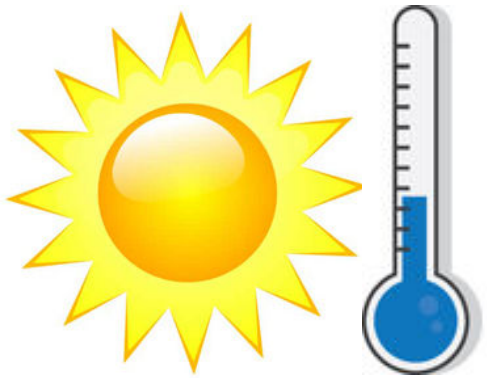
SEMANTICS FOR $\rightarrow$

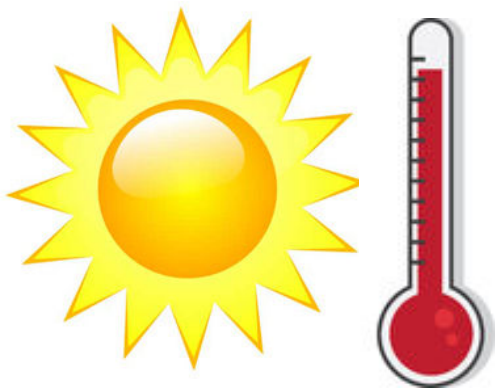
p	q	$p \rightarrow q$
T	T	
T	F	
F	T	
F	F	

If it's sunny, then it's warm

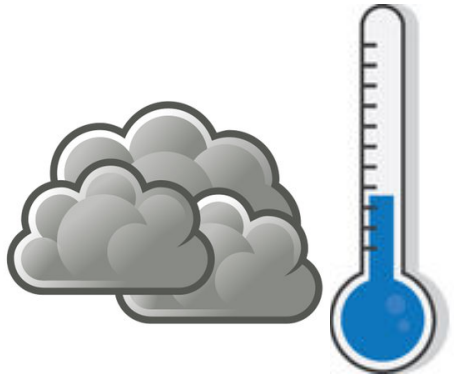


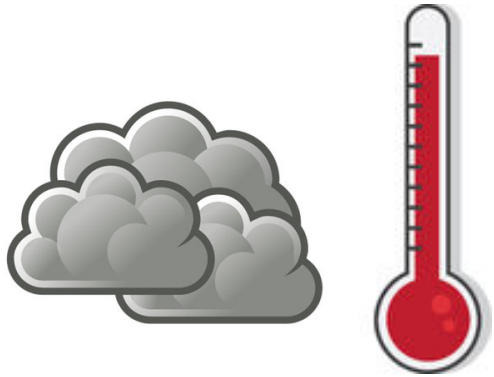


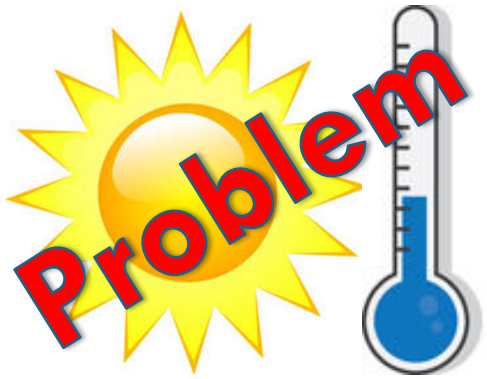


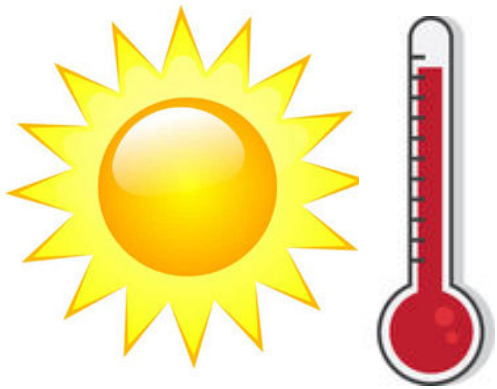


If it's sunny, then it's warm

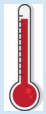
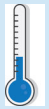
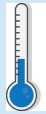




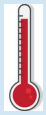
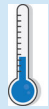
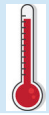
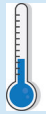




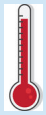
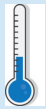
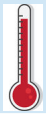
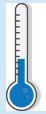
SEMANTICS FOR →

SUNNY	WARM	SUNNY → WARM
T	 T	
T	 F	
F	T	
F	 F	

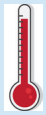
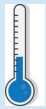
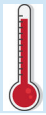
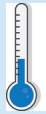
SEMANTICS FOR →

SUNNY	WARM	SUNNY → WARM
T	 T	
T	 F	**Problem**
F	 T	
F	 F	

SEMANTICS FOR →

SUNNY	WARM	SUNNY → WARM
T	 T	
T	 F	F
F	 T	
F	 F	

SEMANTICS FOR $\rightarrow$

SUNNY	WARM	SUNNY $\rightarrow$ WARM
T	 T	T
T	 F	F
F	 T	T
F	 F	T

SEMANTICS FOR $\rightarrow$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

SEMANTICS FOR $\rightarrow$

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

This is called the material conditional.

MODUS PONENS

If Ed is a CreteLing student, he is in Crete.

Ed is a CreteLing student.

Therefore: *Ed is in Crete.*

MODUS TOLLENS

If Ed is a CreteLing student, then Ed is in Crete.

*Ed is **not** in Crete.*

Therefore: *Ed is **not** a CreteLing student.*

MODUS TOLLENS AND MODUS PONENS

MODUS TOLLENS

If p then q

not-q

Therefore:

not-p

MODUS PONENS

If p then q

p

Therefore:

q

MODUS TOLLENS IN SCIENCE

If Hypothesis A is correct, then the solution is acidic.

*The solution is **not** acidic.*

Therefore: *Hypothesis A is **not** correct.*

MODUS TOLLENS IN SCIENCE

If Hypothesis A is correct, then the solution is acidic.

*The solution is **not** acidic.*

Therefore: *Hypothesis A is **not** correct.*

*That's how you **falsify** a hypothesis!*

USING MODUS TOLLENS TO NEGATE

If you're a genius, then I'm a monkey's uncle!

DENYING THE ANTECEDENT

If Ed has cable, then Ed has seen a naked lady.

Ed doesn't have cable.

Therefore: *Ed has never seen a naked lady.*

DENYING THE ANTECEDENT

If p then q

not p

Therefore: *not q*

AFFIRMING THE CONSEQUENT

If Ed is a CreteLing student then Ed is in Crete.
Ed is in Crete.
Therefore: *Ed is a CreteLing student.*

AFFIRMING THE CONSEQUENT

If p then q

q

Therefore: *p*

DESIDERATA FOR A THEORY OF IF-THEN

Valid arguments

Modus Ponens

Modus Tollens

Fallacies

Denying the antecedent

Affirming the consequent

MODUS PONENS TURNS OUT VALID

MODUS PONENS: $p \rightarrow q$; p; *therefore* q

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

MODUS PONENS TURNS OUT VALID

MODUS PONENS: $p \rightarrow q$; p; *therefore* q

p	q	$p \rightarrow q$
T	T	T
T	F	T
F	T	T
F	F	F

FALSE ANTECEDENT → TRUE CONDITIONAL

If the moon is made of green cheese, then I had yogurt for breakfast this morning.

FALSE ANTECEDENT? TRUE CONDITIONAL!

If the moon is made of green cheese, then I had yogurt for breakfast this morning.

Hence, if p is a contradiction, then $p \rightarrow q$ is a tautology!

A COUNTERINTUITIVE TAUTOLOGY

$[p \wedge \neg p] \rightarrow q$

p	q	$\neg p$	$p \wedge \neg p$	$[p \wedge \neg p] \rightarrow q$
T	T	F	F	T
T	F	F	F	T
F	T	T	F	T
F	F	T	F	T

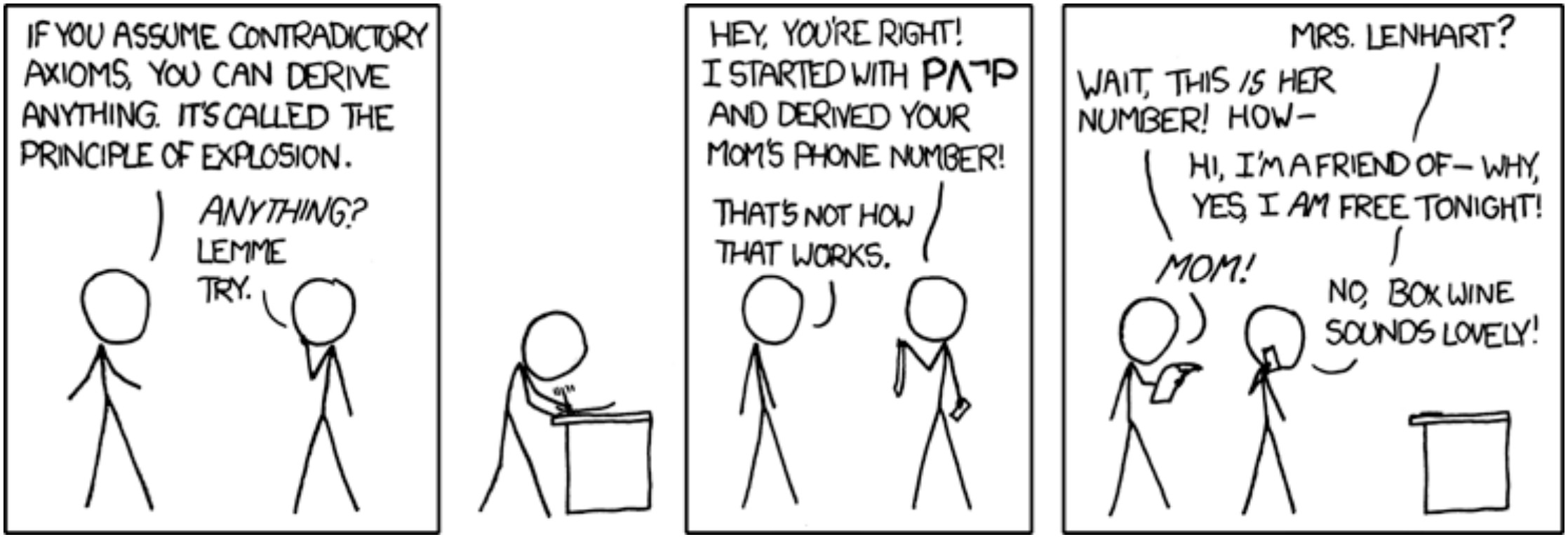
PRINCIPLE OF EXPLOSION

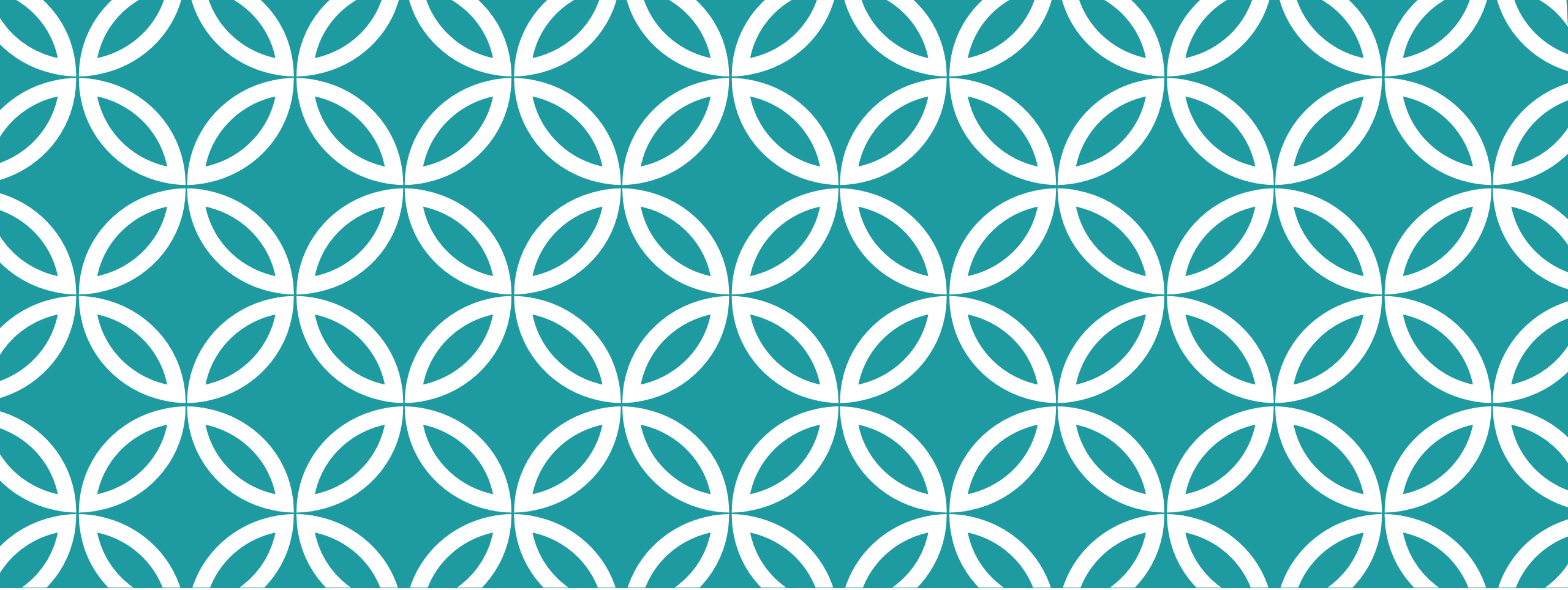
$[p \wedge \neg p] \rightarrow q$

$[p \wedge \neg p]$

Therefore: q

FROM XKCD


IF YOU ASSUME CONTRADICTORY AXIOMS, YOU CAN DERIVE ANYTHING. IT'S CALLED THE PRINCIPLE OF EXPLOSION.
ANYTHING? LEMME TRY.
HEY, YOU'RE RIGHT! I STARTED WITH P∧¬P AND DERIVED YOUR MOM'S PHONE NUMBER!
THAT'S NOT HOW THAT WORKS.
MRS. LENHART?
WAIT, THIS IS HER NUMBER! HOW—
HI, I'M A FRIEND OF— WHY, YES, I AM FREE TONIGHT!
MOM!
NO, BOX WINE SOUNDS LOVELY!



I Predicates and relations

PREDICATE LOGIC

Basic ingredients:

- Predicates and relations
- Names and variables
- Connectives from propositional logic
- Quantifiers

BASIC EXPRESSIONS IN PREDICATE LOGIC

- names: a, b, c, d, e, f, ...
- predicates
  - unary predicates
    - e.g. Man, Violinist, Swedish, Gardener
  - binary predicates
    - e.g. Partner, Husband, Friend

SOME PREDICATE LOGIC EXPRESSIONS

Violinist	(unary predicate)
Swedish	(unary predicate)
j	(name)
Violinist(j)	(formula)
Swedish(j)	(formula)

SETS

Both *Swedish* and *violinist* pick out sets of objects.

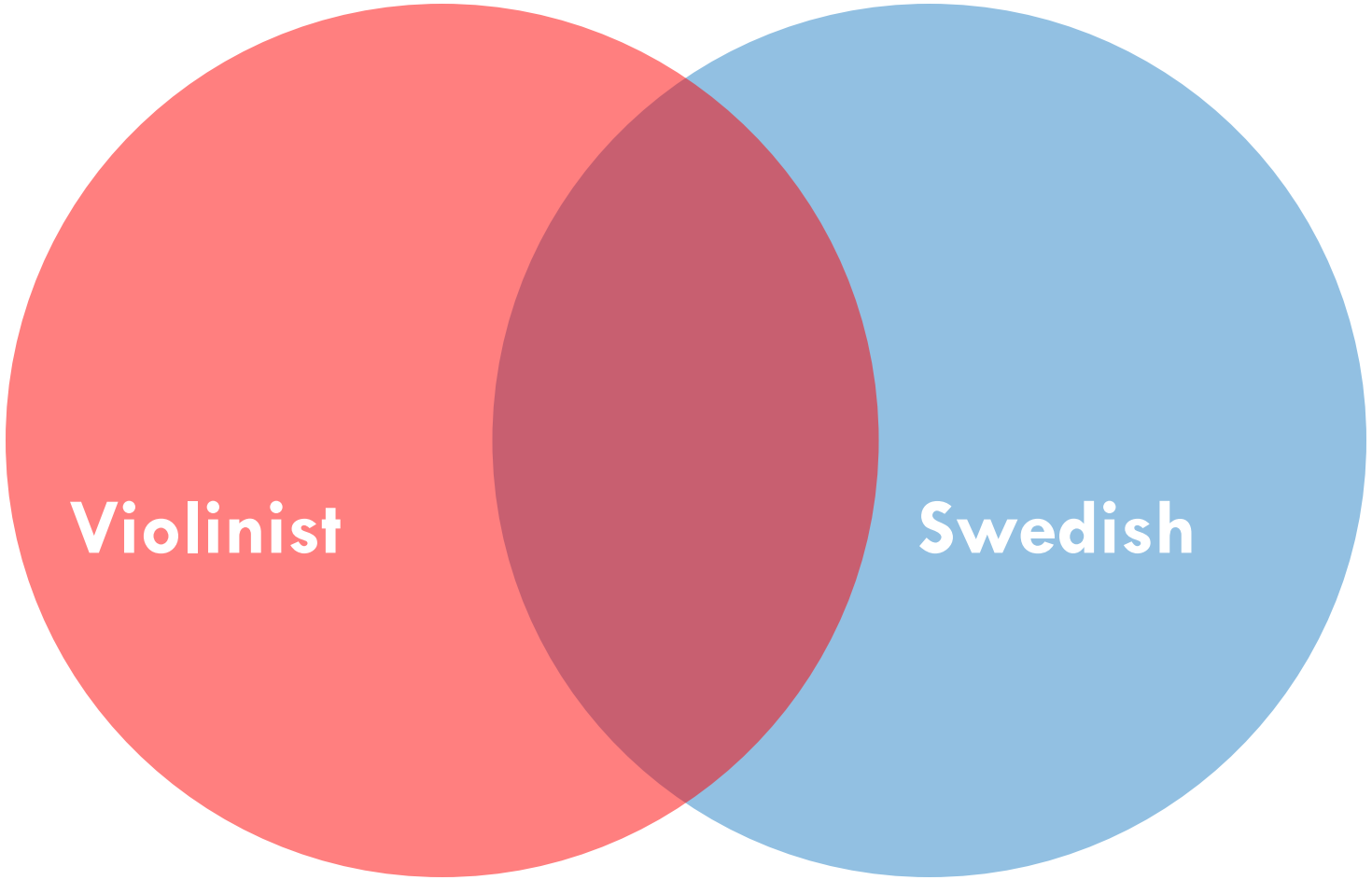
Swedish violinist picks out things that are in the intersection of those two sets.

INTERSECTIVE MODIFICATION

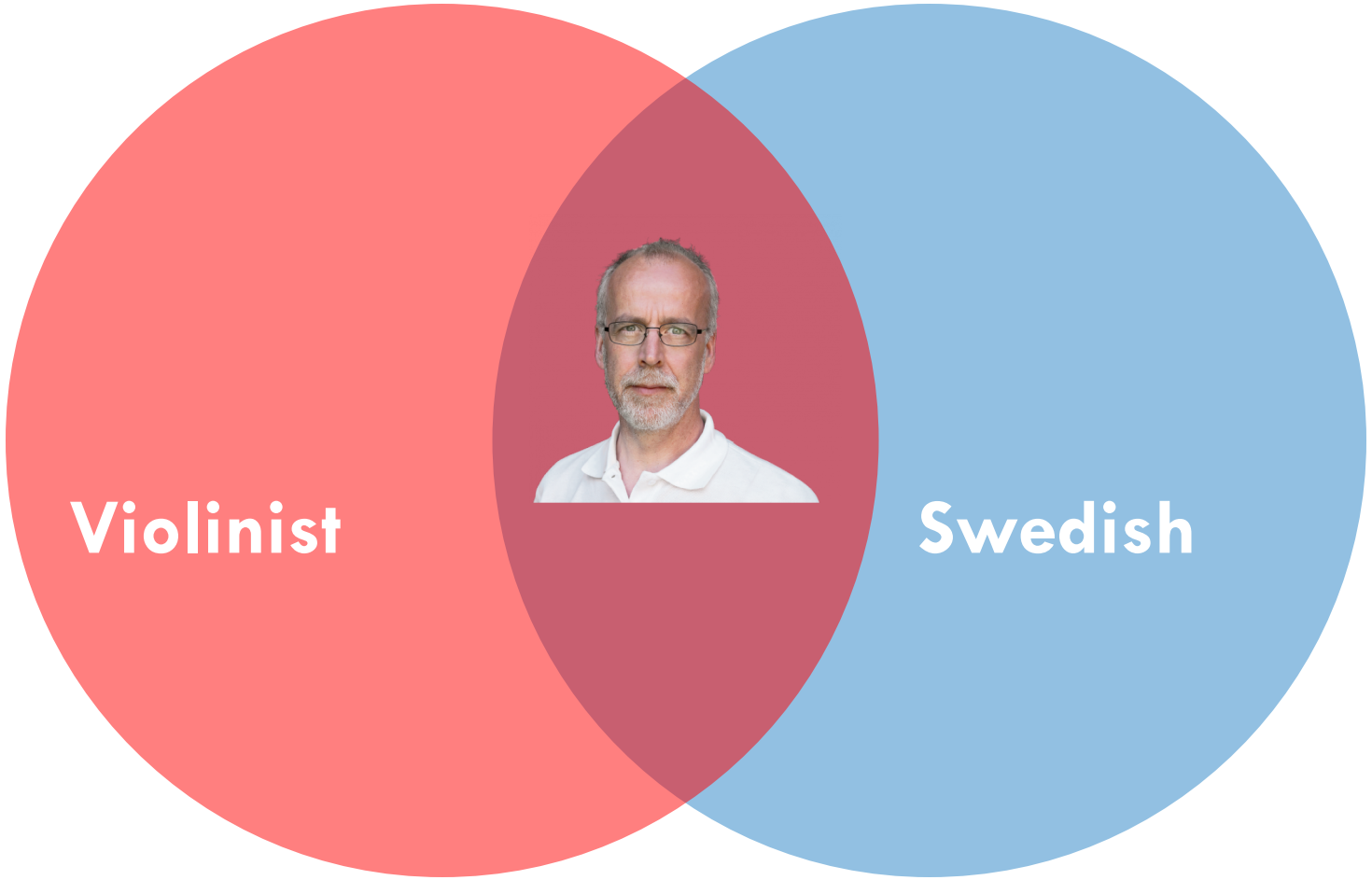
Janne is a Swedish violinist.

Therefore, Janne is Swedish.

INTERSECTIVE MODIFICATION



INTERSECTIVE MODIFICATION



UNARY PREDICATES

Janne is a Swedish violinist.

Therefore, Janne is Swedish.

Violinist(j) & Swedish(j)

Swedish(j)

PREDICATES, SETS, AND RELATIONS

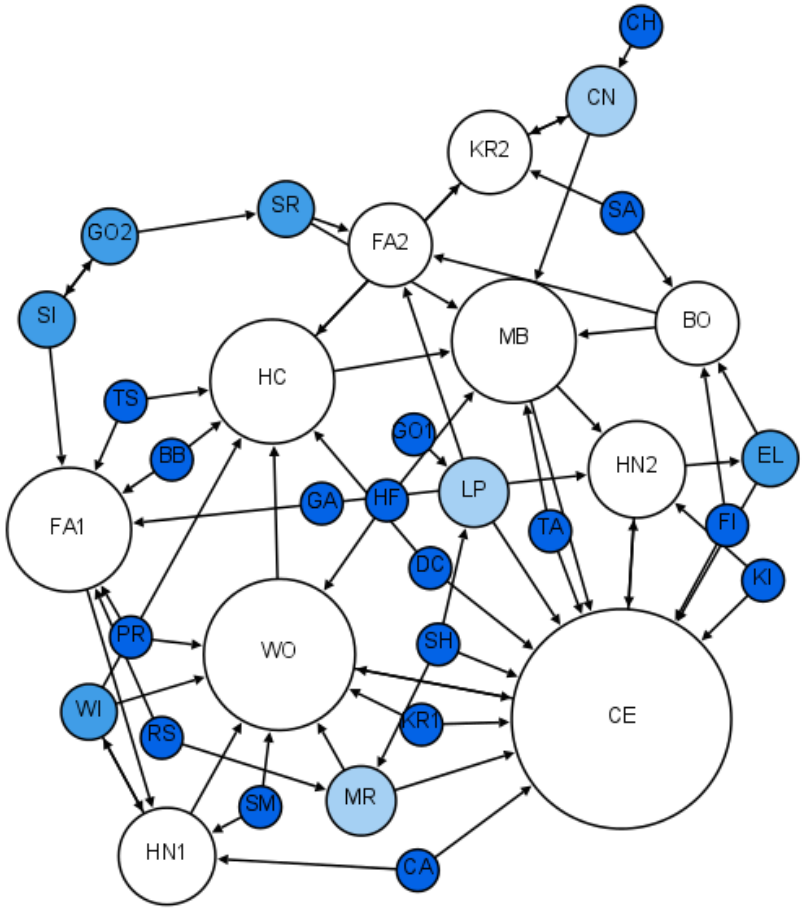
A unary predicate denotes a set.

A binary predicate denotes a binary relation.

BINARY PREDICATES: 2 ARGUMENTS

Partner(h,m)
Partner(m,h)
Husband(m,h)
Brother(l,b)
Sister(b,l)
Sibling(b,l)
Sibling(l,b)

MORENO SOCIOGRAM: 1ST GRADE



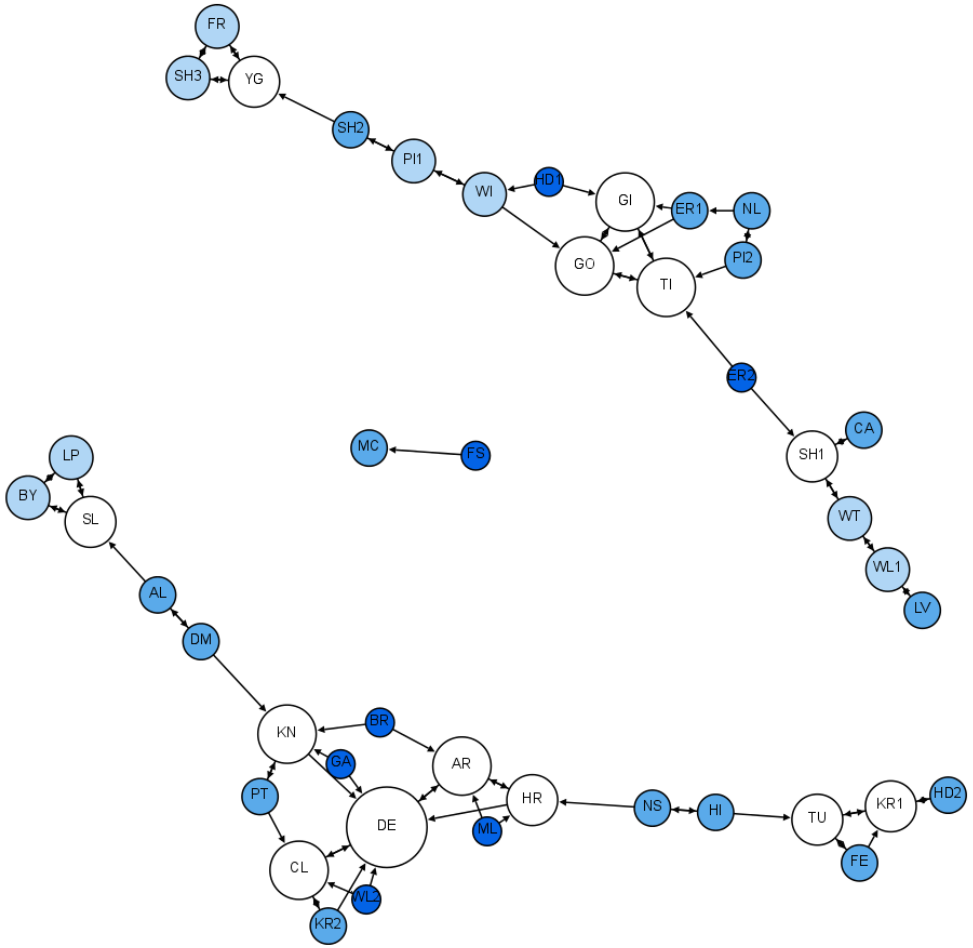
a → b

signifies

a would like to sit next to b

Likes(LP,CE)
Likes(HN2,CE)
Likes(CE,HN2)
Likes(TS,FA1)
…

MORENO SOCIOGRAM: 6TH GRADE



ATOMIC FORMULAS

Unary predicates take one argument:

Man(a)

Violinist(f)

Swedish(f)

Binary predicates take two arguments:

Partner(a,b)

Husband(c,f)

Friend(b,c)

ATOMIC FORMULAS

Unary predicates take one argument:

Man(a)

Violinist(f)

Swedish(f)

Binary predicates take two arguments:

Partner(a,b)

Husband(c,f)

Friend(b,c)

The arity of a predicate is the number of arguments it takes.

WELL-FORMED ATOMIC FORMULAS

Well-formed formulas	Non-well-formed formulas
Violinist(f)	Violinist(f,a)
Husband(c,f)	Husband(c)
Partner(a,b)	Partner(a,b,c)

WELL-FORMED ATOMIC FORMULAS

Well-formed formulas	Non-well-formed formulas
Violinist(f)	Violinist(f,a)
Husband(c,f)	Husband(c)
Partner(a,b)	Partner(a,b,c)

WHICH IS NOT WELL-FORMED?

A. Violinist(x)
B. Husband(b)
C. Husband(a,a)
D. Partner(a,b)

SYNTAX OF PREDICATE LOGIC

- Every well-formed atomic sentence is a wff.
- If X is any wff, then ¬X is also a wff.
- If X and Y are wffs, then so are:

  [X & Y]
  [X ∨ Y]
  [X → Y]

WHICH IS NOT WELL-FORMED?

A. Violinist(a) & Violinist(a)
B. Violinist(a) & Violinist(b)
C. ¬¬¬Violinist(b)
D. Violinist(a&b)

POSSIBLE PROPERTIES OF BINARY RELATIONS

R is **symmetric** iff for all a, b:

R(a,b) entails R(b,a)

R is **reflexive** iff for all a:

R(a,a)

POSSIBLE PROPERTIES OF BINARY RELATIONS

R is **asymmetric** iff it is not the case that for all a, b:

R(a,b) entails R(b,a)

R is **non-reflexive** iff it is not the case that for all a:

R(a,a)

POSSIBLE PROPERTIES OF BINARY RELATIONS

R is **transitive** iff for all a, b, c:

R(a,b) and R(b,c) entails R(a,c)

POSSIBLE PROPERTIES OF BINARY RELATIONS

R is **non-transitive** iff it is not the case that for all a, b, c:

R(a,b) and R(b,c) entails R(a,c)

RECIPROCAL CONSTRUCTIONS

John is Mary's partner.

So, John and Mary are partners. [valid]

RECIPROCAL CONSTRUCTIONS

John is Mary's partner.

So, John and Mary are partners. [valid]

Partner(j,m)

[Partner(j,m) & Partner(m,j)]

RECIPROCAL CONSTRUCTIONS

John is Mary's husband.

#So, John and Mary are husbands.

RECIPROCAL CONSTRUCTIONS

John is Mary's husband.

#So, John and Mary are husbands.

Husband(j,m)

[Husband(j,m) & Husband(m,j)]

RECIPROCAL CONSTRUCTIONS

John is Mary's mailman.

So, John and Mary are mailmen. [not valid]

RECIPROCAL CONSTRUCTIONS

John is Mary's mailman.

So, John and Mary are mailmen. [not valid]

[Mailman(j) & **Poss**(m,j)]

[Mailman(j) & Mailman(m)]

RECIPROCAL CONSTRUCTIONS

Mary is John's sister.

#So, Mary and John are sisters.

Sister(m,j)

[Sister(m,j) & Sister(j,m)]

RECIPROCAL CONSTRUCTIONS

Mary is Sue's sister.

So, Mary and Sue are sisters.

Sister(m,s)

[Sister(m,s) & Sister(s,m)]

RECIPROCAL CONSTRUCTIONS

John is Bill's husband.

?So, John and Bill are husbands.

Husband(j,m)

[Husband(j,m) & Husband(m,j)]

RECIPROCAL CONSTRUCTIONS

Mary is Sue's wife.

??So, Mary and Sue are wives.

Wife(m,s)

[Wife(m,s) & Wife(s,m)]

RELATED READING

Schwarz, Bernhard. 2006. Covert reciprocity and Strawson-symmetry. *Snippets* 13. 9–10.

Winter, Yoad. To appear. Symmetric predicates and the semantics of reciprocal alternations. *Semantics & Pragmatics*.