2 First Order Logic

2.1 REPRESENTATIONS FOR MEANINGS

To discuss the meanings of sentences and other expressions, we need a way to represent them. Sentences written in ordinary writing are not reliable representations of their meanings, as written forms do not always capture sameness and difference of meaning, for example:

(1)a Rameses ruled Egypt.
  b Egypt was ruled by Rameses.
  c Visiting relatives can be boring.
  d Visiting relatives can be boring.

Sentences (1a,b) have different written forms but the same meaning. Sentences (1c,d) have the same written form but different meanings – one means 'Relatives who are visiting one can be boring' and the other means 'It can be boring to visit relatives'. So we need to represent meanings directly, and for this we shall use a notation based on first order logic.

Logic is chiefly concerned with relationships between meanings, particularly the meanings of declarative sentences, in processes of reasoning. The meaning of a declarative sentence – the kind that can be used to make a statement and can be true or false – is a **proposition**. To explore how propositions are related to each other in reasoning, logic analyses their inner structure. **Propositional logic** analyses certain ways of combining propositions to form complex propositions. The expressions which are used to combine propositions are the connectives, discussed in Section 2.2. **Predicate logic** analyses the inner structure of simple propositions, which are formed of predicates and their arguments, discussed in Section 2.3, and may also contain quantifiers, which are discussed in Section 2.4.

2.2 THE LOGICAL CONNECTIVES

The logical connectives combine propositions to form more complex propositions in ways which correspond to certain uses of *and*, *or*, and *if*. We begin with conjunction.

2.2.1 Conjunction

Conjunction is expressed by certain uses of *and*, illustrated below.

(2) Moira left and Harry stayed behind.

In this sentence *and* joins the two sentences *Moira left* and *Harry stayed behind*. The whole sentence is true if both the joined sentences are true, and false otherwise. That is, it is false if they both left, or both stayed behind, or if Harry left and Moira stayed behind. This pattern holds for any two sentences joined by *and*: the truth value for the whole sentence depends on the truth values for the parts.

(3)a Alfred sings alto and Paul sings bass.
b There were lights showing and the door stood open.
c The airport was closed and all ferry trips were cancelled.

This general pattern is characteristic of logical connectives, which are **truth-functional**: the truth value for a complex proposition formed with a truth-functional connective can be calculated simply from the truth values of the joined propositions, without referring to the content of the propositions.

Most natural language expressions for connecting sentences are not truth-functional. The difference can be illustrated with *because*, as in (4).

(4)a Jill was late for work because her car broke down.
b Jill was late for work because she was caught in a traffic jam.

Suppose that Jill was late for work, her car broke down, and she was caught in a traffic jam, so the component propositions in (4a) and (4b) are true. In fact, Jill's car broke down long before she had to leave for work so she took a taxi, and if it hadn't been for the traffic jam she would have arrived on time, so (4a) is false and (4b) is true. We can't calculate the truth or falsity of (4a,b) simply by knowing whether or not the component propositions are true. We have to know the CONTENT of the propositions combined by *because* to judge whether or not the circumstances described in the *because*-proposition really caused the circumstances described in the other proposition. In short, the truth of a proposition with *because* depends on more than just the truth or falsity of the propositions which are combined, so *because* is not truth-functional.

Propositional logic deals with truth-functional expressions. Four of these, including conjunction, are connectives, because they connect two propositions. Propositional logic also deals with negation because it is truth-functional as we shall see below, although it does not combine propositions and therefore is not strictly a connective.

The conjunction connective is written with the symbol '&' or '∧'. The symbol '&' is used in this book. Propositions are represented by **propositional variables**, traditionally *p* and *q*, with *r* and *s* added if needed. Complex propositions formed by conjunction are also called conjunctions. Conjunctions in general are represented by the formula *p&q*, where *p* and *q* stand for any proposition.

We can list all the possible combinations of truth values for p and q and give the corresponding truth value for the conjunction p&q. In effect, this defines the meaning of the conjunction connective. Such a definition is given in the form of a table called a **truth table.**

(5) Truth table for conjunction

p	q	p&q
T	T	T
T	F	F
F	T	F
F	F	F

The order in which p and q are expressed makes no difference to the truth value. The propositions p&q and q&p are **equivalent**: p&q always has the same truth value as q&p, for any combination of truth values for p and q. This doesn't hold for all uses of the word *and*, but it does hold for the examples in (2) and (3), repeated here.

(6)a Moira left and Harry stayed behind.
Harry stayed behind and Moira left.

b Alfred sings alto and Paul sings bass.
Paul sings bass and Alfred sings alto.

c There were lights showing and the door stood open.
The door stood open and there were lights showing.

d The airport was closed and all ferry trips were cancelled.
All ferry trips were cancelled and the airport was closed.

The conjunction connective only connects propositions, expressed by sentences, but the word *and* can connect a wide range of types of expression. Some of the sentences in which *and* connects expressions smaller than sentences can be analysed as conjunction reduction, illustrated below. **Conjunction reduction** is a linguistic abbreviation for what is logically a conjunction of whole propositions.

(7)a [Moira and Harry] left.
b Tom saw [Moira and Harry].
c Moira was [changing her spark plugs and listening to talkback radio].

In (7a) and (7b) *and* connects two names, while in (7c) two verb phrases are joined. Sentences like these can be analysed as instances of linguistic abbreviation:

(8)a *Moira and Harry left* expresses 'Moira left and Harry left'.
b *Tom saw Moira and Harry* expresses 'Tom saw Moira and Tom saw Harry'.
c *Moira was changing her spark plugs and listening to talkback radio* expresses 'Moira was changing her spark plugs and Moira was listening to talkback radio'.

Not all uses of *and* to join non-sentential expressions can be analysed as conjunction reduction. The commonest exception is the use of *and* to form a complex noun phrase which refers to a group, as in these examples.

(9)a Sally and Harry met for lunch.
b Sally, Harry, Jeff and Buzz met for lunch.
c Harry, Jeff and Buzz surrounded Charles.
d The gang met for lunch.
e The forest surrounded the castle.

At first sight it looks as if (9a) could be analysed as a conjunction of propositions, 'Harry met Sally for lunch and Sally met Harry for lunch', but the other examples indicate that this won't work generally. In particular, (9d) indicates that it is the group as a whole which meets, and so the noun phrase *Sally, Harry, Jeff and Buzz* in (9b), for example, should be interpreted as referring to the whole group of people. Similarly, (9c) cannot be understood to mean 'Harry surrounded Charles and Jeff surrounded Charles and Buzz surrounded Charles', because 'Harry surrounded Charles' doesn't make sense – the three people as a group surrounded Charles. In these instances the word *and* is not a connective at all as it doesn't join sentences, but forms a complex noun phrase referring to a group, which as a whole performs the described action.

2.2.2 Negation

As above, negation is generally included with the logical connectives because it is truth-functional, being defined by a truth table. Simply, negation combines with a single proposition to reverse its truth value. The symbol for negation is '$\sim$'.

(10) Truth table for negation

p	~p
T	F
F	T

Negation is expressed in several ways in English, most commonly by *not* or *n't* after the first auxiliary verb. For example, if p represents the proposition expressed by *Moira left*, then $\sim p$ is expressed by *Moira didn't leave*. If 'Moira left' is true, then 'Moira didn't leave' is false, and if 'Moira didn't leave' is true, then 'Moira left' is false.

2.2.3 Disjunction

The **disjunction** connective corresponds to the use of the word *or* which is commonly glossed as 'and/or' or described as 'inclusive disjunction'. The symbol for disjunction is the lowercase letter *v*.

(11) Truth table for disjunction

p	q	pvq
T	T	T
T	F	T
F	T	T
F	F	F

A logical disjunction is true if either or both of the combined propositions is true.

Where two propositions joined by disjunction have some content in common, the sentence expressing the proposition (with the word *or*) is usually abbreviated in the form of **disjunction reduction**, much like conjunction reduction. For example, *That job will take two or three tins of paint, depending on the weather* is interpreted as 'That job will take two tins of paint or that job will take three tins of paint, depending on the weather.'

Inclusive disjunction ('and/or') corresponding to logical disjunction is illustrated in (12).

(12) You can get there by train or bus
('You can get there by train or you can get there by bus')

This sentence is true if there is a bus link and no train link, or a train link and no bus link, or both a bus link and a train link. On the most usual reading both are available, although in any single journey you will choose one or the other.

The sentence in (12) can also be understood to express exclusive disjunction, commonly glossed as 'either/or', Sentence (12) with exclusive disjunction would express something like 'You can get there somehow, either by train or by bus, (but I can't remember which)'. Exclusive disjunction is also illustrated in (13).

(13) The agent arrived in Berlin on the 9.25 or the 11.10.
('The agent arrived in Berlin on the 9.25 or the agent arrived in Berlin on the 11.10').

Here one or the other of the connected sentences is true, but not both. This sentence cannot be understood to mean that the agent arrived on both trains.

As the truth table indicates, logical disjunction is inclusive ('and/or', 'either or both'). The exclusive disjunction use of *or* ('either but not both') can be represented by adding the qualification 'but not both' to logical disjunction, for example:

(14) p = you take the money
q = you take the bag
Either you take the money or you take the bag = (pvq) & ~(p&q).

Note here that brackets are used to indicate which proposition, simple or complex, is combined by a particular connective or combined with negation. The disjunction pvq is itself the first part of the whole conjunction. Negation combines with the conjunction p&q, then the whole negative proposition ~(p&q) is the second part of the whole conjunction.

2.2.4 The Material Implication Connective

Conditionality is mainly expressed by certain uses of *if* or *if... then* – sentences with *if* are called **conditional sentences** or conditionals for short. There are two logical connectives corresponding to conditionals, the material implication connective and the biconditional connective, which is discussed in the next section. As we shall see, these two connectives only partly fit the usual ways we understand *if* sentences.

The material implication connective is represented by the symbol '→'. The proposition p in p→q is the **antecedent**, and q is the **consequent**. In a conditional sentence the antecedent is the sentence to which *if* is attached, although it may appear first or second in the whole sentence. For example, in both sentences in (15) the antecedent is *if Marcia invited John/him* and the consequent is *John/he will go*.

(15)a If Marcia invited John, (then) he'll go.
b John will go if Marcia invited him.

The main point with implication is that where the antecedent is true, the consequent must also be true. If the antecedent is true and the consequent is false, then the whole implication is false. So the first two lines of the truth table for implication are:

p	q	p→q
T	T	T
T	F	F

The remaining lines of the truth table, where the antecedent is false, are not so clearly related to ordinary uses of *if*. Where the antecedent is false the implication is true no matter what the truth value of the consequent, as shown in the full truth table below.

(16) Truth table for material implication

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

Using the example in (15), the lines of the truth table give these truth values:

(17) p = Marcia invited John
q = John will go
p→q = If Marcia invited John, he'll go

Line 1: Marcia did invite John and John will go: the implication is true.

Line 2: Marcia did invite John, but actually John won't go: the implication is false.

Line 3: Marcia didn't invite John, but he will go anyway: the implication is true.

Line 4: Marcia didn't invite John and John won't go: the implication is true.

Lines 1 and 2 give the results we would expect from the ordinary use of *if*. Line 3 seems odd. If John will go (to some understood destination) whether Marcia invited him or not, why bother to say 'if Marcia invited John' at all? All that is communicated here is 'John will go'. In fact, an utterance of *If Marcia invited John, he'll go* is more likely to be intended to mean 'If Marcia invites John he'll go, but not otherwise' – explicitly, 'If Marcia invited John he'll go, and if she didn't invite him, he won't go'. On this reading the whole sentence on line 3 should be false. This use of *if* is more like the biconditional connective, to be reviewed in Section 2.2.5.

The chief general difference between material implication and conditional sentences is that *if* is commonly not simply truth-functional in actual use. Given that material implication is truth-functional, the truth of an implication proposition depends only on a certain combination of truth values for the contained propositions, and the actual content or subject matter of those propositions is irrelevant. Logically, (18) expresses a perfectly fine (and true) implication, but it is odd as a conditional sentence.

(18) If the number 1960 is divisible by 5 then 1960 was a leap year.

antecedent (1960 is divisible by 5)	true
consequent (1960 was a leap year)	true
implication	true

But many of us would dispute the truth of (18), because we don't calculate leap years by dividing by five. The problem here is that we frequently use *if... then* to express some **causal** relationship between the antecedent and consequent – the antecedent describes some event or state of affairs which causes what is described by the consequent – in other words, the consequent describes the consequences. Sentence (18) reads most naturally as stating that the status of 1960 as a leap year depends on the year's number being divisible by five, whereas in fact divisibility by four is the criterion for leap years. For the implication to be true, it is sufficient that the antecedent and consequent are both true. For the conditional sentence to be true as we normally understand it, the status of 1960 as a leap year would have to depend on, or be caused by, the fact that the number 1960 is divisible by 5.

These uses of *if* carry extra aspects of meaning, such as causality, but note that they also include the truth-value combinations given by the first two lines of the truth table. Even with the causal use of *if*, if the antecedent is true the consequent must also be true. For example, *If the number 1960 is divisible by 4 then 1960 was a leap year* expresses the causal connection accurately. In addition, given that the antecedent is true, the conditional is true only if the consequent is also true and 1960 was a leap year – if 1960 was not a leap year the conditional is false. That is, the causal meaning associated with *if* is extra content added to the meaning of logical implication.

There is a common rhetorical use of *if... then* that fits well with the logical analysis, requiring no causal or commonsense connection between the sentences, as illustrated in (19).

(19) If that's a genuine Picasso then the moon is made of longlife food product

p = That's a genuine Picasso
q = The moon is made of longlife food product.

Assume (by conversational conventions discussed in Chapter 11) that a sentence like (19), when uttered, is taken as being true. The rhetorical device requires that the consequent be obviously false. This gives the combination of values:

(20)

p	q	p→q
?	F	T

Checking the truth table for implication, repeated here, we see that this combination of truth values occurs only on line 4, where the antecedent is false.

p	q	p→q
T	T	T
T	F	F
F	T	T
F	F	T

So this rhetorical device is used to convey that the antecedent is false. Here, (19) is used to convey that that's not a genuine Picasso. Routines of this form include the cliché *if... I'll eat my hat.*

The extra aspects of meaning found with *if*, such as causality, are generally analysed as a layer of meaning which is added to the literal meaning of *if* by pragmatic processes. Sentence conjunction with *and* is also commonly interpreted with extra pragmatic content, in addition to the truth-functional meaning of logical conjunction, which is assumed to be the core literal meaning of *and*. This issue is discussed further in Chapter 11. Other kinds of conditional are also discussed in Chapter 3.

2.2.5 Equivalence and the Biconditional Connective

The biconditional connective, represented by the symbol '↔' or '≡', expresses the relation of equivalence between propositions. Two propositions are equivalent if, in any given circumstances, they have the same truth value, either both true or both false. Accordingly, the biconditional p↔q is true if p and q have the same truth value, and otherwise false.

(21) Truth table for equivalence/biconditional connective

p	q	p↔q
T	T	T
T	F	F
F	T	F
F	F	T

The corresponding English expression is *if and only if*, often abbreviated in writing to *iff*. Unlike *and*, *or* and *if*, these paraphrases are not common English expressions, being largely confined to 'philosopher talk'. This relation is commonly used in technical contexts, particularly in statements of truth conditions:

(22)a 'Snow is white' is true if and only if snow is white.
b 'Schnee ist weiss' is true if and only if snow is white.
c 'La neige est blanche' is true if and only if snow is white.

In (22b), for example:

p = 'Schnee ist weiss' is true
q = snow is white

As we saw in Chapter 1, the truth set for q (the set of all possible worlds in which q is true) is the set of all the possible and actual circumstances of snow being white. The whole biconditional in (22b) asserts that 'Snow is white' and '"Schnee ist weiss" is true' are either both true or both false in all circumstances, therefore in all possible worlds. Obviously the truth set for '"Schnee ist weiss" is true' is the same as the truth set for 'Schnee ist weiss'. In effect, the sentence 'Schnee ist weiss' is true in exactly the same possible worlds as 'Snow is white', and therefore it has exactly the same truth set, or intension. In short, it has the same meaning.

The truth table for equivalence also appears to be part of the meaning of *if* in uses like (15), repeated here as (23a), and (23b), where the rider 'but not otherwise' is understood.

(23)a If Marcia invited John he'll go.
b If you kick me again I'll punch you.

Take (23b):

p = you kick me again
q = I punch you

line 1: you kick me again and I punch you	true
line 2: you kick me again but I don't punch you	false
line 3: you don't kick me but I punch you anyway	false
line 4: you don't kick me and I don't punch you	true

What is added to the equivalence relation in this use of *if* is the notion of temporal sequence and causality – that is, the kicking happens before the punching and also causes the punching, as the punching is in response to the kicking. For example, line 1 would also apply if I punched you first and you kicked me in response, but the sentence *If you kick me again I'll punch you* is not understood in this way. As with other uses of *if* mentioned above, the extra content here, in addition to the truth-functional content, is generally considered to be pragmatic.

For review of this section, see the exercises in Part A at the end of the chapter.

2.3 PREDICATES AND ARGUMENTS

The internal structure of the most simple kind of proposition, an **atomic proposition**, consists of a predicate and its argument or arguments. We begin with so-called two-place predicates as an illustration.

(24)a Brigitte is taller than Danny.
b Alex is Bill's henchman.
c Fiji is near New Zealand.

All of these sentences express a relationship between two entities. If we take out the expressions which refer to entities, we are left with the part that expresses the relationship – this part expresses the **predicate**.

(25)a ... is taller than ...
b ... is ...'s henchman.
c ... is near ...

In each of these sequences there is one main word which on its own indicates the nature of the relationship, or the content of the predicate. In the notation to be used here the symbol for the predicate is based on the main word, omitting tense, copula *be*, and some prepositions. The entities bound in a relationship by the predicate are its **arguments**, referred to in these examples by names. By convention, names are represented by lowercase letters. The formulae for the sentences in (24) are

(26) TALLER (b, d)
HENCHMAN (a, b)
NEAR (f, n)

These examples illustrate some of the main points about logical predicates.

First, predicates are semantically 'incomplete' if considered in isolation. It isn't possible to paraphrase or explain the meaning of one of these predicates without including the notion of there being two entities involved in any situation where the predicate applies.

Secondly, each predicate has a fixed number of arguments. These predicates must have exactly two arguments to form a coherent proposition – no more and no fewer – hence they are two-place predicates. The argument 'slots' are part of the predicate's meaning.

Predicates are commonly used elliptically in natural language, with one of the arguments not explicitly mentioned. For example, one might say simply 'Brigitte is taller' or 'Alex is a faithful henchman'. But the second, unmentioned argument in elliptical utterances like these is still understood in the expressed proposition. If Danny is a subject of conversation, 'Brigitte is taller' can be interpreted to mean 'Brigitte is taller than Danny'. In another context, it may be interpreted to mean that Brigitte is taller than she used to be. It isn't possible to be taller in isolation without being taller than some comparison standard, and it isn't possible (in modern English) to be a henchman without being someone's henchman. The second argument is still understood to be present in the proposition expressed.

The elliptical use of predicates found in natural language is not well formed in logic, and both the arguments of a two-place predicate must be represented in a logical formula. Although (27a,b) below can communicate complete propositions (because we can usually understand from the context what elements have been ellipted), (27a,c) are not well formed, and don't express propositions. Logical formulae themselves cannot be elliptical.

(27)a Brigitte is taller.
b Alex is a faithful henchman.
c TALLER(b)
d HENCHMAN(a)

The unmentioned argument is usually clearly identified from the context. Even if no context is supplied with an example to give this information, the argument position can be filled by a general term, for now, *someone* or *something*, as in (28).

(28)a TALLER (b, someone)
b HENCHMAN (a, someone)

The third point about predicates is that the order of the arguments in the formula is significant. Generally (but not always) the order of arguments in a logical representation is taken from the order of the corresponding expressions in the sentence, for example:

(29)a Brigitte is taller than Danny.
TALLER(b, d)

b Danny is taller than Brigitte.
TALLER(d, b)

The predicates we looked at in Chapter 1, such as *dog*, *brown* and *barks*, are all one-place predicates. The most basic subject+predicate sentence of traditional grammar contains a one-place predicate, with the subject of the sentence expressing its single argument, as in (30).

(30)a	Zorba was Greek.	GREEK(z)
b	Moby Grape is purple.	PURPLE(m)
c	Perry is a lawyer.	LAWYER(p)
d	Cyrus coughed.	COUGH(c)

Note that one- and two-place predicates can be expressed by a range of lexical categories, as illustrated in (31).

(31)	adjective:	TALL PURPLE GREEK TALLER
	preposition:	NEAR ON BESIDE
	noun:	LAWYER DOG CORACLE
	verb:	COUGH SEE READ

Three-place predicates (and four-place predicates, if there are any) are expressed by verbs, and perhaps by nouns derived from verbs.

Three-place predicates are commonly expressed by so-called double object verbs, for example:

(32)a	Richard gave Liz a diamond.	(double object)
b	Richard gave a diamond to Liz.	
c	Marcia showed Clive the ad	(double object)
d	Marcia showed the ad to Clive	

other three-place verbs: *tell*, *teach*, *send*, *pass*, *offer*, etc.

Although the two sentences in each pair have different word order, they have the same meaning. For examples like these, one word order must be chosen as the basis for the order of arguments in the logical representation – in this case, the order in (32b, d) is used, and the formulae are as in (33).

(33)a	GIVE (r, a diamond, l)	for (29a, b)
b	SHOW (m, the ad, c)	for (29c, d)

Here two of the arguments are expressed by noun phrases which are not names – *a diamond* and *the ad*. Noun phrases like these are analysed in more detail in the next section and in Chapter 4.

There may not be any real four-place predicates in natural language, although in principle there is no limit on how many arguments a predicate can have. The reason for uncertainty over four-place predicates is covered in the next section.

A couple of likely candidates for four-place predicates are *buy* and *sell*.

(34)a Marcia sold the car to Clive for $200.
SELL(m, the car, c, $200)

b Clive bought the car from Marcia for $200.
BUY(c, the car, m, $200)

2.3.1 Predicates, Verbs and the Number of Arguments

As we saw earlier, every predicate has a fixed number of arguments which must be present in a well-formed proposition, and accordingly, a logical form must represent all the arguments of each predicate. Natural language allows for elliptical forms like those in (27a, b), where an argument of the predicate need not be expressed in the sentence, although its presence in the proposition is still understood. Other examples of ellipsis are in (35).

(35)a	Will you pour out?	(the tea)
b	I gave at the office	(money, to your charity)
c	Add meat to pan and sauté lightly.	(you, the meat)

On the other hand, there is a general axiom in syntactic theory that all syntactic arguments of verbs (and possibly of other predicates) are obligatory, and must be expressed in a well-formed sentence. Ellipsis is a special exception to this general rule. This principle may be used to test whether or not a phrase is an argument of the verb, for example:

(36)a Al put the groceries away/on the bench.
b * Al put the groceries.

The asterisk before (36b) indicates that the sentence is ill-formed. A sentence with the verb *put* requires a locative phrase expressing where something is put. Sentence (36b) lacks a locative expression and is ill-formed, in contrast to (36a). This is generally taken as evidence that the locative expression is obligatory with *put* and therefore is an argument of *put*. Roughly, an expression which can be omitted without making the sentence ill-formed is not an argument of the predicate. The converse is illustrated in (37).

(37)a We planned the weekend the other night.
b We planned the weekend.

Of the two noun phrases *the weekend* and *the other night*, both referring to intervals of time, only the first is an argument of *plan*. The second noun phrase can be left out, and is not an argument.

The general principle that the syntactic arguments of verbs are obligatory has a number of apparent counter-examples falling into two main groups.

The first group are elliptical sentences. A possible strategy for at least some of these sentences is to include a sort of 'silent pronoun' in the syntactic structure of the sentence to fill the argument slot. This allows the obligatory argument principle to be maintained, as the silent pronoun counts as an expression of the argument in question, even though it is not pronounced. What it refers to is provided by the context, as is commonly the case with pronouns like *he*, *she*, *they*, etc.

Counter-examples of the second kind show what is called **variable adicity**. The **adicity** of a predicate is the number of arguments it takes, derived from the terms *monadic* (= one-place), *dyadic* (= two-place), *triadic* (= three-place), and so on. Verbs with variable adicity seem to have variable numbers of arguments in different sentences, for example:

(38)a	<u>They</u> showed <u>the film</u> to <u>the censor</u> on Tuesday	3
b	<u>They</u> showed <u>the film</u> on Tuesday	2
c	<u>He</u> served <u>the soup</u> to <u>the guests</u> first.	3
d	<u>He</u> served <u>the soup</u> first.	2
e	<u>He</u> served <u>the guests</u> first.	2
f	<u>She</u> wrote <u>a letter</u>.	2
g	<u>She</u> wrote him <u>a letter</u>.	3
h	<u>She</u> made <u>a sandwich</u>.	2
i	<u>She</u> made <u>him</u> <u>a sandwich</u>.	3

Discussing data like these, linguists refer informally to optional arguments, although strictly speaking an argument is obligatory by definition. Indispensability is part of what it is to be an argument.

An alternative is to maintain that all arguments are indeed obligatory, and that the sentence groups above do not contain the same verb – for example, the verb *show* in (38a), which has three arguments, is not the same as the two-argument verb *show* in (38b). Although this option protects the obligatoriness principle, it conflicts with the common intuition that the sentence groups do contain the same verb, and it carries the consequence that many common verbs must be classed as highly ambiguous.

For the present purposes, phrases which appear to be so-called optional arguments, in that they are argument-like in meaning but can be omitted, will be analysed as arguments in logical forms.

We are uncertain about the existence of four-place predicates because verbs like *buy* and *sell* have variable adicity.

(39)a Marcia sold the car to Clive for $200.
b Marcia sold the car for $200.
c Marcia sold the car to Clive.
d Marcia sold the car.
e Clive bought the car from Marcia for $200.
f Clive bought the car for $200.
g Clive bought the car from Marcia.
h Clive bought the car.

With *sell* the buyer and the price can be omitted, and with *buy* the seller and the price can be omitted, even though these entities must be present in a buying or selling event. The meaning of *sell* must include the notion of payment, otherwise it isn't distinguishable from *give*. Similarly, the exchange of payment differentiates *buy* from *take* or *receive*.

There is an important difference between the missing arguments in (39) and the ellipsed arguments in (28) and (35), which is that in general, ellipsed arguments are specifically identified from the general context. In contrast to this, the unspecified price in (39c, g), the buyer in (39b, d) and the seller in (39f, h) are understood to exist, but nothing else need be known about them. This raises doubts as to whether or not the omissible phrases in (39) are arguments at the syntactic level.

The main predicate and its arguments comprise an atomic proposition. Ordinary natural language sentences generally contain more than this, for example:

(40)a Seymour will slice the salami carefully in the kitchen tomorrow to make the canapés.
b SLICE (s, the salami)

The basic logical representation introduced so far shows only the main predicate, SLICE, and its two arguments, the slicer and the slicee. We omit the tense marker *will*, the manner adverbial *carefully*, the locative adverbial *in the kitchen*, the temporal adverbial *tomorrow*, and the adverbial clause of purpose *to make the canapés*. Tense is covered in Chapter 7 and adverbials in Chapter 8. Until then they will be omitted in logical forms.

2.3.2 Sentences as Arguments

All the arguments in the discussion so far have been expressed by noun phrases, but arguments can also be expressed by sentences themselves, for example:

(41)a Clive said something
SAY (c, something)

b Clive said [that he gave the car to Marcia]
SAY (c, (GIVE (c, the car, m))

c Clive thinks [the earth is flat]
THINK (c, (FLAT (the earth))

In (41b, c) the proposition expressed by the embedded sentence is the second argument of the main verb – the proposition is what is said or what is believed. (Sentences about thinking and believing are discussed in Section 5.5.)

The clearest examples of sentential arguments are found with verbs, but plausibly members of other word classes can also have sentential arguments.

(42)a Shirley was proud [of the new car]
b Shirley was proud [that she graduated]
c Shirley was proud [to be Miss Lada 1993]

In (42a) it seems that the new car, the source of Shirley's pride, is the second argument of *proud*. In (42b, c) the embedded sentence also expresses the source of pride, the second argument of *proud*, and so the propositions can be represented as in (43).

(43)a PROUD (s, the new car)
b PROUD (s, (GRADUATE(s))
c PROUD (s, (MISS LADA 1993(s))

A sentential argument may also be the only argument of the main predicate in a sentence, for example:

(44)a [That Clive drove the car] is obvious
b It is obvious [that Clive drove the car]
c OBVIOUS (DRIVE (c, the car))

For a review of this section, see the exercises in Part B at the end of the chapter.

2.4 THE LOGICAL QUANTIFIERS

2.4.1 The Universal Quantifier

The atomic propositions we have discussed so far have had **individual arguments**, referred to by names or noun phrases like *the dog*, with logical forms like these:

(45)a	John saw Mary	SEE (j,m)
b	Fido was barking	BARK (f)
c	The dog was barking	BARK (the dog)

Propositions with **quantified arguments** rather than individual arguments (that is, **quantified propositions**) must be treated differently. Take the example below.

(46) God made everything.

The chief point here is that *God* and *everything* have very different kinds of meaning. While *God* is a name referring to an individual, *everything* doesn't refer to a thing, but rather has the potential to refer to any thing considered individually. Suppose it were possible to point to each thing in existence in turn and say 'God made that'. According to (46) each utterance would be true, and (46) can be analysed as a sort of summary of all those propositions about different individuals.

Pronouns like *that*, *this* and *it* are referring expressions which can in principle refer to any individual depending on the circumstances in which they are used – they have **variable reference**, in contrast to names, which have **constant reference**. The logical terms used to translate names are **individual constants**. To analyse quantified propositions we need **individual variables**, comparable to pronouns. Individual variables are traditionally written as *x*, *y* and *z*, with *u*, *v* and *w* added if needed. Like *this* and *that*, individual variables can in principle refer to any individual at all, depending on the context. With an individual variable the logical form for *God made that* can be written as

(47) MAKE (g, x)

As it stands, (47) does not fix the reference for x and so doesn't express a proposition, as it cannot be true or false.

The universal quantifier, written as '∀', fixes how the variable is to be interpreted. The whole logical form for *God made everything* is (48).

(48) ∀x (MAKE (g, x))

The quantifier is paired with a copy of the variable which is its target, and the formula with which the quantifier combines is bracketed to fix the scope of the quantifier. (48) can be read as (49).

(49) For any value of x, God made x
For all values of x, God made x
Whatever x may be, God made x

Here the universal quantifier fixes the value of x as every thing, taken individually. The quantifier **binds** the variable, which is accordingly a **bound variable** in the whole formula '∀x(MAKE (g, x))'. A variable which is not bound by a quantifier is a **free variable**. The x variable is free in the basic formula 'MAKE (g, x)'. A proposition form with a free variable, such as 'MAKE (g, x)', stands for an **open proposition**. An open proposition by itself is incomplete and cannot have a truth value. A formula with no free variables stands for a **closed proposition**, which is complete and has a truth value. Because there are no free variables in (48) it stands for a closed proposition.

Noun phrases expressing universal quantification are usually more complex than *everything*, as in (50).

(50)a Now is the time for <u>all good men</u> to come to the aid of the party.
b <u>Every cloud</u> has a silver lining.
c <u>Every dog</u> is barking.

Take the significance of *dog* in (50c). Suppose 'Every dog is barking' is true. Now you point to each thing in turn and say 'that is barking'. This time the utterance will be false on many pointings, but for any pointing to a dog it will be true. In other words, if the thing pointed to is a dog then 'that is barking' is true. So for 'Every dog is barking', the pointing exercise goes with the utterance of 'If that is a dog then it is barking', and the logical form for (50c) is (51), using the implication connective.

(51) ∀x(DOG(x) → BARK(x))
'For every thing x, if x is a dog then x is barking'

The logical universal quantifier does not express **existential commitment** – that is, a sentence like *Every dog is barking* can be true on the logical analysis even when there are no dogs.

On the logical analysis, if there are no dogs 'Every dog is barking' is true, because the antecedent 'DOG(x)' will be false for any value of x. If there are no dogs, only lines 3 and 4 of the truth table for implication will come into play.

(52)

	DOG (x)	**BARK (x)**	**DOG (x) → BARK (x)**
1	T	T	T
2	T	F	F
3	F	T	T
4	F	F	T

On the logical analysis 'Every dog is barking' is equivalent to 'There is no non-barking dog'.

A universally quantified noun phrase in object position or other sentence positions is analysed in the same way, for example:

(53)a Bill hates all reporters.
$\forall x(\text{REPORTER}(x) \rightarrow \text{HATE}(b, x))$
'For all x, if x is a reporter then Bill hates x'.

b Clive gave a bone to every dog.
$\forall x\ (\text{DOG}(x) \rightarrow \text{GIVE}(c, \text{a bone}, x))$
'For all x, if x is a dog then Clive gave a bone to x'.

c The book was signed by every guest.
$\forall x\ (\text{GUEST}(x) \rightarrow \text{SIGN}(x, \text{the book}))$
'For all x, if x is a guest then x signed the book'.

The expressions *a bone* and *the book* are not fully analysed here, but the way we represent NPs like these will be revised.

2.4.2 The Existential Quantifier

The other logical quantifier, the existential quantifier, is written as '$\exists$' and used to translate noun phrases with *a/an* or *some* and for *there is* sentences. The sequence '$\exists x$' is read as 'there is an x' or 'there is at least one thing x'.

Unlike the universal quantifier, the existential quantifier does explicitly express existential commitment. An existential sentence states the existence of at least one thing of the kind specified, for example:

(54)a A dog barked.
'There is at least one thing x such that x is a dog and x barked'.
$\exists x\ (\text{DOG}(x)\ \&\ \text{BARK}(x))$

b There is an antidote to Huntsman venom.
$\exists x\ (\text{ANTIDOTE}(x, h))$

c Some birds were singing.
$\exists x\ (\text{BIRD}(x)\ \&\ \text{SING}(x))$

d A black limousine awaited Marla.
$\exists x\ (\text{LIMOUSINE}(x)\ \&\ \text{BLACK}(x)\ \&\ \text{AWAIT}(x, m))$

e Louise bought some trashy paperbacks.
$\exists x\ (\text{TRASHY}(x)\ \&\ \text{PAPERBACK}(x)\ \&\ \text{BUY}(l, x))$

As these examples show, the existential quantifier is neutral between singular and plural. Note that unlike the universal quantifier, the existential quantifier is not analysed with '$\rightarrow$'.

The determiner *no* is analysed with the existential quantifier and negation, as in (55).

(55)a There is no antidote to cyanide.
$\sim \exists x\ (\text{ANTIDOTE}(x, c))$
'It is not the case that there is an x such that x is an antidote to cyanide'.
For short, 'there is no x such that...'

b Clive ate nothing.
$\sim \exists x\ (\text{EAT}(c, x))$
'There is no x such that Clive ate x.'

In sentences like (55) the negation cancels the existential quantifier's guarantee of the existence of a thing of the kind described. To affect the interpretation of the existential quantifier in this way, the negation must appear before the quantifier. Reversing the order of the existential quantifier and negation gives a different meaning, as in (56).

(56) $\exists x \sim (\text{EAT}(c, x))$
'There is at least one thing x such that Clive didn't eat x.'

As we saw above, 'Every dog is barking' is equivalent to 'There is no non-barking dog'. For any universally quantified proposition there is an equivalent existentially quantified proposition, and vice versa, as shown in (57) and (58). (57a) is equivalent to (57b), and (58a) is equivalent to (58b).

(57)a $\forall x\ (\text{DOG}(x) \rightarrow \text{BARK}(x))$
'For every x, if x is a dog then x is barking.'

b $\sim \exists x\ (\text{DOG}(x)\ \&\ \sim\text{BARK}(x))$
'There is no x such that x is a dog and x is not barking'.

(58)a $\exists x\ (\text{DOG}(x)\ \&\ \text{BARK}(x))$
'There is an x such that x is a dog and x is barking.'

b $\sim \forall x\ (\text{DOG}(x) \rightarrow \sim \text{BARK}(x))$
'It is not the case that for all x, if x is a dog then x is not barking'

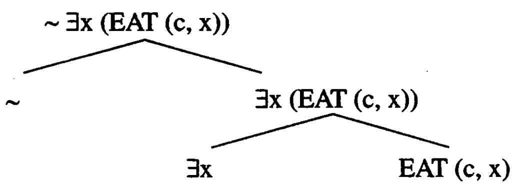
2.4.3 Scopal Ambiguity

The examples above show that the relative order of negation and a quantifier is significant. This was first mentioned with (55b) and (56), repeated in (59).

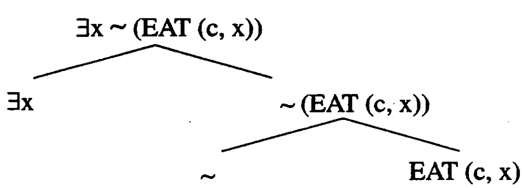
(59)a $\sim \exists x\ (\text{EAT}(c, x))$
'Clive ate nothing'.

b $\exists x \sim (EAT(c, x))$
'There is at least one thing Clive didn't eat.'

These examples illustrate a more general point. The internal structure of a simple proposition always contains at least the predicate and its arguments. With quantifiers and negation, we add to the logical forms symbols which combine with a propositional form as a whole. This is represented by placing the symbols for negation and quantification at the beginning of the propositional form. Expressions like negation and quantification are **scopal expressions**. The interpretation of a scopal expression is combined with, or affects, the interpretation of the whole proposition it combines with, which is its **scope**. This is illustrated in (60) and (61).

(60)


In (60) we say the quantifier is in the scope of negation, or the quantifier has narrow scope with respect to negation, or negation takes scope over the quantifier.

(61)


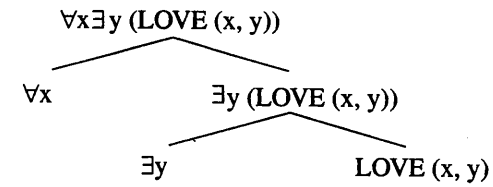
In (61) we say the quantifier has wide scope with respect to negation, or negation has narrow scope.

The representations in (60) and (61) are for the propositions expressed by the sentences *Clive ate nothing* and *There is something Clive didn't eat*, respectively. From the form of the sentences, it is clear which meaning is expressed, and which order is required for negation and the quantifier.

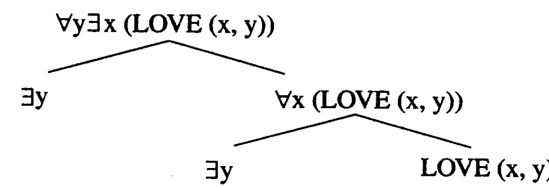
A sentence with two or more scopal expressions doesn't always have a clearly identified single meaning, and is commonly ambiguous between the readings for the different possible scopes – this is called **scopal ambiguity**. Scopal ambiguity with two quantifiers is illustrated in (62).

(62) Everyone loves someone.

The subject *everyone* is represented with a universal quantifier, and the object *someone* with an existential quantifier. There are two possible orders for the quantifiers, with different readings, as shown in (63).

(63)a


'For every person x, there is at least one person y such that x loves y.'

b


'There is at least one person y such that everyone loves y.'

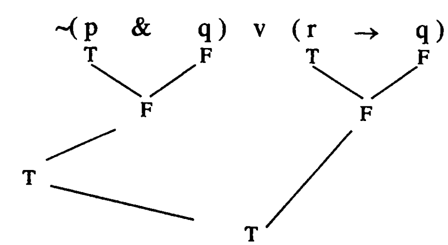
Both of these logical forms represent possible readings of the English sentence, which is ambiguous, depending on which scope is assigned to the scopal expressions, the quantifiers *everyone* and *someone*. A sentence can be ambiguous, but a logical form represents a particular meaning and cannot be ambiguous. An ambiguous sentence is associated with two (or more) logical forms.

EXERCISES

SECTION I: THE LOGICAL CONNECTIVES

(A) Basic Review

Assuming that p is true, q is false and r is true, calculate the truth values for the following formulae. Here is an example: $\sim (p\&q) \vee (r \rightarrow q)$



(1) $(p\&q) \rightarrow q$
(2) $(p \vee (q\vee r)) \rightarrow (p \& (q\vee r))$
(3) $(p\&r) \leftrightarrow (\sim r \vee \sim q)$
(4) $(p\rightarrow q) \& (p\&r)$
(5) $q \rightarrow ((r\vee p) \leftrightarrow (r\&p))$
(6) $((p \leftrightarrow q) \leftrightarrow (r\leftrightarrow q)) \vee \sim r$
(7) $r \rightarrow ((\sim q \& p) \vee ((q \rightarrow r) \vee \sim (r\leftrightarrow p)))$
(8) $((r\vee q) \leftrightarrow (q \rightarrow ((q\&r) \vee p))) \rightarrow \sim r$
(9) $((r\&\sim q) \rightarrow (p\vee\sim r)) \leftrightarrow ((r\vee\sim r) \& (p\leftrightarrow\sim q))$
(10) $((\sim r \rightarrow p) \& (p \rightarrow\sim q)) \& ((q\leftrightarrow\sim p) \& (r \rightarrow q))$

(B) Truth Tables

Construct the truth table for $\sim(p\vee q) \leftrightarrow (\sim p\&\sim q)$.
What can you conclude from this about $\sim (p\vee q)$ and $\sim p\& \sim q$?

(C) Truth Tables

Construct the truth tables for $\sim p \vee q$ and $\sim q \rightarrow\sim p$.
There is a formula with only three symbols which is equivalent to both of these – what is it?

(D) Conditionals and Implications

According to the truth table for implication, what are the truth values for the sentences below? Do these truth values fit with your normal intuitions about these sentences?

(1) If Baltimore is in Singapore then Elvis is dead.
(2) If Baltimore is in Singapore then Elvis is alive.
(3) If Ireland is surrounded by sea then it is an island.
(4) If Ireland is connected to Wales then it is an island.
(5) If Cain killed Abel then Abel is dead.
(6) If Abel isn't dead then Cain didn't kill him.
(7) If humans walk upright then they can use their hands to carry things.
(8) If humans can't use their hands to carry things then they don't walk upright.

(E) Truth Tables

(1) The formula

$(\sim q \& \sim r) \leftrightarrow (r \vee (p\&q))$

is true on only one combination of truth values for p, q and r. What are the values for p, q and r on which the formula is true? (Construct the truth table for the formula to find this out.)
(2) There is a simple formula which just says that the truth values for p, q and r are the values you found as the answer to (1). Can you write this formula? What is the relationship between this formula and the formula in (1)?

SECTION II: PREDICATES AND ARGUMENTS

(F) Basic Review

Give the logical forms for the following sentences.

(1) John gave ten dollars to Mary.
(2) Mary was given ten dollars by John.
(3) Toby was under the table.
(4) Clive showed Maddy the photos.
(5) China is east of Europe.
(6) Sheila is a surgeon.
(7) Max, Clyde and Damien partnered Latoya, Gina and Britt respectively.
(8) Jerry is Ben's brother.
(9) Paul is the brother of Sheila.
(10) Jerry and Ben are brothers.
(11) Clive and Marcia embraced.
(12) Bill was painting in the kitchen.
(13) Bill was painting the kitchen.
(14) Mary finally bought the painting yesterday.
(15) John sat in the chair.
(16) John sat in the hall.
(17) Jason picked at his food.
(18) Clyde told Tom that Bill had left.
(19) 'Bill has left', Clyde told Tom.
(20) 'Ouch!' said Sarah.

(G) Adding Connectives

Give the logical forms for the following sentences. Some of the sentences are ambiguous. For each ambiguous sentence give two logical forms showing the different readings.

Example: Dorothy saw Bill or Alan.
SEE (d, b) v SEE (d, a)

(1) Either Sydney or Canberra is the capital of Australia.
(2) Audrey went to Motueka and visited Rangi or interviewed Cameron.
(3) Alice didn't laugh and Bill didn't either.
(4) Alice didn't laugh and nor did Bill.
(5) Neither Bill nor Alice laughed.
(6) Frank is not both rich and generous.
(7) If Adam trusts Eve he's stupid.
(8) Sue will be rich if Lenny dies.
(9) If David is Audrey's brother then Fanny's his aunt or Bob's his uncle.
(10) Claire will hire Burt and Ethel will resign if Lenny leaves Taiwan.

(H) *Only if*: Discussion

Compare the uses of *only if* in the sentences below. Does *only if* have a constant meaning? Write the logical forms for the sentences.

(1) Combustion occurs only if oxygen is present.
(2) Bill will leave only if Mary resigns.
(3) Bill will leave only if Mary doesn't resign.
(4) Mary will resign only if Bill leaves.

SECTION III: THE LOGICAL QUANTIFIERS

(I) Basic Review

Using ∀ and ∃ where appropriate, write logical forms for the sentences below.

(1) A young woman arrived.
(2) Ida saw something sinister.
(3) All roads lead to Rome.
(4) Utopia welcomes all travellers from Spain.
(5) There's a castle in Edinburgh.
(6) Someone murdered Clive.
(7) Clive got murdered.
(8) The boat got sunk.
(9) The boat sank.
(10) Nobody saw Charles.
(11) Maxine sent every letter John had written her to Ruth.
(12) Gina or Boris fed every puppy.

(J) Negation

Using ∀ and ∃, write logical forms for the sentences below. If the sentence is ambiguous, give a form for each reading.

(1) Everyone doesn't like Bob.
(2) Not everyone likes Bob.
(3) Bob doesn't like everyone.
(4) Bob doesn't like anyone.

(K) Adding Connectives

Using ∀ and ∃, write logical forms for the sentences below. Note that (6) is ambiguous so there are two forms for it.

(1) Grammar A generates all and only well-formed formulae.
(2) Clive gave every child a biscuit or a Batman comic.
(3) Zoe read all the death notices but nothing else.
(4) There's no business like show business. (Treat *show business* as a name.)
(5) Chairman Miaou is heavier and meaner than any spaniel.
(6) Every prize was won by some high school kid.

FURTHER READING

Allwood, Andersson and Dahl's (1977) *Logic in Linguistics* is an accessible introduction to first order logic and is particularly recommended.