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Superlative Modifiers as Modified Superlatives*

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Abstract The superlative modifiers *at least* and *at most* are quite famous, but their cousins *at best*, *at the latest*, *at the highest*, etc., are less well-known. This paper is devoted to the entire family. New data is presented illustrating the productivity of the pattern, identifying a generalization delimiting it, and showing that the cousins, too, have the pragmatic effects that have attracted so much attention to *at least* and *at most*. To capture the productivity, I present a new decomposition of *at least* into recombinable parts. Most notable is the *at*-component (silent in some languages), which takes advantage of the comparison class argument of the superlative to produce the set of possibilities involved in the ignorance implicatures that superlative modifiers are known for. A side-effect is a new view on gradable predicates, accounting for uses like *88 degrees is too hot*.

Keywords: superlative modifiers, superlatives, alternative semantics, degee semantics

1 Introduction

As Geurts & Nouwen (2007) observed, the superlative modifiers *at least* and *at most* are intriguingly different from the comparative modifiers *more than* and *less than*. For example, *at least* is felt to be an odd choice in the following example for a speaker who knows how many sides a hexagon has, while *more than* is not problematic in the same way (Nouwen 2010):

(1) a. #A hexagon has **at least** six sides.
b. A hexagon has **more than** five sides.

Support for the existence of such a contrast in out-of-the-blue contexts comes from an informal experiment I have conducted several times while giving talks on this subject: When English speakers are informally asked to guess which of the following examples is attested, and which is constructed, they typically guess correctly:

* Thanks to Chris Kennedy for extensive discussion on a prior draft of this system that was presented as part of a joint talk at the *Two Days at Most* workshop in Utrecht, September 2015; thanks also to the participants of that workshop.

(2) a. I am in my sixties and had **at least three** parents growing up.

b. I am in my sixties and had **more than two** parents growing up.

The *at least* variant in (2a) furthermore typically elicits a chuckle, as it suggests that the author of the statement does not know how many parents he or she had growing up (or doesn't know how to define 'parent'). This ignorance implicature, along with a number of other properties, has been placing high demands on the ink supply.[1,2]

But, as Krifka (2007) points out, despite all the attention, the internal structure of superlative modifiers, containing a *superlative*, has rarely been recognized. Recognizing this structure makes it possible to account for the productivity of the pattern: We have not only *at least* and *at most*, but also *at best*, *at the latest*, *at the highest*, and others. To my knowledge, only a few have recognized this, including Krifka (2007) (handout; brief mention), Penka (2010) (handout) and Solt (2011: pp. 6-10). The present paper critiques this prior work, and offers an alternative solution.

The solution is built on two observations, one new in this paper (as far as I know) and one not quite as new. The new observation is that the phrase being modified must be interpretable as a measure along the dimension indicated by the gradable adjective to which the superlative attaches. The less new observation is that all superlative modifiers have the interesting pragmatic properties of *at least* and *at most*, such as ignorance implications.

Based on these observations, the paper offers a decomposition of *at least* into recombinable parts, including a superlative ending and a special meaning for *at*. The *at*-component takes advantage of the comparison class argument of the superlative to produce a set of possibilities involved in the ignorance implicatures that superlative modifiers are known for. In a nutshell, *at least three* denotes a set of Hamblin alternatives containing numbers that are as high as three or higher, and these alternatives 'fan outwards' to yield a disjunction-like proposition of the kind used in inquisitive semantics.[3]

1 Krifka 1999, Geurts & Nouwen 2007, Krifka 2007, Büring 2008, Cohen & Krifka 2011, Nakanishi & Rullmann 2009, Cummins & Katsos 2010, Nouwen 2010, Penka 2010, Solt 2011, Mayr 2013, Biezma 2013, Schwarz 2013, Coppock & Brochhagen 2013, Nouwen 2013, Penka 2014, Westera & Brasoveanu 2014, McNabb & Penka 2014, Kennedy 2015, Schwarz to appear, Schwarz 2016, Alexandropoulou, Dotlacil, McNabb & Nouwen 2015, Alexandropoulou 2015, Rett 2014, Blok 2015, Mendia to appear, and certainly others to whom I owe an apology.

2 Westera & Brasoveanu (2014) show that some explicit QUDs, namely yes/no questions, can eliminate the ignorance implicature for superlative modifiers. However, the difference remains in place in some contexts, including 'how many' questions. Comparative modifiers also signal ignorance implicatures in such contexts, but to a lesser degree, so it seems that the ignorance inferences triggered by comparative modifiers are of a somewhat different, less obligatory nature than those triggered by superlative modifiers, given both Westera and Brasoveanu's data and the contrasts observed in out-of-the-blue contexts.

3 See www.illc.uva.nl/inquisitivesemantics.

2 Data

2.1 Productivity: A generalization

Evidence that *at least* and *at most* are not fixed expressions comes from the productivity of this pattern. Penka (2010) discusses the case of *frühestens* and *spätestens* 'at the earliest/latest' in German. For English, corpus uses can be found of at least the following: *at (the) best*, *at (the) worst*, *at the earliest*, *at (the) latest*, *at the longest*, *at the highest*, *at the lowest*, and *at the fastest*.[4]

(3) [T]he officers have arguments which I feel are fatuous, fallacious, erroneous and **at best** equivocal.

(4) I hope to be back in action for the second race or, **at the worst**, the third.

(5) I must be away by eleven **at the latest**, though.

(6) [A] final decision is not expected until the end of the month **at the earliest**.

(7) The natural life of a hen is seven years, **at the oldest**, while a battery hen may last two and a half years.

(8) ... the court saying it was bound to limit the period to one year **at the longest**.

(9) It is worth noting that even if there is some discrepancy between government numbers and real unemployment, economists still put the number at 15.6 percent **at the highest**, according to PolitiFact.

(10) Capital would return 15 percent **at the lowest**.

(11) These groups move much more slowly than trains—only several miles per hour **at the fastest**.

However, that there seem to be some restrictions on what can be modified. The following strike me (and those I have consulted) as anomalous to the point of being nearly uninterpretable, so I label them with an asterisk.

(12) *You may invite Sarah **at the oldest**.
Intended: You may invite (someone as old as) Sarah or someone younger.

(13) *The room will fit this table **at the longest**.
Intended: The room will fit this table or something shorter.

The problem, intuitively, is that Sarah is not an age, and no table is a length. I therefore suggest the following generalization: *Superlative modifiers only modify expressions that occupy a position on the scale indicated by the gradable predicate*.

4 English also has *at (the) minimum*, and *at (the) maximum*, which will not be covered by the present analysis, as well as variants involving possessives such as *at his worst*, *at his finest*, which seem less restricted: *He was 200 pounds at his heaviest* is perfectly acceptable, in contrast to (1).

This is more straightforward in some cases than others. It is no problem to consider 'the end of the month' as a position on the earliness scale, or 'seven years' as a position on the oldness scale (for a hen). But is'equivocal' a level of goodness (rather than something that *has* a distinct level of goodness)? If we allow that it is, in this context, then we can rule out (12) and (13) on the grounds of a general constraint that the modified element must constitute a degree on the scale named by the gradable predicate to which the superlative attaches.[5]

2.2 Ignorance effects

Crucially, all of the superlative modifiers identified above give rise to the ignorance effects that have been shown for *at least* and *at most*. This was observed for temporal superlative modifiers by Penka (2010). Consider also the following contrast after checking one's watch:

(14) a. It's **after 5 o'clock** now, so we should stop.

b. #It's **5 o'clock at the earliest** now, so we should stop.

Or imagine informing your co-worker that the time is 4:45, and hearing one of the following responses:

(15) a. Well, it's **before 5**, so we should keep going.

b. #Well, it's **5 o'clock at the latest**, so we should keep going.

The examples with superlative modifiers are not appropriate in these non-ignorance contexts.

This extends beyond temporal superlative modifiers; consider the following statements after opening one's report card:

(16) a. ?Oh no, I got **worse than a B** in Calculus.

b. #Oh no, I got **at best a C** in Calculus.

Although (16a) is not completely natural, (16b) is quite distinctly more odd. Or suppose that you have just learned that your friend's mother is turning 67 years old tomorrow. Then only the first of the following variants would be appropriate as a response:

5 This restriction would still overgenerate somewhat, though with less severe consequences; the following examples involve cases where a measurement is modified by the *at*-expression and they still don't sound completely natural:

(i) ?I suspect that he weighs 200 pounds **at the heaviest**.

(ii) ?My algorithm is $O(n^2)$ **at the slowest**.

(17) a. Oh, she's **older than 65**, so she must be eligible for Social Security.
b. #Oh, she's **65 at the youngest**, so she must be eligible for Social Security.

Thus, superlative modifiers generally carry an ignorance implication, and this fact is not limited to *at most* and *at least*.

Another difference that has been observed between comparative and superlative modifiers is that the latter have been observed to give rise to strong upper-bounding inferences under ability modals. As Geurts & Nouwen (2007) observe, *at least* and *fewer than* give rise to different inferences under weak deontic modals:

(18) a. You may read **at most 3** books. $\rightsquigarrow$ *no more*
b. You may read **fewer than 4** books. $\not\rightsquigarrow$ *no more*

The same holds for *at the latest*, *at the worst*, and *at the oldest*:

(19) a. You may arrive at **5pm at the latest**. $\rightsquigarrow$ *no later*
b. You may arrive **earlier than 5pm**. $\not\rightsquigarrow$ *no later*

(20) a. You may earn **a B at the worst**. $\rightsquigarrow$ *no worse*
b. You may earn **better than a C**. $\not\rightsquigarrow$ *no worse*

(21) a. You may be **12 at the oldest**. $\rightsquigarrow$ *no older*
b. You may be **younger than 12**. $\not\rightsquigarrow$ *no older*

These judgments appear quite clear and consistent across all superlative modifiers; I conclude that the inferences should be attributed to *at*, not to *at least* and *at most* specifically.

3 Analyses of superlative modifiers

As mentioned above, superlative modifiers have been keeping the ink producers in business. For the purposes of situating the present proposal, let us divide up the existing proposals for how to analyze them along two axes. The first axis is what kind of scale superlative modifiers are thought to make reference to. Kennedy (2015), following Nouwen (2010), evokes a scale over degrees, as one can infer from the following lexical entries:

(22) a. *at least* $\rightsquigarrow \lambda m_d \lambda P_{\langle d,t\rangle}.\max(P) \geq m$
b. *at most* $\rightsquigarrow \lambda m_d \lambda P_{\langle d,t\rangle}.\max(P) \leq m$

Thus, assuming that *three* denotes a degree, *at least three* denotes a generalized quantifier over degrees (type $\langle dt,t\rangle$). This type of analysis is capable of accounting for intricate data involving modals, and relates *at most* and *at least* to the grammar

of degrees, but does not appear to cover uses of *at least* such as the following from Cohen & Krifka (2011) involving an *if not* continuation, revealing what lies on higher positions on the scale:

(23) This is at least misleading, if not wrong.

(24) The agent who bills such expenses is at least unethical, if not criminal.

(25) This is at least confusing, if not conflicting.

The modified element in these cases (e.g. *misleading*) does not appear to denote a degree.

What we might call 'discourse-based analyses' invoke scales of pragmatic strength, which are not restricted to numerals, and need not even respect entailment (Beaver & Clark 2008). Examples in this category include Krifka 1999, Geurts & Nouwen 2007, Büring 2008, and Coppock & Brochhagen 2013. Letting $\trianglerighteq$ stand for 'is pragmatically stronger than' (following Geurts & Nouwen (2007)), the following is an example lexical entry following that approach (τ is any type, and p is an abbreviation for $\langle s,t\rangle$):

(26) a. *at least* $\rightsquigarrow \lambda\alpha_{\langle\tau,p\rangle}\lambda\beta_{\tau}\lambda w.\exists q[q \trianglerighteq \alpha(\beta) \wedge q(w)]$
'some alternative as strong as the prejacent is true'
(prejacent = $\alpha(\beta)$)

b. *at most* $\rightsquigarrow \lambda\alpha_{\langle\tau,p\rangle}\lambda\beta_{\tau}\lambda w.\forall q[q \triangleright \alpha(\beta) \rightarrow \neg q(w)]$
'no alternative stronger than the prejacent is true'

This lexical entry, which effectively implements Beaver & Clark's (2008) MIN operator (argued to be presupposed by *only*), is more flexible than the degree-based one, straightforwardly covering cases like *at least misleading*. The observations in the foregoing section, however, suggest that with a sufficiently liberal conception of the degrees with which one measures along various dimensions, one which allowed 'misleading' to constitute a degree on some contextually relevant scale below 'wrong', we would be able to circumscribe the felicitous uses of this *at*-construction and rule out nearly uninterpretable cases like (12) and (13). So it is not clear that the discourse-based analysis has a clear advantage over the degree-based analysis purely on the basis of this kind of evidence.

Another dimension along which analyses of superlative modifiers vary is how the ignorance implicature is derived. The field of approaches is too broad to cover in its entirety here, so I will just mention three. Büring's (2008) idea was to see superlative modifiers as being akin to disjunctions; in the same way that 'ϕ or ψ' conveys that both ϕ and ψ are epistemically accessible options, 'at least n' conveys that both n and more than n are epistemically accessible. Similar intuitions have been implemented using (neo-)Gricean reasoning about alternative expressions to the superlative modifier that the speaker could have used. Representatives of this strategy

include Mayr (2013), Rett (2014), Kennedy (2015), and Schwarz (2016). Schwarz assumes that *at least* and *only* form a Horn scale, and that the set of numerals does as well, so among the expression alternatives for *at least three* are *at least four*, *at least five*, etc., as well as *only three*, *only four*, etc. The resulting structure of alternatives yields ignorance implicatures through Fox's (2007) method of innocent exclusion.

According to Nouwen (2010), the ignorance implicature with *at most* relies on a "reinterpretation" process introducing a speaker epistemic possibility modal. Nouwen's semantics generates a reading for (27) on which it is equivalent to (28) on an "exactly" reading.

(27) Jasper invited at most 10 people.

(28) Jasper invited 10 people.

Nouwen claims that this reading is blocked because it can be expressed through a simpler form. The hearer therefore reinterprets (27) as a statement about what the speaker holds possible, and an epistemic possibility modal for the speaker is introduced into the interpretation through a non-compositional rescue operation.

A third strategy for generating the ignorance implicature sees superlative modifiers as alternative-introducing expressions akin to *who* as analyzed by Hamblin (1973), *irgendein* as analyzed by Kratzer & Shimoyama (2002), or *any* as analyzed by Aloni (2002). Coppock & Brochhagen (2013) couch this latter strategy in the framework of inquisitive semantics, where declaratives and interrogatives alike denote sets of ordinary propositions, in the style of Hamblin's treatment of questions. Schematically, this analysis for *John read at least three books* might be represented as follows:

$$\text{John read} \left\{ \begin{array}{c} \ldots \\ 5 \\ 4 \\ 3 \end{array} \right\} \text{books.}$$

These alternatives 'expand outwards' as Kratzer & Shimoyama (2002) put it, so that the denotation of *John read at least three books* is a set of propositions, one for each number: {'John read three books', 'John read four books', ...}. For Coppock & Brochhagen (2013), the ignorance implicature is generated through a generalization of Groenendijk & Roelofsen's (2009) maxim of 'Inquisitive Sincerity', which they call 'Interactive Sincerity'. In slogan form, it says: 'Don't raise an issue that you know how to resolve'—and asserting a sentence whose denotation is a set of propositional alternatives amounts to 'raising an issue'.[6]

6 It has been observed for *wh-ever* constructions that ignorance is just one in a family of inferences, including indifference, agreement-to-disagree, and others (Condoravdi 2014). The same seems to apply to *at least* and *at most*, so this maxim should be generalized appropriately.

The present paper follows a strategy involving Hamblin alternatives, and builds up a semantics of superlative modifiers from a semantics for superlatives. The basic idea is that the comparison class for the superlative corresponds to the set of Hamblin alternatives. (It is not clear to me that such a natural connection between superlatives and superlative modifiers would be available under the approach involving expression-alternatives to *at least*, but this is for future research to determine.) The proposal will deviate from Coppock & Brochhagen's (2013) in restricting superlative modifiers to degree-type arguments, but in principle the analysis should nevertheless be flexible enough to maintain the advantages of the discourse-based approach.

4 Previous decompositions

The only existing works that provide a compositional derivation of the semantics of superlative modifiers from the semantics of superlatives that I am aware of are Penka 2010 and Solt 2011. Both of these adopt an approach to ignorance implicatures in the style of Nouwen (2010), where an epistemic uncertanty operator is inserted through a non-compositional emergency repair operation. As the solutions are generally quite similar, and Solt 2011 is a paper while Penka 2010 is only a handout, I will focus on Solt's (2011) solution.

Solt's (2011) analysis is built on the following insight, which I fully endorse although I will offer a different analysis: "What distinguishes the acceptable uses [of superlative modifiers] is that there is a range of actual or possible values under consideration, and not just a single value. This constraint mirrors a restriction on the superlative to situations where the comparison class has multiple members." Thus, "the non-singleton requirement is captured as a presupposition on -est that the comparison class it introduces have multiple members."

Formally, under Solt's (2011) analysis, *at most* is analyzed as *most*, where *most* is decomposed into *-est* and *much*, following Hackl (2001). She assumes Heim's (1999) lexical entry for *-est*, according to which it denotes a function that accepts three arguments: a comparison class C, a gradable predicate G, and an individual x. The function is defined if x is in C and C has multiple members, and returns true if x has a greater degree of G-ness than any distinct member of C. She assumes furthermore that, in a case like *Fred has read at most 15 Shakespeare plays*, the members of the comparison class C are *sets of degrees*, rather than mere degrees or individuals. So for the Shakespeare example, the comparison class contains all sets of degrees I where I is a set of degrees d such that Fred has read at least d plays. Since this is a singleton set, the presupposition that C have multiple members is not satisfied. Emergency insertion of an epistemic possibility modal will save the day, though: If C consists of all sets of degrees I containing *possible* numbers of Shakespeare plays that Fred has read, then C may contain multiple members. As she

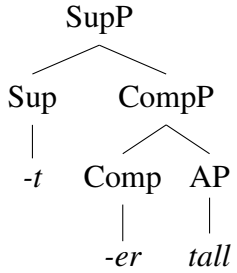
puts it, "Informally speaking, the comparison class C might be taken to be the set of numbers n such that Fred *might have* read n Shakespeare plays." (As she shows, this strategy will not work if *C* is allowed to contain mere degrees rather than sets of degrees, because then the comparison class will contain multiple members from the start, and there will be no need for emergency insertion of the epistemic modal.)

There are several challenges for this account, all inherited from Nouwen 2010. First, the conditions under which an epistemic operator is inserted are not spelled out, nor is the process by which it gets inserted. Second, as a matter of fact, speaker uncertainty is not part of the truth conditions for superlative modifiers; indeed it is not always implied, as shown by for example by cases like *Computers of this kind have at most 2GB of memory* (Nouwen 2010), where the superlative modifier is associated with a range of values rather than ignorance. The alternatives may also be distributed across conversational participants in an "agree to disagree" situation. If you are certain that the house has 9 foot ceilings and I am certain that it has 10 foot ceilings, we can agree that the ceilings are "at least 9 feet high". Ignorance is only one member in a "family of implications" (Condoravdi 2014) that can include variation, agreeing to disagree, and others. What is needed, therefore, is a framework that produces ignorance implications in certain contexts but not others, ideally in a compositional manner.

5 Proposal

The strategy pursued here is to derive the uncertainty/variation from the introduction of alternatives, as under Coppock & Brochhagen's (2013) analysis (henceforth C&B) in Inquisitive Semantics; however, we build on Penka/Solt's insight that the range is related to the comparison class of the superlative. The analysis to be derived is one which gives as a meaning for an *at least* sentence the set of possibilities that are as strong as or stronger than the prejacent according to the contextually salient pragmatic strength ranking $\trianglerighteq$. Since the sentence denotes a non-trivial set of possibilities, an ignorance implicature is derived (see Coppock & Brochhagen 2013 for details).

The proposed decompositional analysis of *at least* builds on Bobaljik's (2012) Containment Hypothesis, according to which superlatives contain comparatives:



To foreshadow how the connection between degree semantics and pragmatic strength rankings will be made: The *taller* component in *tallest* is analogous to the *less* component in *least*, which I suggest encodes the pragmatic strength ranking in the case of *at least*.

In order to allow *at least* to project alternatives, I assume a semantic framework in which natural language expressions are translated to expressions of a formal logic that denote a set of intensions.[7] Intensionality is handled using explicit quantification over worlds, and the world argument comes first. So the translation for *tall* would be as follows:

(29) $tall \rightsquigarrow \{\lambda w \lambda d \lambda x . \mathbf{tall}_w(d)(x)\}$

On the right-hand side of the squiggly arrow (indicating the "translates to" relation) is an expression denoting a singleton set containing a function from worlds to relations between degrees and individuals, following the standard line on gradable predicates (e.g. Heim 1999). Nothing new regarding the analysis of gradable adjectives here; this is just to illustrate the framework.

To get a meaning for something like *tallest*, we need a meaning for *taller*, which in turn is derived from *tall* plus a semantics for the comparative marker *-er* that yields phrasal comparatives. This phrasal semantics for *-er* can be stated in our framework as follows:

(30) $\textit{-er} \rightsquigarrow \{\lambda w . \lambda G_{\langle d,\tau t\rangle} \lambda s_\tau \lambda t_\tau . \max(\lambda d . G_w(d)(t)) > \max(\lambda d . G_w(d)(s))\}$

Thus *-er* denotes a singleton set containing a function that expects, along with a world argument, a gradable predicate G, a standard s, and a target t, and returns true if the target is G to a greater extent than the standard. I assume that this is derived via type-shifting from a more basic version of *-er* that forms the basis of an *-er* that can in clausal comparatives, but I will not spell out how that works here.

I also assume a matching version of *less* that can be used in phrasal comparatives:

(31) $less \rightsquigarrow \{\lambda w . \lambda G_{\langle d,\tau t\rangle} \lambda s_\tau \lambda t_\tau . \max(\lambda d . G_w(d)(t)) < \max(\lambda d . G_w(d)(s))\}$

Ultimately I would hope to derive this from a decomposition of *less* into *little* and *-er* (Rullmann 1995; Heim 2006; Büring 2007; Heim 2008), and maybe even divide *little* into *not* and *much* (Wellwood 2014), but I am not aware of any existing decompositional analysis of phrasal comparatives of inferiority, and it would take us too far afield to develop one here.

7 In terms of Ciardelli & Roelofsen's (2015) typology of frameworks, this is an 'alternative semantics', where all expressions denote sets, as opposed to what they call a 'possibility semantics', where sentences denote sets of propositions but composition involves standard Functional Application and Predicate Abstraction. I do this because I find it more intuitive in the present setting and the only rule needed for the examples considered here is Functional Application. Nothing hinges on this choice though; the theory could just as well have been stated in a possibility semantics.

For the superlative, I assume the following lexical entry (setting aside the presuppositions). The types of the variables are indicated using subscripts; τ is any type, and the parentheses are used to indicate set-types.

(32) $\textit{-t} \rightsquigarrow \{\lambda w.\lambda R_{\langle\tau,\tau p\rangle}\lambda C_{(\tau)}\lambda x_{\tau}.\forall x'_{\tau} \in C.x \neq x' \rightarrow R_w(x,x')\}$

Thus the superlative effectively saturates the 'standard' argument of the comparative with a universal quantifier. It denotes a function which expects, besides a world w, a relation R between two elements of type τ, and a comparison class C, which is a set of things of type τ, hence type (τ). The relation R might be saturated by something like *taller*, and the result would mean 'taller than everything in C'. Note that this is quite similar to Szabolcsi's (2012) entry except that it is placed inside set-brackets, because we are working in alternative semantics.

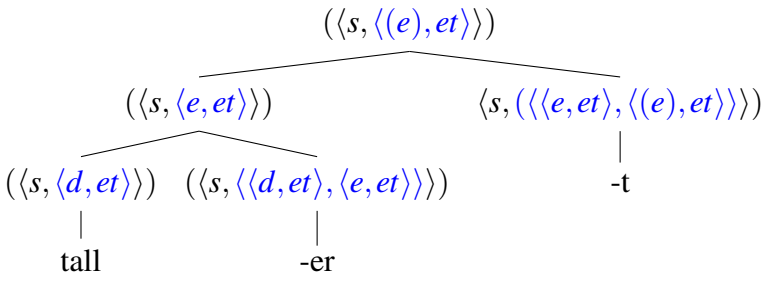
To put meanings together, I assume the following general composition rule, a pointwise, intension-friendly version of functional application that is compatible with the present style of formalization using translations:

(33) **Functional Application**
Let α and β be the only sub-trees of the tree γ. If:

- $\alpha \rightsquigarrow \alpha'$, where α' is of type $(\langle s,\langle\sigma,\tau\rangle\rangle)$
- $\beta \rightsquigarrow \beta'$, where β' is of type $(\langle s,\sigma\rangle)$

Then: $\gamma \rightsquigarrow \{\lambda w.f(w)a(w) \mid f \in \alpha' \wedge a \in \beta'\}$.

Thus the analysis tree will have the following structure. As the types are rather complicated, the fundamental types — corresponding to extensions at particular worlds – are highlighted in blue:



The meaning for *taller* that we derive, then, is as follows:

(34) $\textit{taller} \rightsquigarrow \{\lambda w\lambda s_{\tau}\lambda t_{\tau}.\max(\lambda d.\mathbf{tall}_w(d)(t)) > \max(\lambda d.\mathbf{tall}_w(d)(s))\}$

Feeding this into superlative *-t*, we have

(35) *tallest* $\rightsquigarrow$
$\{\lambda w.\lambda C_{(\tau)}\lambda x_{\tau}.\forall x'_{\tau} \in C.x \neq x' \rightarrow \max(\lambda d.\mathbf{tall}_w(d)(x)) > \max(\lambda d.\mathbf{tall}_w(d)(x'))\}$

Before coming to superlative modifiers, it will be necessary to introduce a supplementary non-standard treatment for gradable predicates (which will also work with the analysis of comparatives and superlatives given so far). Recall the generalization observed in §2.1 above: that superlative modifiers are only acceptable when modifying something that can be construed as a measure along the dimension indicated by the gradable adjective. This suggests an analysis of, for example, *at least equivocal*, according to which it denotes a set of degrees of goodness of which 'equivocal' is the best. Likewise, *5 o'clock at the earliest* denotes a set of times of which 5 o'clock is earliest, *5mph at the fastest* denotes a set of speeds of which 5mph is the fastest, and *15 percent at the highest* denotes a set of percentage rates of which 15 percent is the highest. But this raises an interesting philosophical question: If the things to be measured are themselves degrees on the relevant scale, then what does it mean for them to be measured by a gradable predicate? Does it make sense to see, for example, 5mph as faster than 4mph, or the fastest of a given set of speeds, given that 5mph is the *value* of the function that measures speed? Does it make sense to say that 6 feet is taller than 5 feet, or that 30 degrees Celcius is warmer than 25 degrees Celcius? People do appear to speak this way; consider the following attested examples:

(36) -30 degrees is warmer than -40 degrees. Neither would be considered warm temperatures.

(37) I'm pretty sure 90mph is faster than the legal maximum speed on any UK road.

(38) We know that one o'clock is later than 9'oclock on the same day if we know that the first time is PM and the second is AM.

It is also possible to find attested examples of the following kind, with positive *hot* being predicated of temperatures:

(39) 88 degrees is too hot for most New Jerseyans, survey finds.

But how hot is 88 degrees? How fast is 60mph? I suggest that the degree to which 88 degrees is hot is 88 degrees, and the degree to which 60mph is fast is 60mph. But this relies on a slightly different interpretation of *hot* and *fast*, one which merely presupposes that the subject of predication is a degree on the relevant scale, and encodes an identity relation. So if the ordinary extension of a gradable adjective is $G_{d,et}$, then there is a corresponding extension of the following form:

(40) $\lambda d \lambda d' . d = d' \wedge \partial(d \in \mathbf{dom}(G))$

where $d \in \mathbf{dom}(G)$ encodes the constraint that d is in the domain of the function G. Thus, alongside (29), we have also:

(41) $tall^{\circ} \rightsquigarrow \{\lambda w \lambda d \lambda d' . d = d' \wedge \partial(d \in \mathbf{dom}(\mathbf{tall}_w))\}$

If a shift from an ordinary gradable predicate denotation to this kind of denotation were implemented as an operation that imposes a certain cost, then the relative rarity of the construction and the limited productivity of the superlative modifier construction could potentially be accounted for.

We then have to allow comparatives and superlative to apply to these odd gradable predicates, to form, along with superlative modifiers, kinds of constructions we have just seen, with degree-denoting expressions in subject position. A comparative like *taller* would be formed as a combination of (41) with the regular comparative marker (30), which boils down to the following:

(42) $taller^{\circ} \rightsquigarrow \{\lambda w.\lambda d_d \lambda d'_d.d' > d \wedge \partial(d' \in \mathbf{dom}(\mathbf{tall})) \wedge \partial(d \in \mathbf{dom}(\mathbf{tall}))\}$

And the superlative is:

(43) $tallest^{\circ} \rightsquigarrow$
$\{\lambda w.\lambda C_{(d)} \lambda x_d.\forall x'_d \in C.x \neq x' \rightarrow x > x'$
$\wedge \partial(x \in \mathbf{dom}(\mathbf{tall})) \wedge \partial(x' \in \mathbf{dom}(\mathbf{tall}))\}$

Now for the famous *at*-operator, promised at the beginning. We assume that *at* takes as an argument some function (like *best*) which is expecting both a comparison class argument and an argument that will serve as the subject of the superlative predication, presupposed to be part of the comparison class. The alternatives that are projected are members of that comparison class. In other words, the idea is that *at best x* denotes a set of alternatives *y* whose best member is *x*.

This can be implemented with the following lexical entry.

(44) $at \rightsquigarrow \{\lambda w.\lambda S_{\langle(\tau),\tau t\rangle} \lambda x_\tau.y_\tau \mid y \in \mathbf{C} \wedge S_w(\mathbf{C})(x)\}$
'the set of things *y* in a comparison class **C** s.t. *x* is *S* [least/most/etc.] in **C**'

Note that this is an abuse of notation. Any expression of the form

$$\{\lambda v_1 ... \lambda v_n.\alpha \mid \psi(v_1, ...v_n, \alpha)\}$$

should be interpreted as an abbreviation for:

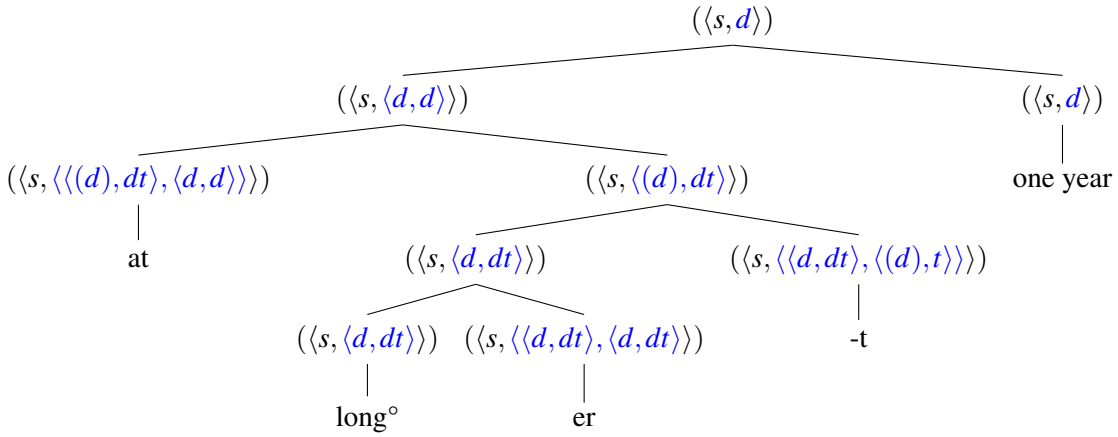
$$\{f \mid \forall v_1 ... \forall v_n \forall \alpha : f(v_1, v_n) = \alpha \rightarrow \psi(v_1, ...v_n, \alpha)\}$$

So the proper notation for (44) is:

(45) $at \rightsquigarrow \{f \mid \forall w \forall S_{\langle(\tau),\tau t\rangle} \forall x_\tau \forall y_\tau [f(w,S,x) = y \rightarrow S_w(\mathbf{C})(x) \wedge y \in \mathbf{C}]\}$

but I find this less intuitive so I will stick to the abuse.

Let us see this in action, using *one year at the longest*. This phrase will have the following structure:



The logical representation we obtain for this is as in (46), based on a following meaning for *longest*° parallel to the one given above in (43) for *tallest*.

(46) $\{\lambda w\,.\,y_\tau \mid y \in \mathbf{C} \wedge \forall x'_d \in C\,.\,\mathbf{years}(1) \neq x' \rightarrow \mathbf{years}(1) > x'$
$\wedge \partial((\mathbf{dom}(\mathbf{long}_w))(\mathbf{years}(1))) \wedge \partial((\mathbf{dom}(\mathbf{long}_w))(x'))\}$

Note that I am representing the meaning of '1 year' using the expression $\mathbf{years}(1)$, which is supposed to denote a degree on the (relevant) 'long' scale. I am of course glossing over any polysemy and/or multidimensionality in the word *long*. Note finally that, given the density of time, the set of alternatives generated must be dense, and therefore infinite. Nothing in the theory of alternatives prevents this, as far as I know, but it is not the kind of phenomenon we are used to. All this aside, we have a good result: the meaning of *one year at the longest* turns out to be a set of degrees of length upper-bounded by one year, as desired. Embedded in a sentence, this phrase will function exactly as *one year* does, in terms of types. But the alternatives that are projected can yield ignorance implications, upper-bounding inferences under certain modals, and related properties.

Now, in the case of *at least*, it is not immediately obvious what (if anything) plays the role of *long* in 'at the longest'. We will assume that *least* is composed of *less* and *-t*. But if *less* is like *-er*, then it needs to take a gradable predicate as an argument: *less what?* I assume that it takes as an argument a contextually-specified gradable predicate **m** which measures the magnitude of things in a way that correlates with the pragmatic strength ranking $\trianglerighteq$ from Beaver & Clark (2008). So in the case of *at least*, the comparative that is fed to the superlative ending is what I call $less_m$, analyzed as follows:

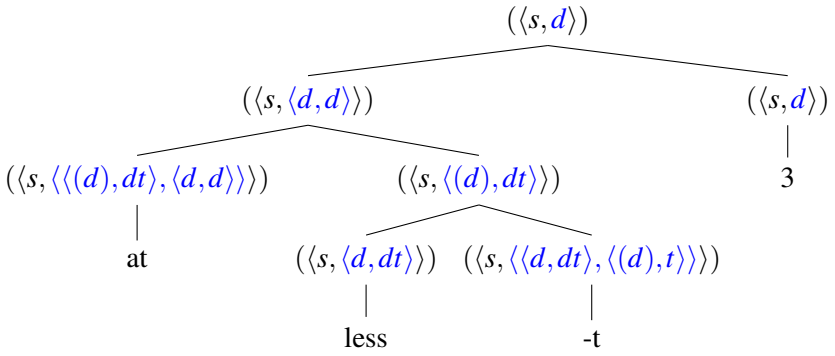
(47) $less_m \rightsquigarrow \{\lambda w\,.\,\lambda x_\tau \lambda x'_\tau\,.\,\max(\mathbf{m}_w(x')) < \max(\mathbf{m}_w(x))\}$

The result of combining $less_m$ and *-t* is, after simplifying:

(48) $\textit{least} \rightsquigarrow \{\lambda w.\lambda C_{(\tau)}\lambda x_{\tau}\forall x' \in C.x' \neq x \rightarrow \max(\mathbf{m}_w(x)) < \max(\mathbf{m}_w(x'))\}$

For *at least an assistant professor*, this gives the set of ranks at least as great as 'assistant professor' – great in some contextually-determined sense. If we assume that **m** functions in such a way that the alternatives produced by the clause are ranked by $\trianglerighteq$, then we can derive the discourse-based analysis.

In the case of numerals, we obtain the rather intuitive result that a phrase like *at least 3* denotes the set of numbers that are as high as or higher than 3. Assume a version of *less* that expresses a relation between two degrees, and assume that numbers denote degrees (fundamentally $\langle d,dt\rangle$). Then we have an analysis with the following types:



The meaning we derive for this is $\{\lambda w.d \mid \exists C[d \in C \wedge \forall d' \in C[d' \neq 3 \rightarrow \max(\mathbf{m}_w(3)) < \max(\mathbf{m}_w(d))]]\}$, i.e., something like the set $\{3,4,5,...\}$, ignoring intensionality, and limiting our attention to integers. Embedded in a larger sentence, each one of these numbers will correspond to a possibility in the Hamblin set for the whole proposition.

6 Summary

This paper has given an alternative-based analysis of superlative modifiers, starting from more basic initial assumptions than Coppock & Brochhagen (2013). In this system, a phrasal semantics for the comparative is the input to the superlative morpheme *-t*, and the *at* in superlative modifiers introduces alternatives in the comparison class of the superlative. The inroduction of alternatives accounts for ignorance/variation effects without recourse to emergency insertion of an epistemic operator, and the proposed lexical entry for *at* can be productively recombined to produce other superlative modifiers with the same discourse properties.

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