Sophisms and insolubles

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Just prior to the spread of universities across Europe in the fourteenth century, a systematic method for training the minds of young future leaders to think rationally began to crystallize through the practice of logical disputations. The oldest centres of logic in Europe were Oxford and Paris, both originating in the eleventh century, and rich traditions were built up there over the subsequent years. In fourteenth-century Oxford, before earning a Bachelor of Arts, a student was required to earn the title of *sophista generalis* (Leader, 1989, p. 96). As such, he was allowed to participate in structured disputations involving a respondent and an opponent, and would have learned the art of considering a sentence called a *sophism* (Latin *sophisma*) against a hypothetical scenario or given set of assumptions, called a *casus* in Latin.[1] Typically, it was not trivial to decide whether the sentence was true or false, and arguments could be made on both sides. Sophisms thus presented a puzzle to be solved. In medieval texts, the discussion of a sophism follows a more or less strict outline that includes arguments both for and against the truth of the sentence under the assumption that the casus is true, and a resolution of the puzzle.

[1] To avoid ambiguity and potentially misleading implications, we will use the technical term *casus* rather than 'scenario' or 'set of premises' in this article.

The etymologically related word *sophistry* has a connotation of gratuitous obfuscation not shared by *sophism*. The arguments for and against the same sentence reflect puzzlement, but the aim was not to invite trickery. Rather, skills achieved were in the disambiguation of Latin expressions, in the exact formulations of their truth conditions, and in the recognition of inferential connections between them. In short, rather than sophistry, sophisms led to the treatment of Latin as a precise, logical language. This program bears some similarity to that of Richard Montague, the founder of modern formal semantics, who wrote, "I reject the contention that an important theoretical difference exists between formal and natural languages" (Montague, 1974).

Examples of sophisms, with accompanying discussion, can be found both in independent collections of sophisms and within treatises on logical topics, such quantificational words like 'every' or exclusives like 'only'. One much-discussed thirteenth century sophism is 'Every Phoenix exists' (Tabarroni, 1993), which occurs in many different sources, including Walter Burley's *Questions on Aristotle's 'On Interpretation'* (Brown, 1974, pp. 260-2) In discussions of this example, it is assumed, in accordance with the myth, that there is only one Phoenix at a time, although over time, there are many. It is then examined whether this state of affairs warrants the use of the sign of universal quantity (namely 'every') and the present tense. If 'every' must range over at least three particulars – as some authors suggest – and the present tense requires a limited time reference, the sentence is to be judged false. Authors willing to analyze the linguistic items differently, including Burley, gave different evaluations. As a result, the function of the word 'every' was spelled out with great theoretical clarity and exactness, though not with unanimity.

Among the different sophismata, one group stands out: the insoluble (*insolubilia*). 'Insoluble' was a technical term adopted as early as the twelfth century, applying to sophisms that are particularly difficult to resolve, including paradoxes of self-reference. For example, given the casus that Socrates only says 'Socrates says something false', it appears impossible to give the sentence 'Socrates says something false' any truth value. No solution to the sophism appears acceptable.

The range of topics dealt with in the sophism literature is very wide, but can be divided into four broad categories. We have already alluded to two of these: those concerning the interpretation of so-called 'syncategorematic' terms such as 'every' and 'only', and the semantic paradoxes ('insolubles'). Sophisms also dealt with mathematical physics through the study of terms like 'begins', 'ceases', and 'infinite'. These contributed substantially to the development of mathematics and physics, and had a considerable influence on work by early modern natural philosophers like Galileo Galilei (Duhem, 1913) (Glagett, 1959). Sophisms also dealt with questions related to knowledge and belief, including when exactly a person can be said to know something, the nature of what one believes or knows, reference *de dicto* and *de re*, and the relationship between knowledge, belief, and doubt. We will address each of these four categories of sophisms below, after a brief historical overview.

1. From the twelfth to the sixteenth century

The practice of constructing sophisms arose from certain twelfth century trends in learning. As analysis of language and logic gained a central role in education, authors began constructing artificial examples rather than considering existing quotations from classical sources. These gained increasing complexity, allowing for very delicate differentiation of meaning (Dronke, 1992, pp. 240-241). The practice of constructing artificial examples is also found in modern-day linguistics, where discussions often centre around constructed example sentences, sometimes in connection with artificial hypothetical scenarios, although it is not common nowadays to argue both for and against the truth of the example sentence with respect to the scenario, as in the presentation of a sophism.

The dominant original use of sophisms was educational, and so collections of sophisms started to circulate as teaching aids that were not tied to any particular theoretical approach or school. Possibly the earliest surviving collection is a manuscript written by several hands from the twelfth century containing, in addition to treatises on logic, a collection of some 80 sophisms (Kneepkens, 1993). The section has the Latin title *Sophismata* and the sophisms contained in it appear to be presented in no particular order. Most of these sophisms have however a clear connection to the issues discussed in the various topical treatises in so-called 'old logic' (see the chapter on the *logica vetus* in this volume) contained in the same manuscript. The collection of sophisms in the book presents examples which elucidate logical problems discussed in the main texts. Even more generally, from the end of the twelfth century until at least the end of the middle ages, sophisms were used in the presentation of theoretical viewpoints, applying the theses and the rules making up the core of the theory to solve sophisms. Burley's 'On Obligations' (written c. 1300), for example, gives rules concerning the respondent's duty to grant [Latin *concedo*] or deny [Latin *nego*] sentences consistently in a dynamic disputation (see chapter on obligations in this volume). The workings of these rules are illustrated through a series of sophisms containing words like 'grant' and 'deny'.

Towards the end of the thirteenth century, theoretical content within the discussions of sophisms increased, and the basic structure became more regimented. Thus, the presentation of a sophism typically consists of six parts:

(1) The sophism sentence itself;
(2) The *casus* (a hypothetical scenario or set of assumptions);
(3) The proof of (1) given (2);
(4) The disproof of (1) given (2);
(5) The reply, which determines whether (1) is true or false given (2), and explains why;
(6) Depending on the direction of the reply, the author's reply to the opposing proof, i.e. to (3) or to (4).

In its outline, a sophism thus follows roughly the same structure as a standard medieval *quaestio* found, for example, in Thomas Aquinas's *Summa Theologiae*. There is however one significant structural difference. As the point in a standard medieval *quaestio* was to discuss what is really true, there is no hypothetical scenario, or stipulation of the supposed facts of the matter – i.e. a *casus*. Quite particularly, the casus is the part that makes a sophism artificial rather than real. In most sophisms, the casus is an obviously hypothetical, constructed case.

The dominant type of sophism in the thirteenth century dealt with issues of logic and semantics. The fourteenth century saw new developments in the circle that is known by the name 'Oxford Calculators', so called because of their work in mathematical physics. Among the main characters of this group were Thomas Bradwardine, Richard Kilvington, and William Heytesbury, who also worked in epistemology and the semantic paradoxes, as discussed below. Richard Kilvington's *Sophismata* may be taken as a work of paradigmatic importance for this group, developing mathematical physics in genuinely new directions. At that time, sophisms seem to have been very important as a systematic part of the bachelor course.

After the work of the Oxford Calculators, the most influential author to write a collection of sophisms is John Buridan (Klima, 2001). His *Sophismata* forms the last section of his *Summulae de dialectica*, and it has to be counted as one of the most innovative collections of sophisms surviving from the middle ages, considering the richness of the logic developed there.

After Buridan, the currents of intellectual history brought so-called Renaissance humanism to the forefront, a movement that was not particularly friendly to the sophismatic tradition. Later fourteenth century sophisms did contain interesting new developments such as Albert of Saxony's sophisms related to *de se* reference discussed below. Both Kilvington's and Buridan's *Sophismata* were widely used as textbooks (Markowski, 1993). But sophisms, and the peculiar Latin structures found in the literature, were one of the most explicit targets of humanist criticism. They admired the eloquent rather than logical Latin of classical authors like Cicero and Seneca.

2. Grammatical sophisms

What modern medievalists sometimes call 'grammatical sophisms' served to elucidate what medieval logicians called 'syncategorematic' terms (as opposed to 'categorematic' terms), and dealt with some of the problems that are treated in modern formal semantics. Among the syncategorematic terms are words like 'not', 'and', 'or', and 'therefore', as opposed to 'man', 'stone', 'whiteness', and 'white', which are categorematic (Buridan, 2001, p. 232). A standard characterization of syncategorematic terms, given for example by the ancient grammarian Priscian, is as *consignificantia*, i.e. terms which signify something only in combination with other terms (Courtenay, 2008, p. 32). In modern formal semantics, 'syncategorematic' is used in a similar sense, characterizing not a word but a style of analysis in which a term is given a meaning in combination with other terms, rather than in isolation.

Some authors supplemented this definition with a semantic characterization of the syncategorematic terms. John Buridan, for example, characterized a 'categorematic' word as one that serves to pick out existing things in the world while syncategorematic words did not (Buridan, 2001, p. 232). This way of making the distinction bears some similarity to the characterization of 'logical constants' as those whose interpretation is constant across models (Westerståhl, 1985), where models determine the extensions of predicates and relations, although the relationship between syncategoremata and logical constants is subject to debate (Dutilh Novaes & Spruyt, forthcoming). There is a sense, then, in which grammatical sophisms dealt with the 'logical words' of Latin.

Not all sophisms that might be called 'grammatical' dealt with particular words. For example, as Terence Parsons discusses, medieval logicians produced innovative analyses of relative clauses in sophisms such as 'Everything that will be is' (Parsons, 2014). Possession was another theme treated in this literature that can be brought under the heading of 'grammatical sophisms', although, like relative clauses, the issue is not connected to a particular word. Here the question concerns the interpretation of the genitive case. One popular example in this category is the sophism: 'That dog is yours; That dog is a father; So that dog is your father' from Aristotle's Sophistical Refutations, ch. 24, drawn from Plato's Euthydemus. But particular words constitute the bulk of this category, especially in the thirteenth century.

One particularly widespread collection of grammatical sophisms was written in the mid-thirteenth century by an otherwise unknown Richard known as the '*magister abstractionum*'. Among the over 300 sophisms in this collection, two large groups of over 60 sophisms concern respectively two specific kinds of syncategorematic words. One of these consists of words expressing exclusion or exception (such as *tantum* 'only', *solus* 'alone', and *praeter* 'except'), and the other consists of words like *omnis* 'every', which signal universal quantification of a sentence. Thus, exclusion, exception, and universal quantification seem by this quantitative measure to be the most important topics discussed in sophisms. There are further groups of about 30 sophisms each, related to conditionals (words like *si* 'if' and *nisi* 'unless'), to negations (*ne* 'not', *nullus* 'nobody', and *nihil* 'no'), to alethic logical modalities (*necessarium* 'necessary', *impossibile* 'impossible', and *possibile* 'possible') and to beginning and ceasing (*incipit* 'begins', and *desinit* 'ceases'). Yet further syncategorematic words are considered, but not as extensively. For example, the collection has eleven sophisms on 'in as much' (*inquantum*) and just three on 'or' (*vel*). The collection should not however be taken to reflect the whole scene. Certain important syncategorematic words that are found in other collections of sophisms seem to be missing from the *magister abstractionum* collection. Thus the collection has no sophisms concerning the word 'infinite', for example. The selection and variety of topics in the *magister abstractionum* collection also shows how linguistic, logical and even physical analyses were not separated in the sophismata literature (Streveler, 1993).

At the turn of the fourteenth century, Walter Burley wrote distinct treatises on exclusives (*De exclusivis*) and on exceptives (*De exceptivis*), and both topics are also discussed as chapters of his *On the Purity of the Art of Logic* (Burley, 2000). Burley proceeds through rules, distinctions, doubts – and sophisms. The discussion on exclusives opens with an important technique often used in various kinds of sophisms, and thus worthy of attention here. Burley says that an exclusive proposition like

'Only Socrates runs'

can be expounded or 'unpacked' (*exponitur*) as a conjunction

'Socrates runs and nothing other than Socrates runs'.

Such a technique was called 'exposition', and it could be applied to a wide variety of sentences. Among them, those containing the word 'begin' or 'cease' were prominent cases. Thus,

'Socrates begins to be white'

was to be analyzed by the exposition

‘Socrates is not white and immediately after now Socrates is white’.

Burley’s main focus in the treatise is on how the exclusive particle ‘only’ functions in relation to the structures of the standard Aristotelian predication that yields the syllogistic system. ‘Only’ can be attached to either the subject or to the predicate, and in each place the exposition will be somewhat different. Also, Burley considers how rules of conversion turning the predicate into the subject and vice versa work with exclusive propositions. From this, it is natural to investigate how exclusives work in structures resembling the syllogistic figures. In effect, Burley is building a logic of exclusives as an extension of the Aristotelian syllogistic system.

Comparison to contemporary theory of linguistic exclusion shows that the approach of distinguishing between the positive and the negative component in a sentence containing an exclusive expression is an accepted practice even nowadays, sometimes even explicitly connected to the medieval practice of ‘exposition’, although details of the analyses differ. There is a rich ongoing debate about how to analyze exclusives, but modern semanticists almost all agree that there is an asymmetry between the positive component and the negative component, the former commonly seen as being presupposed (Coppock & Beaver, 2013). Burley and other medieval logicians, including Peter of Spain and William of Sherwood were, in contrast, what Lawrence Horn calls “symmetricalists”, treating the two exponents as having equal status (Horn, 2011).

Burley's symmetricalism plays an important role in his treatment of the sophism 'If nothing runs then something runs', which goes, in part, as follows (Burley, 2000, pp. 223-224):

> -- It is proved as follows: 'If nothing runs, not only Socrates runs; and if not only Socrates runs, something other than Socrates runs; and it follows: something other than Socrates runs; therefore, something runs; therefore, from first to last: if nothing runs, something runs'.
> -- It is disproved as follows: The antecedent is possible; but a possible proposition never implies its contradictory; therefore, 'If nothing runs something runs' is false.
> -- Solution: The sophism-proposition is false, and there is a fallacy of the consequent in its proof, when it argues like this: 'Not only Socrates runs; therefore, something other than Socrates runs'. For 'Not only Socrates runs' has two causes of truth, one of which is 'Another than Socrates runs' and the other 'Socrates does not run'.

As in a modern proof, the proof explicitly mentions a rule of inference, namely "from first to last", which chains three conditionals (p□q & q□r & r□s □ p□s). The rule is given by Burley in the section on conditionals as follows (Burley, 2000, p. 155):

> When many inferences occur between the first antecedent and the last consequent, if in each inference the same thing that is the consequent in the preceding conditional is the antecedent in the following conditional, then an inference 'from first to last' holds, so that the last consequent follows from the first antecedent.

To show that the proof is fallacious, Burley rejects this premise: 'If not only Socrates runs, something other than Socrates runs' but accepts this one: 'If nothing runs, not only Socrates runs'. The rejection of the first crucially depends on the assumption that

'Not only Socrates runs' can be true either because Socrates does not run or because someone other than Socrates runs (a direct consequence of the symmetricalist thesis, as Burley points out).

A modern asymmetricalist would do the opposite, accepting the first but not the second premise. According to one common analysis (originally due to Lawrence Horn), the positive component of an exclusive sentence is presupposed and the negative component is part of the ordinary semantic content (Beaver & Clark, 2008). In the standard Fregean theory of presupposition, this presupposition survives when the sentence is negated, and the ordinary content – the negative component, that nobody other than Socrates runs – is targeted by the negation. Hence 'Not only Socrates runs' implies 'Something other than Socrates runs' (and the first premise is valid). But 'Nothing runs' does not imply 'Not only Socrates runs' (the second premise), because 'Not only Socrates runs' presupposes that Socrates runs, and this cannot be true if nothing runs. The modern asymmetricalist thus agrees with Burley that the proof is not valid, but disagrees about why.

This is one of several phenomena dealt with in the sophism literature that are standardly treated using the concept of presupposition in modern semantics. Other cases involve quantifier domain restrictions. As mentioned above, 'Every phoenix exists' becomes puzzling under the assumption that 'every' must range over at least three objects. This kind of restriction is treated as a presupposition under standard modern accounts (Heim & Kratzer, 1998). A related problem shows up in discussions concerning the sentence 'Every lunar eclipse takes place by the interposition of the earth between the sun and the moon'. Under the not-so-unusual circumstance that there is currently no eclipse of the moon, there is nothing in the domain of 'every'. John Buridan writes that the sentence is false in that case, strictly speaking, though we might get the feeling that it is true because this sentence is actually a loose way of saying 'Whenever there is a lunar eclipse, it takes place by the interposition of the moon between the sun and the earth' (Buridan, 2001, pp. 725-726). If the requirement that the domain is non-empty is a presupposition, as a typical modern semanticist would say, then the sentence is not straightforwardly false; it is common nowadays to claim that its truth value is 'undefined' in case that condition is not fulfilled. (This does not help to explain why the sentence might be felt to be true.)

3. Sophisms on mathematical physics

In the first quarter of the fourteenth century, as we see in Richard Kilvington's *Sophismata* (Kilvington, 1990), a new kind of subject matter begins to be considered. Sophisms in natural philosophy were a flourishing oral practice at Oxford at the time (Sylla, 2010), and Kilvington's work builds upon this practice.

Of Kilvington's 48 sophisms, 44 deal with problems related to movement and change, both quantitative and qualitative. The central words occurring in these sophisms are 'begins' and 'ceases'. Kilvington takes the discussion to new heights as he embeds 'begins' and 'ceases' in sentences having rich structures of different tenses, in many cases combined with comparatives and superlatives expressing greater or lesser speeds, or greater or lesser intensities of whiteness (whiteness being just a placeholder for a continuously quantifiable quality). This results in elaborate discussions concerning the mathematical properties of continuous quantities, including speed.

For example, in sophism 23 we are to suppose the following casus. The body A (the reader might think of a slow paintbrush) is steadily moving across the body B (a plank), making B white until it reaches the endpoint C after an hour. At the half hour, a blackener D will start to move over B twice as fast as A, changing the whiteness generated by A into blackness. In this casus, only the part of the plank between the two moving paintbrushes will be white. Given the speeds of the two paintbrushes, they will reach the end of the plank at the same time, and thus at the end of the plank there is no space between them. The sophism to be considered is:

> A will generate whiteness up to point C, and no whiteness will be immediate to point C.

After work by Newton, Leibniz, and others in the seventeenth century, infinitesimal calculus has become part of mathematics. Thus, we can say that when the whitener A moves infinitesimally close to C, the blackener D also moves infinitesimally close to C, and the distance between A and D becomes infinitesimally small. D is nevertheless always twice as far from C as A is. Because D moves twice as fast as A, D and A will arrive at C at the same instant. Thus, the area that is whitened before it is blackened will first grow and then diminish, becoming infinitesimally small as the distance from the endpoint becomes infinitesimal. At any finite distance from C, some whiteness will be generated before it is blackened, but will there be any whiteness generated immediate to C? Does the expression 'immediate to C' refer to anything? Kilvington's solution, on which we cannot go into detail here, is based on how 'up to' and 'will generate' are to be treated in different word orders, and especially in the word order in which the sophism is actually put. He is thus providing rules of linguistic usage to aid in mathematical precision.

The crucial theme of this sophism and many others in Kilvington's collection is continuous motion, either uniform or uniformly accelerating or decelerating. Infinitesimal quantities and in some cases infinitesimal proportions also play an important role. Kilvington's younger colleagues and followers in Oxford also wrote sophisms with such themes. One work of particular importance is William Heytesbury's *Rules for Solving Sophismata*, a thematically organized guide for the student in handling various kinds of sophisms. This work is known as the first to explicate the so-called 'mean speed theorem', according to which, in a uniformly accelerating motion, a body moves in a given time the same distance as it would move if it moved the same time with the mean speed of the accelerating motion (Sylla, 2010). A particularly notable point here is that this result required recognition that it is possible to attribute a speed to a body at an instant despite the obvious fact that no body moves anywhere in an instant.

4. Sophisms on knowledge and doubt

The final four of Richard Kilvington's *Sophismata* concern the verb 'to know' as an epistemic operator. Like mathematical physics, this theme too was relatively marginal in thirteenth-century sophisms, but gained importance in the fourteenth century. Among Oxford Calculators, William Heytesbury dedicated a full chapter of his *Rules on Solving Sophisms* to the problems of epistemic logic and the nature of knowledge. Furthermore, while the Parisian logician John Buridan seems not to have been interested in mathematical physics, a number of his sophisms deal with problems of knowledge ascriptions. Sophisms concerning knowledge and doubt are sometimes subsumed under a larger class dealing with so-called 'officiable' or 'functionalisable' terms (*officiabilis* in Latin, roughly translatable into modern formal semantics terminology as 'operators'), which include deontic and alethic modalities and belief-related propositional attitudes (Bos, 2007).

One of the issues arising in connection with knowledge and doubt is known to modern linguists and philosophers as the distinction between *de dicto* and *de re* interpretations (Quine, 1956). The sophism literature makes a similar distinction between 'composite' and 'divided' senses. This distinction plays a role in the solution to the following two of Kilvington's sophisms, which involve a demonstrative pronoun ('this'):

S45. You know this to be everything that is this.

Casus: You see Socrates from a distance and do not know that it is Socrates.

S46. You know this to be Socrates.

Casus: You see Socrates and Plato at the same time, and Socrates and Plato are altogether alike, and you are a little confused, so you don't know which is Socrates and which is Plato. By 'this' is indicated the one who is in the location where Socrates was before you became confused.

For S46, Kilvington puts forth the proposal that in the 'divided' sense ('about this thing, you know it to be Socrates', a *de re* interpretation) the sophism is false, but in the ‘compounded’ sense, the sophism should be doubted, because you know the sentence ‘Socrates is Socrates’, and you doubt whether ‘This is Socrates’ is the same sentence in the language of thought. It is not clear whether this constitutes a satisfactory resolution of the issue, but the discussion at least brings out important differences between demonstratives and proper names in epistemic contexts.

The distinction between composite and divided senses was an important tool in solving epistemic sophisms for Kilvington and Heytesbury, and John Buridan also addresses this theme quite extensively some years later in Paris. Consider Buridan’s discussion of the sophism ‘You know the one approaching’ (Buridan, 2001, p. 892):

> I posit the case that you see your father approaching from afar, so that you cannot tell whether he is your father or someone else.
>
> P.1 Then [the sophism] is proved as follows: you know your father well; and your father is the one approaching; therefore, you know the one approaching.
>
> P.2 Again, you know the one who is known by you; but the one approaching is known by you; therefore, you know the one approaching.
>
> P.2.1 I prove the minor: for your father is known by you, and your father is the one approaching; therefore, etc.
>
> O.1 The opposite is argued: you do not know the person concerned when [he is such that], if asked who he is, you would truly say: ‘I do not know’; but about the one approaching you will say this; therefore, etc.

The sophism sentence is argued to be true on what modern semanticists would call a *de re* reading, but false on a *de dicto* reading.[2]

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[2] The terms *de dicto* and *de re* are medieval in origin but not commonly used then

Note that issues related to opaque contexts were not limited to discussions of knowledge. For example, in Buridan's sophism,

'I owe you a horse'

part of the problem is that there is no particular horse that is owed. This case will remind the modern formal semanticist of Richard Montague's 'John seeks a unicorn' (Montague, 1974), which, of course, does not entail the existence of a unicorn.

Alongside *de dicto* versus *de re*, we also find discussions of reference *de se*. John Buridan's student Albert of Saxony includes in his *Sophismata* (c. 1359) a number of sophisms where anaphoric pronouns occur in the scope of the knowledge operator (Biard, 1989). Included are two sophisms with the pronoun 'himself' (Latin *se*) used to express the kind of knowledge David Lewis called knowledge *de se* (Lewis, 1979). Consider the sophism II, 34 (Saxony, 1490),

'Socrates can know what God cannot know'.

The disproof is straightforward: God can know and indeed knows everything that is true, and Socrates cannot know anything that is not true; therefore, anything Socrates knows, God knows too. The proof of the sophism is more interesting in this case. According to the proof, Socrates can know that someone is better than he (himself), but God cannot know that someone is better than he (himself). Thus, Socrates can

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(Dutilh Novaes, 2004).

know something that God cannot know. As Albert explains more fully in the solution of the previous related sophism II, 33, the propositional objects considered by Socrates and God differ in what the personal pronoun 'he' (Latin *se*) refers to.

The epistemic sophism of Kilvington's that is longest and which has raised most attention among modern scholars devises a case where it appears that the respondent must admit that he both knows and doubts the same proposition. It is as follows:

S47. You know that the king is seated.

Casus: You know that the king is seated, if he is, and you know that he is not seated, if he is not.

To solve the sophism, Kilvington engages in a discussion of the rules of responding in a sophism, apparently assuming that sophisms should be understood as obligational disputations. However, in Kilvington's view, these rules need to be modified to serve the purpose. (See chapter on obligations in this volume.)

In the chapter on knowledge and doubt in his *Rules for Solving Sophisms*, Heytesbury also considers whether one can simultaneously know and doubt the same sentence. He is thus taking up the theme of Kilvington's sophism S47, but as a substantive issue on the nature of knowledge. To some extent, his discussion also goes into the problem whether it is possible to have a second-order doubt concerning whether one knows, or whether the so-called KK-principle of Hintikka (that knowledge entails that one knows that one knows) is true. Heytesbury defines knowledge as follows: 'to know is nothing other than unhesitatingly to apprehend the truth' (Pasnau, 1995). It can be argued that his choice not to require justification of knowledge is a conscious one (Martens, 2010), but the main point that he wants to make is that knowledge is an 'unhesitating' propositional attitude. Thus, it is different from, and contrary to, doubt.

Another of Kilvington's epistemic sophisms has features that make it arguably classifiable as an 'insoluble'.

S48. A is known by you.

Casus: A is one or the other of these: 'God exists' or 'Nothing granted by Socrates is known by you', and Socrates grants this and nothing else: 'A is known by you'.

In this case, we have an indirect form of self-reference: What Socrates grants is 'A is known by you', and another premise concerns your knowledge of what Socrates grants. These kinds of meta-semantic claims are characteristic of the 'insolubles', which we discuss in the next section.

5. Insolubles

Among the so-called 'insolubles' are the liar paradox ('This sentence is false'), and other sophisms containing meta-semantic terms such as 'true', 'valid', 'grant', 'deny', and 'lying', such as:

'This argument is valid, so you are an ass' (where 'this argument' refers to the argument made by the very sentence in question). This particular sophism, found in William Heytesbury's *Rules for Solving Sophisms*, was rediscovered as Curry's paradox (Read, 2001) – evidence that such sophisms dealt with issues of deep logical significance. Heytesbury takes issue with the customary title of such sophisms, 'insolubles', claiming that they may not be really impossible to solve, but that providing a solution is very difficult. He writes, "although the insoluble can be solved, nevertheless they have not yet been solved" (Heytesbury, 1979, p. 18). Unlike the other types of sophisms discussed above, insolubles were perhaps more often treated in separate treatises, and they were not a common topic in the thirteenth-century collections of sophisms. However, the basic structure of a sophism as it settled over the thirteenth century proves to suit the medieval way of discussing the liar paradox and similar paradoxes very well.

As a genre, insolubles literature was in Heytesbury's time almost two centuries old. It seems that earlier Ancient Greek, Byzantine (Gerogiorgakis, 2009), and Arabic (Alwishah & Sanson, 2009) treatments of the Liar paradox did not have much direct influence on the Latin tradition, which is considered to start with the so-called *Insolubilia Monacensia*, dated to the end of the twelfth century (Martin, 1993). For a summary of the types of solutions found in the discussion up to Bradwardine, we can take the classification from his own treatise *Insolubles* (Bradwardine, 2010), which dates from about the same years as Kilvington's *Sophismata*, a decade before Heytesbury's *Rules for Solving Sophisms*. As Bradwardine saw it, there were four basic types of solutions differing from his.

(1) Firstly, there were restrictionists, who wanted to pose restrictions on what terms

can stand for in a sentence, or for which time they can stand for the denoted things. Simply put, self-reference is to be banned in such a way that the paradoxes cannot be produced. Socrates cannot refer to his own speech act when he says 'Socrates says something false'. As Bradwardine elaborately points out, the solution appears to be *ad hoc*, since there seems to be no natural general way of describing what exactly is to be banned.

(2) Second, 'nullifiers' claimed that 'no one can say that he is uttering a falsehood', in Bradwardine's formulation. What exactly is impossible to do is rather difficult to spell out given that 'a man [can] open his mouth and form the words' as Bradwardine says.

(3) Third, the principle of bivalence had been denied. It had been claimed that in insolubles we find propositions that are neither true nor false. Bradwardine's straightforward counterargument is to reformulate the paradoxes through reverting to a proposition either being or not being true rather than being true or false.

(4) Fourth, one could distinguish between utterance in act (*exercitus*) and the thought (*conceptus*). This would make the actual formation of the spoken sentence as a truth-bearer different from that for which the term 'falsehood' stands for when I say 'I am uttering a falsehood.' Then the utterance in act would be true but the corresponding thought would be false. Bradwardine remarks that this solution only applies to those versions of the paradox that are based on utterances that are distinct from thoughts.

The core of Bradwardine's own solution to the paradoxes of self-reference is that a sentence that claims itself to be false entails and thus signifies not only that it is false but also that it is true. On the basis of such a signification, it is unproblematic to judge that the sentence is false because the sentence cannot be both true and false, and is thus impossible (and thus false). This solution was highly influential for decades.

Bradwardine's solution concentrated on the relation between a sentence and the claim that the sentence is true. This relation is taken under scrutiny by other authors of the time. In his *Sophismata*, Richard Kilvington argues that a sentence and the claim that it is true are equivalent only under two crucial conditions (Kilvington, 1990). Consider the following two sentences:

> 'You are in Rome' is true.
>
> You are in Rome.

According to Kilvington, the logical relation between these sentences depends on what the sentence quoted in the first one signifies. The second does not follow from the first, if 'you are in Rome' means that man is an animal. Furthermore, Kilvington points out that the first follows from the second only if the second sentence is actually formulated, spoken or written out. That is, even if you were in Rome, 'you are in Rome' would not be true if no one makes the claim. Kilvington relies here on a generally accepted medieval understanding that the truth-bearers are actually uttered sentence tokens rather than abstract types. (See the chapter on propositions in this volume.)

Heytesbury adopts these distinctions as the basis for his treatment of insolubles. He approaches the paradox as a problem of how the respondent should answer in a disputation. Take the *casus*,

Socrates only says 'Socrates says something false'

The sophism to be evaluated is:

Socrates says something false.

Heytesbury tells the respondent to deny the sophism and then also deny, if asked, that the sophism signifies exactly what the words usually would signify. As the respondent is only answering by granting or denying, he cannot be forced to explain what Socrates' sentence exactly signifies in the casus. He should deny any exact formulation of what the sentence signifies, and thus leave room for the sentence having some other unspecified abnormal signification. Thus, Heytesbury argues, the respondent can remain coherent in the disputation indefinitely without having to explain why the sophism is false. Such a solution may not be a satisfactory explication of the paradox, but it does allow the respondent to conduct an actual disputation coherently.

In any case, Heytesbury's solution follows Bradwardine in locating the problem in the exact signification of the insoluble sophism. This is to some extent true of John Buridan as well. In his *Sophismata,* he renounces the view that all sentences signify their own truth, admitting that he had earlier held it. But he does not change his mind completely. He opts for saying that every sentence virtually implies rather than signifies the sentence saying that the sentence is true, and that this implication belongs to the truth conditions (Buridan, 2001, pp. 966-7). Thus, consider the following consequence:

Man is an animal; therefore, *a* is true,

where the subject term of the consequent '*a*' refers to the antecedent 'Man is an animal'. Because such a consequence is in Buridan's view valid for all sentences, the antecedent can be true only if things are as the consequent signifies. In this sense, the truth of any sentence is partly dependent on its own truth-claim. In the case of insolubles, this requirement clashes with other requirements imposed by the sentence, yielding falsity.

Here is how the clash comes about. Buridan asks us to consider the sentence 'no sentence is true', and the associated consequence deriving the truth-claim from the sentence itself (Buridan, 2001, p. 967):

No sentence is true; therefore, *c* is true,

where '*c*' refers to the antecedent. In this case, the consequence is problematic. As Buridan sees it, the consequence is formally valid but the antecedent and the consequent cannot both be true. Thus, the antecedent must be false. And generally, any similar insoluble must be evaluated as false.

Buridan has a number of further examples of insolubles in his *Sophismata*, some circular in a mediated way, and even more interestingly, some practical ones. Thus, in his 17th sophism on insolubles, Plato promises to throw Socrates into a river if (and only if) he speaks falsely, and Socrates replies saying 'You will throw me into the river'. Plato has thus made an apparently unproblematic promise that turns out to be impossible to fulfill. It is a sign of Buridan's great influence as a logician that this sophism found its way to Cervantes' *Don Quixote*.

6. Final remarks

The study of sophisms contributed substantially to the development of logic during the middle ages. While the Aristotelian system of syllogistics remained important for logic, sophisms concentrated on topics outside Aristotelian syllogistics, as we have seen in the various examples discussed above. A number of important contributions arose out of these investigations, including, in addition to the analysis of syncategorematic terms, new ideas related to reference and propositional attitudes. The literature connected to grammatical sophisms also contains quite advanced ideas concerning the interpretation of relative clauses, as Terence Parsons argues in a new exposition of medieval logic using modern logical notation (Parsons, 2014).

The 'proofs' and 'disproofs' of sophism sentences were literally (semi-formal) *proofs*, connecting a set of premises (the casus), with a conclusion (the sophism sentence), using inference rules. The inference rules specified consequence relations of a proof-theoretic nature, providing a syntactic characterization of validity, just as Aristotelian syllogisms and the sequents of modern proof theory do.[3] Among these can be counted

[3] It is debatable whether a rule like 'from inferior to superior', as in "A man is running, therefore an animal is running", is syntactic. In *On the Essence of Logic*, Burley writes that such rules are of a special nature but nevertheless 'formal':

the rules of exposition and the rule 'from first to last', which we saw above.[4] Crucially, all of this was happening in Latin, so the medieval authors were essentially developing a *proof system for Latin,* albeit a highly regimented form of Latin.

In this endeavor, medieval authors used Latin in two ways: first, as an object of empirical study, whose properties are discoverable truths, and second, as the language for which a proof system is to be developed. The nature of the investigation was thus both linguistic and logical. Indeed, the thirteenth-century logician Robert Kilwardby writes that the study of logic can be seen in these two ways: "And thus logic is for us in one way a science of word usage (Latin 'scientia sermonicinalis'), and in this way it contains grammar, rhetoric and logic strictly taken. In another way it is science of reason, and in this way it is a trivium science distinct from grammar and rhetoric" (Kilwardby, 1976, p. 167).

Under one common style of analysis in formal semantics, originating with Montague (Montague, 1974), natural language expressions are translated into expressions of a formal language, and consequence relations between sentences in natural languages derive from the consequence relations among their formal counterparts, for which consequence relations are stipulated by definition. Natural language remains an object

"Formal consequence is of two kinds: one kind hold by reason of the form of the whole structure… another kind… holds by reason of the form of the incomplex terms, e.g. a consequence from an inferior to a superior affirmatively is formal, but holds by reason of the terms" (Burley, 2000, p. 173).

4 The rule "from first to last" allows an inference from the first antecedent to the last consequent in an arbitrarily long sequence of conditionals. It follows, as Burley points out, from repeated application of either of these more basic rules: "The second main rule is that whatever follows from a consequent follows from the antecedent. There is another rule too, almost the same as this one: Whatever is antecedent to an antecedent is antecedent to the consequent" (Burley, 2000, p. 4).

of empirical study, and the logical properties of basic expressions remain discoverable truths.

The usual separation of natural and formal languages in formal semantics may be a natural resolution of the tension between these two roles. The convenience of having an unambiguous, regimented language when defining a proof system led fourteenth-century logicians to introduce certain modifications to their Latin. Above, we alluded to a case where Kilvington stipulated special usage conventions for “up to” and “immediate” in Latin. For another example, Burley wrote that “a negation has scope over what follows, not over what precedes” (Burley, 2000, p. 15). This is not exactly the case in classical Latin. In modern semantics, all such creative language-construction is relegated to the development of the formal representation language, and the natural language is taken as given. From a modern perspective, changing the object of study is tampering with the evidence, while changing the formal language is developing a theoretical tool.

While the modern duality between natural and formal languages may be the natural resolution of a tension, it also seems natural that thirteenth-century authors did not separate the two roles of Latin. The logical tradition at the time was not very rich, and no artificial languages had been developed. This changed over the course of the middle ages, thanks to the work of the scholastic tradition.

Another salient contrast between modern formal semantics and the *sophismata* literature is in the use of presupposition. Although it was not as widely used then as it is today, the notion of presupposition was not entirely foreign to medieval scholars.

Buridan writes for example that "the question *propter quid* ['why'] presupposes a proposition to the effect that the predicate truly inheres in the subject, and what is asked for is the cause of the inherence" (Buridan, 2001, p. 816). The Latin word *praesupponunt* 'presupposes' can be found in discussions of declarative sentences as well. Peter of Spain writes regarding the "reduplicative" expression *inquantum* 'insofar as', as in 'Man, insofar as he is an animal, is sensitive', that "such a particle presupposes [*praesupponit*] that a given predicate inheres in the subject and denotes [*denotat*] that the term to which it is attached causes that inherence" (Horn, 1996, p. 300). Another case of presupposition in a declarative sentence is as follows (Burley, 2000, p. 143):

> But there are certain predicates that presuppose being simply, such as predicates that denominate accidents and signify an act or a form in act, like 'white', 'black', 'hot', 'cold'. In such cases the inference does hold from 'is' as a third component to 'is' as a second component. For it follows: 'Socrates is white; therefore, Socrates is'. And it follows: 'Socrates is hot; therefore, Socrates is'.

(A predicate like 'is possible' or 'is a thinkable thing' would not have the same kind of existence presupposition.) This discussion seems to imply a notion of presupposition that is modern insofar as it licenses inferences.

However, for Burley, presupposition failure would cause falsity, or truth under negation, and was therefore much like ordinary entailment. In this respect it was very different from the modern conception of presupposition, on which presupposition failure often means that a sentence is neither true nor false. There are some examples within medieval logic where the principle of bivalence was questioned, but these are not connected to presupposition, and for the most part do not involve a truth value corresponding to nonsense. As mentioned above, one strategy for dealing with insolubles was to deny the principle of bivalence, but this seems to be the only candidate for a use of a truth value of nonsense.[5]

Common to medieval logic and modern formal semantics is that the study of natural language is an engine for the development of logic. Furthermore, while we have observed a number of differences between the medieval and modern analytical frameworks, the potential for such comparisons between them underscores the closeness in orientation of the enterprises. If a medieval logician and a modern formal semanticist were seated next to each other at a dinner party, they would not run out of things to discuss.

Bibliography

[5] Other cases where bivalence was questioned appear to have been limited to under-determination ('unknowable' rather than 'nonsense'), and over-determination (true and false at the same time). One group of candidates for under-determination included future contingents like Aristotle's 'There will be a sea battle tomorrow'. Another included sophisms like Kilvington's S47 discussed above ('you know that the king is seated'). In the latter case, a respondent in a disputation was to answer 'doubt', rather than 'grant' or 'deny', which implies that the sentence is in fact either true or false, but the respondent needs more information in order to decide. These are both potential cases of unknowability (as captured by the strong Kleene interpretations of the connectives in multi-valued logic), rather than nonsense (as captured by the weak Kleene connectives). One candidate for over-determination was the possibility that two contradictory sentences might simultaneously hold at a moment of change, suggesting that a sentence might simultaneously be *both* true and false, rather than *neither*. This idea would be captured with the truth-value 'both' in a multi-valued logic with truth-values 'true', 'false', 'neither' and 'both' (Muskens, 1995).

Alwishah, A., & Sanson, D. (2009). The Early Arabic Liar: Liar Paradox in the Islamic World from the Mid-Ninth to the Mid-Thirteenth Centuries CE. *Vivarium , 47* (1), 97-127.
Andrews, R. (1993). The Sophistria of Petrus Olai. In R. Andrews, & S. Read (Ed.), *Sophisms and Medieval Logic and Grammar* (pp. 3-30). Dordrecht: Kluwer.
Beaver, D. I., & Clark, B. Z. (2008). *Sense and Sensitivity: How Focus Determines Meaning.* Malden, MA: Wiley-Blackwell.
Biard, J. (1989). Les sophismes du savoir: Albert de Saxe entre Jean Buridan et Guillaume Heytesbury. *Vivarium , 17*, 36-50.
Bos, E. P. (2007). Richard Billingham's Speculum puerorumm, Some Medieval Commentaries and Aristotle. *Vivarium , 45* (2), 360-373.
Buridan, J. (2001). *Summulae de Dialectica: An annotated translation, with a philosophical introduction by Gyula Klima.* New Haven: Yale University Press.
Burley, W. (2000). *On the Purity of the Art of Logic: The Shorter and the Longer Treatises.* (P. V. Spade, Trans.) New Haven: Yale University Press.
Coppock, E., & Beaver, D. (2013). Principles of the Exclusive Muddle. *Journal of Semantics , 31* (3), 371-432.
Courtenay, W. J. (2008). *Ockham and Ockhamism.* Leiden: Brill.
Dronke, P. (Ed.). (1992). *A History of Twelfth-Century Philosophy.* Cambridge: Cambridge University Press.
Duhem, P. (1913). *Études sur Léonard de Vinci. Troisième Série. Les précurseurs parisiens de Galilée.* Paris: A. Hermann et fils.
Frege, G. (1948). Sense and Reference. *The Philosophical Review , 57* (3), 209-230.
Gerogiorgakis, S. (2009). The Byzantine Liar. *History and Philosophy of Logic , 30*, 313-330.
Glagett, M. (1959). *The Science of Mechanics in the Middle Ages.* Madison: University of Wisconsin Press.
Heim, I., & Kratzer, A. (1998). *Semantics in Generative Grammar.* Oxford: Blackwell.
Heim, I., & Kratzer, A. (1998). *Semantics in Generative Grammar.* Oxford: Blackwell.
Horn, L. R. (2011). Only XL: The assertoric asymmetry of exponibles. In E. Cormany, S. Ita, & D. Lutz (Eds.), *Proceedings of Semantics and Linguistic Theory (SALT) 19* (pp. 198-22). eLanguage.
Horn, L. R. (1996). Presupposition and Implicature. In S. Lappin (Ed.), *Handbook of Contemporary Semantic Theory* (pp. 299-319). Blackwell.
Kamp, H., van Genabith, J., & Reyle, U. (2011). Discourse Representation Theory. In D. M. Gabbay, & F. Guenthner (Eds.), *Handbook of Philosophical Logic* (Vol. 15, pp. 125-394). Dordrecht: Springer.
Kilvington, R. (1990). *The Sophismata of Richard Kilvington.* (N. Kretzmann, & B. Kretzmann, Trans.) Cambridge: Cambridge University Press.
Kilwardby, R. (1976). *De orto scientarium.* (A. G. Judy, Ed.) London: British Academy.
Klima, G. (2001). Introduction. In J. Buridan, *Summulae de Dialectica* (pp. xxvii-lxii). New Haven: Yale University Press.
Kneepkens, C. H. (1993). Orleáns 266 and the Sophismata Collection: Master Joscelin of Soissons and the infinite words in the early twelfth century. In S. Read, *Sophisms in Medieval Logic and Grammar* (pp. 64-85). Dordrecht: Kluwer.

Leader, D. R. (1989). *A History of the University of Cambridge* (Vol. I: The University to 1546). Cambridge: Cambridge University Press.
Lewis, D. (1979). Attitudes De Dicto and De Se. *The Philosophical Review , 88* (4), 513-543.
Markowski, M. (1993). Die Rolle der Sophismata im unterricht der Krakauer Universität im 15. Jahrhundert. In S. Read, *Sophisms in Medieval Logic and Grammar* (pp. 116-127). Dordrecht: Kluwer.
Martens, D. B. (2010). William Heytesbury and the conditions for knowledge. *Theoria , 76* (4), 355-374.
Montague, R. (1974). English as a Formal Language. In R. Thomason (Ed.), *Formal Philosophy: Selected Papers of Richard Montague* (pp. 188--221). New Haven: Yale University Press.
Montague, R. (1974). The Proper Treatment of Quantification in Ordinary English. In R. H. Thomason (Ed.), *Formal Philosophy.* New Haven: Yale University Press.
Muskens, R. (1995). *Meaning and Partiality.* Stanford: CSLI Publications.
Parsons, T. (2014). *Articulating Medieval Logic.* Oxford: Oxford University Press.
Pasnau, R. (1995). William Heytesbury on Knowedge: Epistemology without Necessary and Sufficient Conditions. *History of Philosophy Quarterly , 12* (4), 347-366.
Quine, W. (1956). Quantifiers and propositional attitudes. *Journal of Philosophy , 53*, 177-187.
Read, S. (Ed.). (1993). *Sophisms in Medieval Logic and Grammar.* Dordrecht: Kluwer Academic Publishers.
Saxony, A. o. (1490). *Sophismata.* n/a: n/a.
Strawson, P. (1950). On Referring. 320-344.
Streveler, P. (1993). A Comparative Analysis of the Treatment of Sophisms in MSS Digby 2 and Royal 12 of the Magister Abstractionem. In S. Read (Ed.), *Sophisms in Medieval Logic and Grammar* (pp. 144-184). Dordrecht: Kluwer.
Sylla, E. (2010). The Oxford Calculators' Middle Degree Theorem in Context. *Early Science and Medicine , 15*, 338-370.
Tabarroni, A. (1993). 'Omnis phoenix est': Quantification and Existence in a new Sophismata-collection (MS Clm 14522). In S. Read, *Sophisms in Medieval Logic and Grammar* (pp. 185-201). Dordrecht: Kluwer.
Westerståhl, D. (1985). Logical constants in quantifier languages. *Linguistics and Philosophy , 8*, 387-413.