

What do comparative and superlative modifiers have to do with comparatives and superlatives?

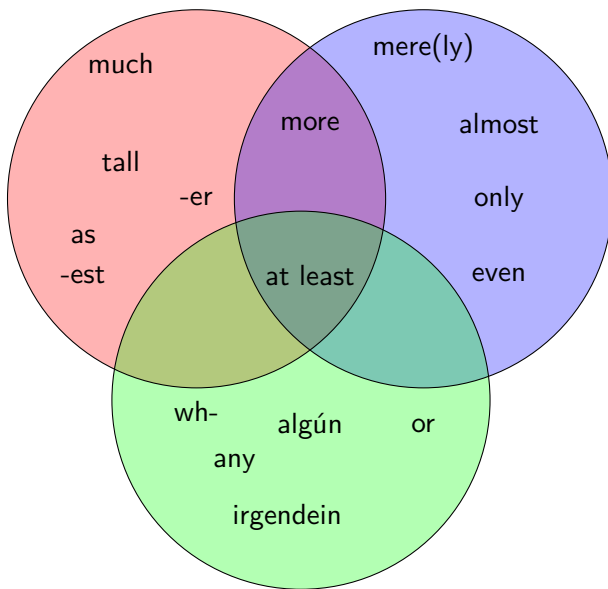
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'Two days at least'

Kasteel de Hooge Vuursche, 11 September 2014

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Question I: Comparatives and comparative modifiers

Is it the same *more* in all of the following?

- (1)
 - a. This airplane has more than five emergency exits.
 - b. Fred is more intelligent than Gloria is.
- (2)
 - a. She had more than three highballs; she also had several beers.
 - b. She is more than an assistant professor.

If so what does it mean?

Question II: Superlatives and superlative modifiers

What do superlative modifiers have to do with ordinary superlatives?

- (3) a. Gloria climbed the *tallest* mountain.
b. Gloria ate the *fewest* popsicles.
- (4) a. This airplane has *at least* six emergency exits.
b. Gloria is *at least* an assistant professor.
- (5) a. We will arrive at 3 o'clock *at the earliest*.
b. Do not fear your enemies. *The worst* they can do is kill you. Do not fear friends. *At worst*, they may betray you. (Bruno Jasiński)

Little work we know of except Krifka 2007 (handout), Penka 2010 (handout) Solt (2011:pp. 6-10).

Question(s) III: Comparatives and superlatives

What is the relation between comparatives and superlatives, and are comparative and superlative modifiers related in the same way?

Why do comparative/superlative modifiers differ in the ways that they do?

(6) *Distribution*

- Betty had three martinis, {at most/*fewer than}
- {At least/*More than}, Betty had three martinis.
- Wilma danced with {at most/*fewer than} every second man who asked her.

(7) *Ignorance implications*

- This airplane has more than five emergency exits.
- This airplane has at least six emergency exits.

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Two kinds of analyses: Degrees vs. discourse

In terms of degrees (the usual suspects):

- (8) a. *more than* $\rightsquigarrow \lambda m_d \lambda P_{\langle d,t \rangle} . \max(P) > m$
 b. *less than* $\rightsquigarrow \lambda m_d \lambda P_{\langle d,t \rangle} . \max(P) < m$

In terms of discourse strength/alternatives (e.g., Krifka, Geurts and Nouwen, Coppock and Brochhagen):

- (9) a. *more than* $\rightsquigarrow \lambda \alpha_{\langle \tau,p \rangle} \lambda \beta_{\tau} \lambda w . \exists p [p \triangleright \alpha(\beta) \wedge p(w)]$
 b. *less than* $\rightsquigarrow \lambda \alpha_{\langle \tau,p \rangle} \lambda \beta_{\tau} \lambda w . \forall p [p(w) \rightarrow \alpha(\beta) \triangleright p]$

Two kinds of analyses: Degrees vs. discourse

The former approach works very well for things like *more than five* and *more intelligent than Gloria is* — i.e., for **comparatives**.

The latter approach works very well for things like *more than five BEERS* and *more than an assistant professor* — i.e., for **comparative modifiers**.

We would like to ask whether it's possible to (in effect) derive both of these kinds of meanings from a single starting point.

Comparatives as degree relations

We begin with a very simple hypothesis about the meaning of comparative morphology (cf. Heim 2006, Beck 2012):

(10) a. *more* $\rightsquigarrow \lambda s_d \lambda t_d . t > s$

b. *less* $\rightsquigarrow \lambda s_d \lambda t_d . t < s$

(11) a. Five is more than three.

b. Three is less than five.

Caveat: we will ignore further (de)compositional possibilities — *more*; *er much*; *than*; *er little*; etc. — possibly to our peril.

Comparatives as degree relations

In simple cases of composition with degree predicates, this together with some appropriate syntactic assumptions (about argument linking) and compositional operations (function composition, existential closure) gives us what we need to get the truth conditions right:

- (12) a. more than five exits
 b. $\lambda x. \exists d [d > 5 \wedge \#(x) = d \wedge \mathbf{exits}(x)]$
- (13) a. more intelligent than Gloria is
 b. $\lambda x. \exists d [d > d_G \wedge \mathbf{intelligent}(x) \geq d]$

This won't work for *less* ("van Benthem's problem"), and it won't account for all of the interesting properties and kinds of comparatives, but we still think this is a good starting point because....

Comparatives as degree relations **and more**

The core meaning can be straightforwardly mapped to various other interesting and useful meanings, including:

“Quantificational” comparative meaning(s)

$$(14) \quad \lambda S_{\langle d,t \rangle} \lambda T_{\langle d,t \rangle} . \mathbf{more}(\max(S))(\max(T)) \equiv \\ \lambda S_{\langle d,t \rangle} \lambda T_{\langle d,t \rangle} . \max(T) > \max(S)$$

This kind of meaning can explain scope ambiguities in comparatives, and has been used by CK and others to account for properties of modified numerals. It also avoids van Benthem’s problem for *less*.

Comparatives as degree relations **and more**

“Phrasal” comparative meaning(s)

$$\begin{aligned}
 (15) \quad & \lambda f_{\langle d, \langle \tau, t \rangle \rangle} \lambda s_{\tau} \lambda t_{\tau}. \mathbf{more}(\max(\lambda d.f(d)(s)))(\max(\lambda d.f(d)(t))) \\
 & \equiv \lambda f_{\langle d, \langle \tau, t \rangle \rangle} \lambda s_{\tau} \lambda t_{\tau}. \max(\lambda d.f(d)(t)) > \max(\lambda d.f(d)(s)) \\
 & \equiv \mathbf{more}_p
 \end{aligned}$$

This type of meaning is the one that will be of most interest to us, since it in effect provides a means of building a general comparative relation between arguments of arbitrary types, provided we can find an appropriate mapping from that type to degrees.

Phrasal comparatives

Some typical cases:

(16) Kim is taller than Lee.

a. $\mathbf{more}_p(\mathbf{tall})(\mathbf{l})(\mathbf{k})$

b. $\max(\lambda d.\mathbf{tall}(d)(\mathbf{k})) > \max(\lambda d.\mathbf{tall}(d)(\mathbf{l}))$

(17) Kim read more books than Lee.

a. $\mathbf{more}_p(\lambda d \lambda y. \exists x [\#(x) = d \wedge \mathbf{books}(x) \wedge \mathbf{read}(x)(y)])(\mathbf{l})(\mathbf{k})$

b. $\max(\lambda d. \exists x [\#(x) = d \wedge \mathbf{books}(x) \wedge \mathbf{read}(x)(\mathbf{k})]) >$
 $\max(\lambda d. \exists x [\#(x) = d \wedge \mathbf{books}(x) \wedge \mathbf{read}(x)(\mathbf{l})])$

A more interesting case

When the standard is “contained in” the target, there is a presupposition that the sentence holds of the standard (Grant 2013):

- (18) Kim read more (books) than *The Idiot* and *The Devils*.
- Kim didn't read more (books) than *The Idiot* and *The Devils*.
 - Did Kim read more (books) than *The Idiot* and *The Devils*?
 - If Kim read more (books) than *The Idiot* and *The Devils*, then she must have had more time than I thought.
 - If Kim read *The Idiot* and *The Devils*, then she read (books) than those two books.
- (19) # K
im read more books than *The New York Times* and *The Wall Street Journal*.

A more interesting case

Aparicio-Terrasa (2014) shows that this follows from the interaction of the regular phrasal semantics and the particular target/standard configuration:

- (20) Kim read more books than *The Idiot* and *The Devils*.
- $\mathbf{more}_p(\lambda d \lambda x. \#(x) = d \wedge \mathbf{books}(x) \wedge \mathbf{read}(x)(\mathbf{k}))(\mathbf{i} + \mathbf{d})$
 - $\exists x [\max(\lambda d. \#(x) = d \wedge \mathbf{books}(x) \wedge \mathbf{read}(x)(\mathbf{k})) > \max(\lambda d. \#(\mathbf{i} + \mathbf{d}) = d \wedge \mathbf{books}(\mathbf{i} + \mathbf{d}) \wedge \mathbf{read}(\mathbf{i} + \mathbf{d})(\mathbf{k}))]$

If Kim didn't read *The Idiot* and *The Devils* (or if they are not books), the standard degree set is empty; he presuppositions follow from the assumption that max requires a non-empty input.

NB: Target argument saturated by existential closure. More on this later.

Presuppositions of comparative modifiers

Comparative modifiers introduce to presuppositions similar to standard-in-target comparatives:

- (21) Jesus is MORE than a good teacher. Jesus is MORE than a miracle worker. **Jesus is MORE than a prophet.** Jesus is the SON of God.
- Jesus is no(t) more than a prophet.
 - Is Jesus more than a prophet?
 - If Jesus is more than a prophet, then ...
 - If Jesus is a prophet, then he's no more than that.

We're not sure if anyone has noticed this before, but it doesn't follow from a G&N/C&B-style semantics for comparative modifiers.

- (22) $\lambda w. \exists p [p \triangleright \mathbf{prophet}(j) \wedge p(w)]$

Comparative modifiers: Standard-in-target comparatives?

Maybe comparative modifiers are nothing more than standard-in-target phrasal comparatives! Suppose that inside (23a) is a function that maps properties to degrees. Maybe the thing pronounced as *much* in (23b)?

- (23) a. Jesus is more than a prophet.
 b. John isn't much of a prophet.

Composition parallel to what we saw with with arguments derives something very much like the discourse-based semantics, but with the presupposition that we want.

- (24) a. $\mathbf{more}_p(\lambda d \lambda P. \mathbf{m}(P) = d \wedge P(\mathbf{j}))(\mathbf{prophet})$
 b. $\exists P[\max(\lambda d. \mathbf{m}(P) = d \wedge P(\mathbf{j})) > \max(\lambda d. \mathbf{m}(\mathbf{prophet}) = d \wedge \mathbf{prophet}(\mathbf{j}))]$

Summary

- Simple semantics for comparatives as degree relations
- Various extensions, including “parameterized” relations between arbitrary types (“phrasal” comparatives)
- A hypothesis about the relation between comparative modifiers and comparatives: comparative “modifiers” are just (a special case of) phrasal comparatives

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Two kinds of analyses: Degrees vs. discourse (again!)

Degree-based (Kennedy 2014):

- (25) a. *at least* $\rightsquigarrow \lambda m_d \lambda P_{\langle d,t \rangle} . \max(P) \geq m$
 b. *at most* $\rightsquigarrow \lambda m_d \lambda P_{\langle d,t \rangle} . \max(P) \leq m$

Discourse-based (Krifka, Büring, Coppock & Brochhagen):

- (26) a. *at least* $\rightsquigarrow \lambda \alpha_{\langle \tau,p \rangle} \lambda \beta_{\tau} \lambda w . \exists q [q \sqsupseteq \alpha(\beta) \wedge q(w)]$
 'some alternative as strong as the prejacent is true'
 b. *at most* $\rightsquigarrow \lambda \alpha_{\langle \tau,p \rangle} \lambda \beta_{\tau} \lambda w . \forall q [q \triangleright \alpha(\beta) \rightarrow \neg q(w)]$
 'no alternative stronger than the prejacent is true'

Neither approach engages with the current state of the art on superlatives.

A superlative analysis of superlative modifiers

Solt (2011) (similar to Penka 2010):

- “What distinguishes the acceptable uses [of superlative modifiers] is that there is a range of actual or possible values under consideration, and not just a single value. This constraint mirrors a restriction on the superlative to situations where the comparison class has multiple members.”
- “What sort of comparison class might we have in the case of superlative quantifier *most*? [In *Fred has read at most 15 Shakespeare plays*] the most obvious possibility is that it is a comparison class of numbers.”
- “Informally speaking, the comparison class C might be taken to be the set of numbers n such that Fred **might have** read n Shakespeare plays.”

Penka (2010) / Solt (2011)

Composition of *est* and *much/many*:

- (27) a. John is at most 2m tall.
 b. $2m$ [-est_C [λd [d-much] [$\lambda d'$ John d'-tall]]]

- (28) a. $\exists d[2m \geq d \wedge \mathbf{height(j)} \geq 2m \wedge \forall d' \in C[d' \neq 2m \rightarrow \neg[d' \geq d \wedge \mathbf{height(j)} \geq d']]]$
 b. Presuppositions:
 $2m \in C$; C is heights of John; C has multiple members.

The resulting meaning is “defective,” but can be fixed by inserting a covert epistemic operator, à la Nouwen (2010).

- (29) a. $\exists d[2m \geq d \wedge \blacklozenge \mathbf{height(j)} \geq 2m \wedge \forall d'[d' \in C \wedge d' \neq 2m \rightarrow \neg[d' \geq d \wedge \blacklozenge \mathbf{height(j)} \geq d']]]$
 b. Presuppositions:
 $2m \in C$; C is **possible** heights of John; C has multiple members.

Can we do better?

Challenges for Penka/Solt:

- We would prefer a non-magic-based account.
- Speaker uncertainty is not part of the truth conditions; indeed it is not always implied (Nouwen's range-of-variation cases).

Our strategy:

- The uncertainty/variation comes from the introduction of alternatives, as under Coppock & Brochhagen's (2013) analysis in Inquisitive Semantics.
- But we build on Penka/Solt's insight that the range is related to the comparison class of the superlative.

Inquisitive analysis: *at least*/*most* raises an issue

“fewer than 5”

0

1

2

3

4

5

6

7

“at most 4”

0

1

2

3

4

5

6

7

Coppock & Brochhagen's lexical entry for *at least*

Propositional version (simplified formalization):

$$(30) \quad \textit{at least} \rightsquigarrow \{\lambda p.p' \mid p' \sqsupseteq p\}$$

Type: (pp) – a set of functions from possibilities to possibilities

Geached version:

$$(31) \quad \textit{at least} \rightsquigarrow \{\lambda\alpha_{\tau p} \lambda\beta_{\tau}.p \mid p \sqsupseteq \alpha(\beta)\}$$

Type: $(\langle\tau p, \tau p\rangle)$

Also useful: Backwards-Geached version:

$$(32) \quad \textit{at least} \rightsquigarrow \{\lambda\beta_{\tau} \lambda\alpha_{\tau p}.p \mid p \sqsupseteq \alpha(\beta)\}$$

Type: $(\langle\tau, \langle\tau p, p\rangle\rangle)$

Hamblin-style composition

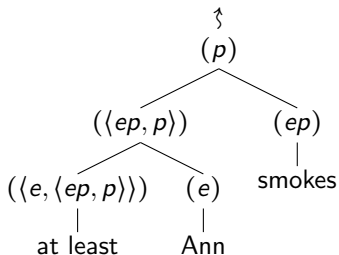
Pointwise Function Application

If $\alpha \rightsquigarrow \alpha'_{(\sigma\tau)}$ and $\beta \rightsquigarrow \beta'_{(\sigma)}$ then $\alpha\beta \rightsquigarrow \{A(B) \mid A \in \alpha' \wedge B \in \beta'\}$

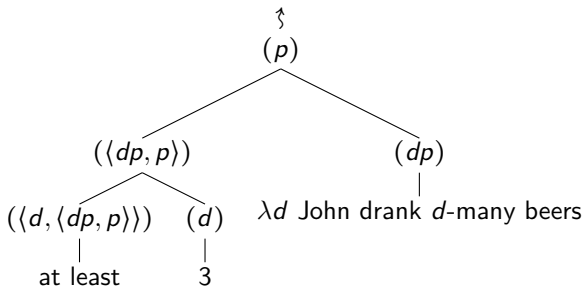
- $John_{(e)} \rightsquigarrow \{\mathbf{j}\}$
- $snores_{(ep)} \rightsquigarrow \{\lambda x.\lambda w.\mathbf{snores}_w(x)\}$
- $John\ snores_{(p)} \rightsquigarrow \{\lambda w.\mathbf{snores}_w(\mathbf{j})\}$

- $John\ or\ Mary_{(e)} \rightsquigarrow \{\mathbf{j}, \mathbf{m}\}$
- $John\ or\ Mary\ snores_{(p)} \rightsquigarrow \{\lambda w.\mathbf{snores}_w(\mathbf{j}), \lambda w.\mathbf{snores}_w(\mathbf{m})\}$

{Ann smokes, Ann and Bill smoke, ...}



{John drank 3 beers, John drank 4 beers, ...}



Successes of this analysis

- truth conditions
- focus-sensitivity of *at least/most*
- distribution (not just scalar modifiers, non-entailment scales)
- lack of scalar implicatures
- ignorance implicatures
- authoritative readings
- 'missing readings' under permission modals

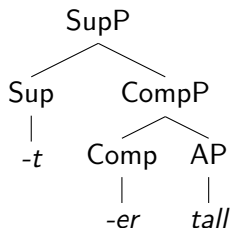
(Coppock & Brochhagen 2013)

Breaking it down

Now we would like to see if we can derive this analysis of superlative modifiers (or something like it) from the morphemes they contain.

Superlatives are derived from comparatives

Bobaljik's (2012) Containment Hypothesis:



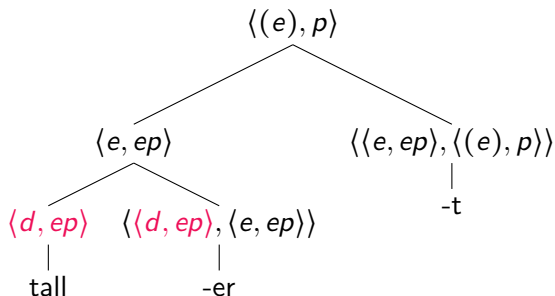
Szabolsci's (2012) semantics of *-t* (with world variables):

- (33) $-t \rightsquigarrow \lambda R_{\langle \tau, \tau, p \rangle} \lambda C_{(\tau)} \lambda x_{\tau} \lambda w [\forall x'_{\tau} \in C [x \neq x' \rightarrow R_w(x, x')]]$
 'x is greatest among the elements in C according to R'

Superlative saturates the 'standard' argument of the comparative.

Phrasal comparative as input to superlative

Since $-t$ is expecting a relation between two objects of type τ , we need to use the phrasal semantics of $-er$ to build the input to the superlative.



Phrasal semantics (with world variables):

$$(34) \quad -er \rightsquigarrow \lambda f_{\langle d, \tau t \rangle} \lambda s_{\tau} \lambda t_{\tau} \lambda w. \max(\lambda d. f_w(d)(t)) > \max(\lambda d. f_w(d)(s))$$

least

Assumption: an implicit measure function \mathbf{m} can instantiate the ‘phrasal’ meaning of *less*, yielding a relation between objects of arbitrary type τ .

$$(35) \quad \textit{less} \rightsquigarrow \lambda x_{\tau} \lambda x'_{\tau} \lambda w. \max(\mathbf{m}_w(x')) < \max(\mathbf{m}_w(x))$$

Combined with *-t*, we have:

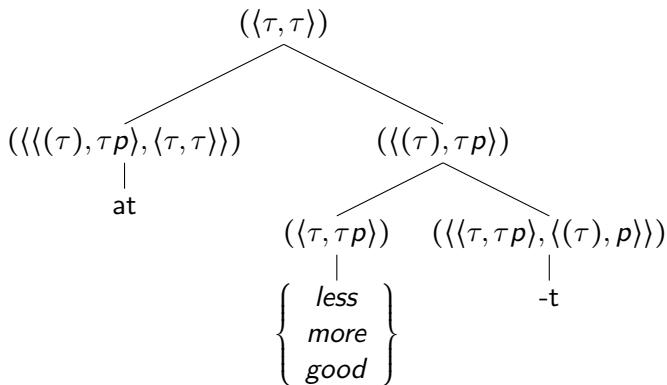
$$(36) \quad \textit{least} \rightsquigarrow \lambda C_{(\tau)} \lambda x_{\tau} \lambda w. \forall x' \in C [x' \neq x \rightarrow \max(\mathbf{m}_w(x)) < \max(\mathbf{m}_w(x'))]$$

(Alternative semantics versions: singleton sets containing these meanings.)

From *least* to *at least*

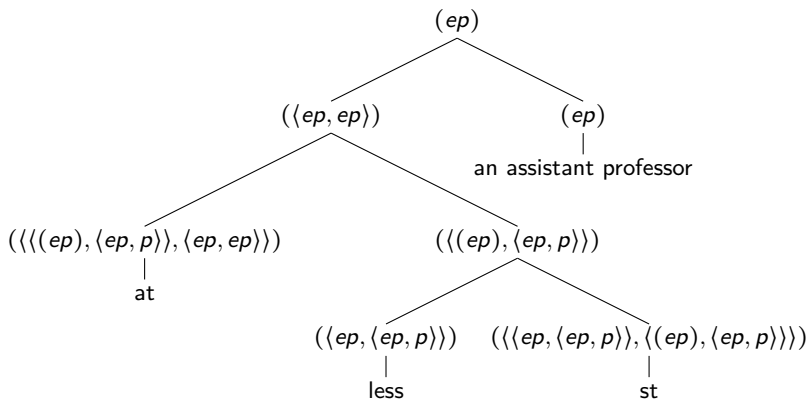
Idea: *at least* x denotes a set of alternatives y whose least member is x .

- (37) $at \rightsquigarrow \{\lambda S_{\langle(\tau), \tau p\rangle} \lambda x_{\tau} \cdot y_{\tau} \mid \exists C [y \in C \wedge S_{w*}(C)(x)]\}$
 'the set of things y in a class C s.t. x is S [least/most/etc.] in C '



Property-modifying *at least*

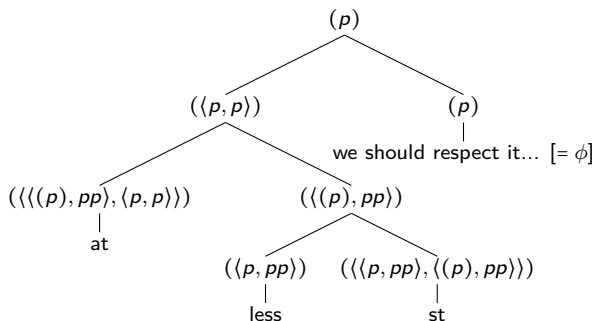
(38) at least an assistant professor



$\leadsto \{P_{ep} \mid \max(\mathbf{m}(P)) \geq \max(\mathbf{m}(\mathbf{asstprof}))\}$

Sentence-modifying *at least*

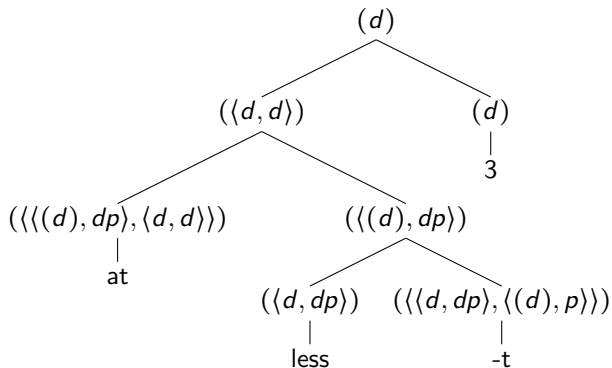
(39) At least we should respect it as a law that was in existence.



$$\begin{aligned} &\rightsquigarrow \{p \mid \exists C[p \in C \wedge \forall p' \in C[p' \neq \phi \rightarrow \max(\mathbf{m}(\phi)) < \max(\mathbf{m}(p))]]\} \\ &\equiv \{p \mid p = \phi \vee \phi \triangleleft p\} \quad \leftarrow \text{assuming } \max(\mathbf{m}(p)) < \max(\mathbf{m}(p')) \text{ iff } p \triangleleft p'. \end{aligned}$$

Numeral-modifying *at least*

Both phrasal *less* and 'raw' *less* can be type $\langle d, dp \rangle$. Regardless:



$$\begin{aligned} &\leadsto \{d \mid \exists C[d \in C \wedge \forall d' \in C[d' \neq 3 \rightarrow \max(\mathbf{m}(3)) < \max(\mathbf{m}(d))]]\} \\ &\equiv \{3, 4, 5, \dots\} \end{aligned}$$

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Distribution

Recall the distribution problem:

- (40)
- a. Betty had three martinis, {at most/*fewer than}
 - b. {At least/*More than}, Betty had three martinis.
 - c. Wilma danced with {at most/*fewer than} every second man who asked her.

Geurts & Nouwen (2007:p. 543): “while the argument of a comparative modifier must be a first-order predicate, superlative modifiers freely take a wide range of argument types.”

Distribution

On our analysis, comparative modifiers are target-in-standard phrasal comparatives. In order for (41a) to be interpretable, English would have to include a propositional **m**, and allow existential closure over the propositional target, which would give us (41b).

- (41) a. * More than [Betty had three martinis]
 b. $\exists p[\max(\lambda d.\mathbf{m}(p) = d) > \max(\lambda d.\mathbf{m}(\mathbf{b}) = d)]$

So evidently this is not a coherent meaning, or there's no proposition-taking **m** (unlikely), or there's no \exists -closure at the propositional level.

Relevant: comparatives actually *can* modify propositions, but only when the proposition is the target:

- (42) More than that, she had five highballs.

The core distinction

Comparative “modifiers” not really modifiers; they’re comparatives, and need to interact with a degree predicate.

(43) Jesus is more than a prophet.

- The standard is *a prophet*.
- The target is an existentially quantified property that applies to Jesus.

Superlative modifiers are really modifiers, which can modify any expression that can be the argument of a comparative relation:

(44) Jesus is at most a prophet.

- *a prophet* is modified by *at most*, not a complement of it.

Ignorance implications

We have derived a system that's equivalent to Coppock and Brochhagen, starting from more basic initial assumptions and without stipulating a difference between comparatives and superlatives.

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Summary of proposal

- Comparatives are degree relations, extendable to a ‘phrasal’ semantics which can take an implicit measure function.
- “Comparative modifiers” are really standard-in-target comparatives.
- The phrasal semantics for the comparative is the input to the superlative morpheme *-t*.
- The *at* in superlative modifiers introduces alternatives in the comparison class of the superlative.

Results

- Unified analysis of comparatives and comparative modifiers
- Compositional analysis of superlative modifiers (*at* + *less* + *-t*)
⇒ broader coverage
- Accounts for distributional difference between comparative and superlative modifiers.
- Contrasts between comparative and superlative modifiers wrt ignorance implicatures, scalar implicatures, behavior under modals, etc. follows from introduction of alternatives by *at*, which makes use of the superlative's comparison class.

Remaining questions

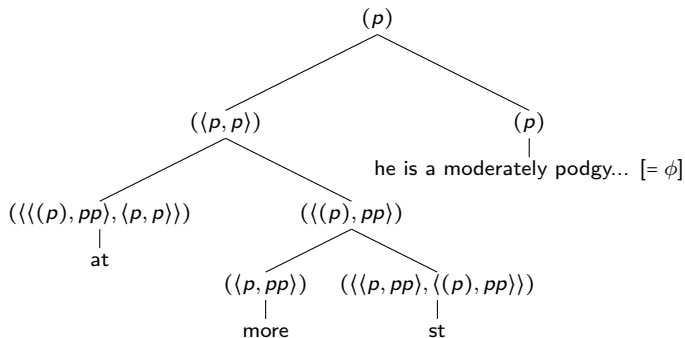
- van Benthem's problem
- *less* = *little* + *more*?
- Independent contributions of *more* and *than*?
- Do we really need the alternatives to be part of the denotation? Can we get away with ordinary denotations and general (neo-)Gricean principles? In some cases (e.g., for modified degree terms)? In all cases??

Thank you!

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Sentence-modifying *at most*

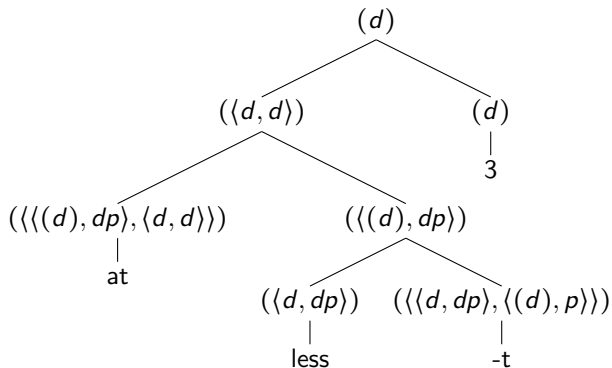
(45) At most, he is a moderately podgy man in a short-sleeved shirt.



$$\begin{aligned} &\leadsto \{p \mid \exists C[p \in C \wedge \forall p' \in C[p' \neq \phi \rightarrow \max(\mathbf{m}(\phi)) > \max(\mathbf{m}(p))]]\} \\ &\equiv \{p \mid p = \phi \vee \phi \triangleright p\} \quad \leftarrow \text{assuming } \max(\mathbf{m}(p)) > \max(\mathbf{m}(p')) \text{ iff } p \triangleright p'. \end{aligned}$$

Numeral-modifying *at most*

With *at most*, not all parses rule out 'more than':



$\leadsto \{d \mid \exists C[d \in C \wedge \forall d' \in C[d' \neq 3 \rightarrow \max(\mathbf{m}(3)) > \max(\mathbf{m}(d))]]\}$
 $\equiv \{0, 1, 2, 3\} + \text{many} + \exists\text{-closure} \not\rightarrow \text{no more than}$

van Benthem's problem

Under this analysis, *at most* ϕ will denote the set of alternatives to ϕ that are ranked lower than it. If those do not exclude higher-ranked alternatives, then we falsely predict that higher-ranked alternatives are allowed.

That was why C&B “chopped off” the alternatives for *at most*:

$$(46) \quad \textit{at most} \rightsquigarrow \{\lambda\alpha_{\tau p}\lambda\beta_{\tau}.\text{MAX}(p) \mid p \succeq \alpha(\beta)\}$$

where $\text{MAX}(p)$ means that no alternative stronger than p is true

Hard to derive compositionally.

Possible solutions:

- Disallow nested alternatives, following the latest developments in Inquisitive Semantics. Raises certain issues.
- Use quantificational analysis of numerals. General enough?