Elizabeth Coppock
coppock@phil.hhu.de
Wed. November 30th, 2011
Time: 14:30–16:00

Compositional Semantics
Heinrich Heine University
Winter Semester 2011/12
Room: 25.22-U1.72

Top Down and Bottom Up Composition

1 Notes on Notation and Terminology

Denotation brackets

How do you make denotation brackets? Several options:

1. ⟦[$_{\text{NP}}$ [$_{\text{D}}$ **the**] [$_{\text{N}}$ **elevator**]]⟧
2. ||[$_{\text{NP}}$ [$_{\text{D}}$ **the**] [$_{\text{N}}$ **elevator**]]||
3. [[[$_{\text{NP}}$ [$_{\text{D}}$ **the**] [$_{\text{N}}$ **elevator**]]]]
4. **[** [$_{\text{NP}}$ [$_{\text{D}}$ **the**] [$_{\text{N}}$ **elevator**]] **]**

If you choose the third option, then it gets really really confusing unless you put a space in between the tree bracket and the denotation bracket! If your word processor does not support option 1, then maybe the best plan is to go with option 2.

Notice that I am putting the object language in **bold**, because as we discussed before, ⟦**Ann**⟧ = Ann, whereas ⟦Ann⟧ is nonsense. But it is not a terrible crime to use normal font for object language.

Trees, not strings, inside denotation brackets!

Bad: ⟦**the negative square root of 4**⟧

Good: ⟦[$_{\text{NP}}$ [$_{\text{D}}$ **the**] [$_{\text{N}}$ [$_{\text{A}}$ **negative**] [$_{\text{N}}$ **square root**] [$_{\text{PP}}$ [$_{\text{P}}$ **of**] [$_{\text{NP}}$ **4**]]]]]⟧

Wait, aren't I being hypocritical? Why did I write things like:

- ⟦**opera by Beethoven**⟧

Because I was tacitly assuming that *opera by Beethoven* was a **lexical item**! That *is* cheating, but I'm not being hypocritical!

Two senses of "functional application"

This is the really confusing part. *Functional application* has two meanings!

1. The process of applying a function to an argument (a.k.a. "β-reduction")
2. The *composition rule* that allows us to compute the semantic value of a *phrase* given the semantic values of its parts:

   Functional Application (FA)
   If α is a branching node, $\{\beta, \gamma\}$ is the set of α's daughters, and $[\![\beta]\!]$ is a function whose domain contains $[\![\gamma]\!]$, then $[\![\alpha]\!] = [\![\beta]\!]([\![\gamma]\!])$.

When we do semantic derivations, we reach a certain point where the tree is completely broken down, and it's just a matter of applying functions to arguments in the first sense of functional application. These steps in the derivation should *not* be labelled FA! We can leave those steps un-labelled. **Composition rules are only used to break down trees.**

2 Top-down and Bottom-Up Composition

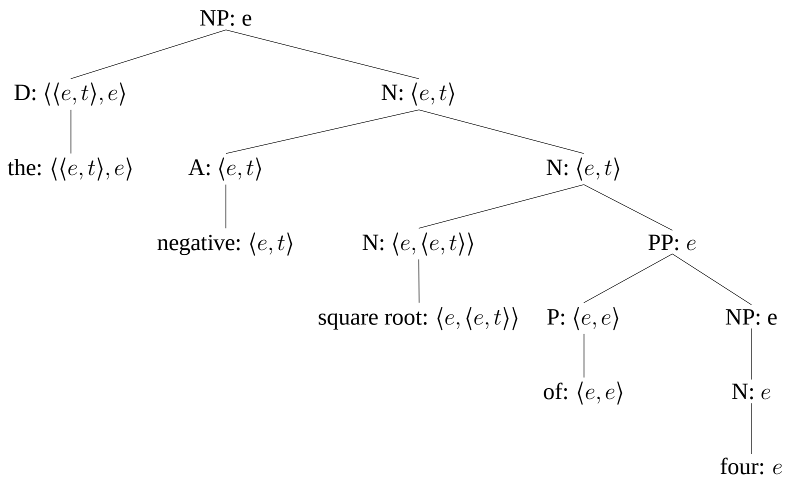
The negative square root of 4

Regarding *the negative square root of 4*, Frege says, "We have here a case in which out of a concept-expression [i.e., an expression whose meaning is of type $\langle e, t\rangle$] a compound proper name is formed [that is to say, the entire expression is of type e] with the help of the definite article in the singular, which is at any rate permissible when one and only one object falls under the concept."

To flesh out Frege's analysis of this example, Heim and Kratzer suggest that *square root* is a "transitive noun", with a meaning of type $\langle e, \langle e, t\rangle\rangle$, and that "*of* is vacuous, [[**square root**]] applies to 4 via Functional Application, and the result of that composes with [[**negative**]] *under predicate modification.*"

Predicate Modification (PM)
If α is a branching node, $\{\beta, \gamma\}$ is the set of α's daughters, and $[\![\beta]\!]$ and $[\![\gamma]\!]$ are both in $D_{\langle e,t\rangle}$, then $[\![\alpha]\!] = \lambda x \in D_e . [\![\beta]\!](x) = [\![\gamma]\!](x) = 1$

So the constituents will have denotations of the following types:

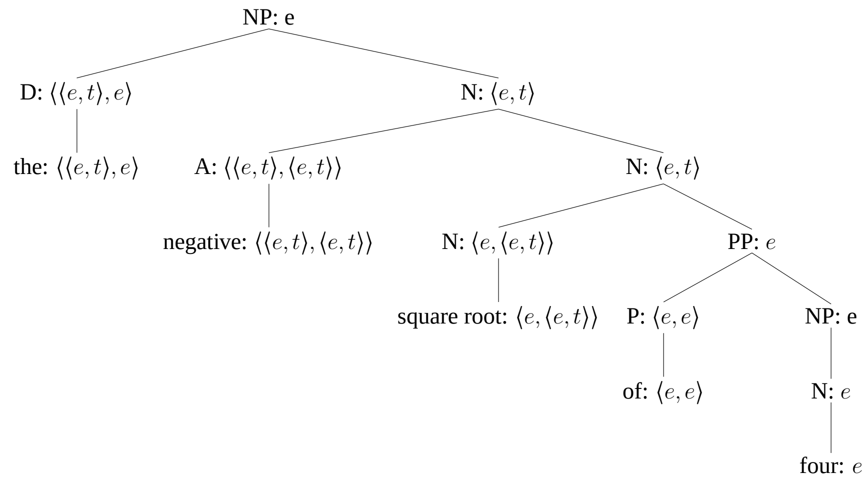


So our lexical entries will be:

- ⟦the⟧ = λf $\epsilon D_{\langle e,t\rangle}$: there is exactly one x such that f(x) = 1 . the unique y such that f(y) = 1
- ⟦negative⟧ = λx ϵD_e . x is negative
- ⟦square root⟧ = λy ϵD_e . λx ϵD_e. x is the square root of y
- ⟦of⟧ = λx ϵD_e . x
- ⟦four⟧ = 4

We could try it using Functional Application instead of Predicate Modification – that would work if we had a lexical entry for *negative* of type $\langle\langle e,t\rangle,\langle e,t\rangle\rangle$. Here is an $\langle\langle e,t\rangle,\langle e,t\rangle\rangle$ analysis of *negative*:

- ⟦negative⟧ = λf $\epsilon D_{\langle e,t\rangle}$. λx ϵD_e . f(x) = 1 and x is negative



But this is not so nice, because then how do we analyze a sentence like *-2 is negative*?

⟦is⟧ = λf $\epsilon D_{\langle e,t\rangle}$. f

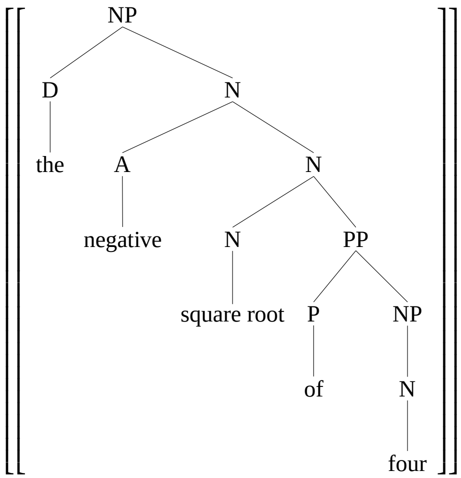
Now ⟦is⟧(⟦negative⟧) is undefined!

But with the $\langle e,t\rangle$ analysis:

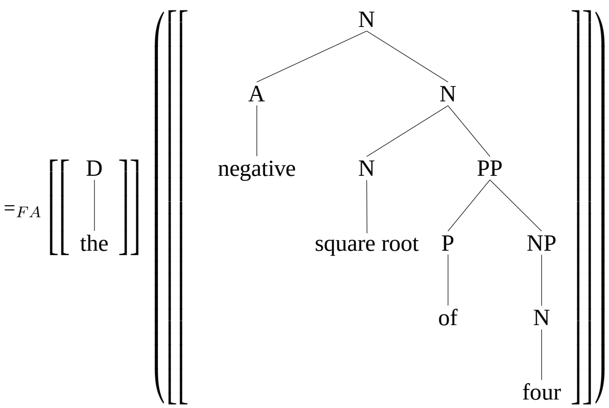
⟦is⟧(⟦negative⟧) = [λf $\epsilon D_{\langle e,t\rangle}$. f](λx . x is negative)
= λx . x is negative

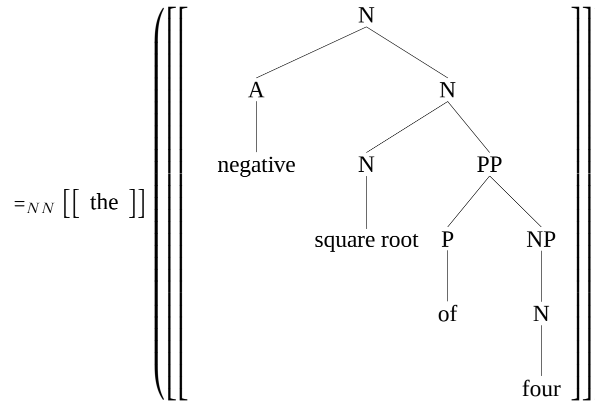
So we should stick to the suggestion and use Predicate Modification.

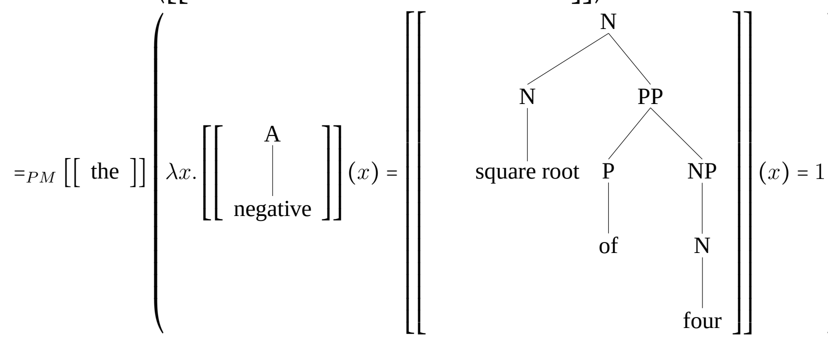
Top-down evaluation. To compute the value “top-down”, we put the whole tree in one big old pair of denotation brackets:

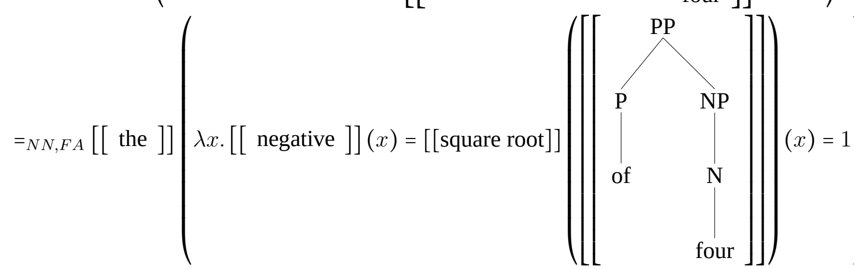


and use composition rules to break down the tree. Here I am putting the name of the rule I used as a subscript on the equals sign, because I don't have enough room to put them off to the right.









$$=_{FA} [\![\text{ the }]\!] \left(\lambda x . [\![\text{ negative }]\!] (x) = [\![\text{square root}]\!] \left([\![\begin{array}{c} \text{P} \\ | \\ \text{of} \end{array}]\!] \left([\![\begin{array}{c} \text{NP} \\ | \\ \text{N} \\ | \\ \text{four} \end{array}]\!] \right) \right) (x) = 1 \right)$$

$$=_{NN} [\![\text{ the }]\!] (\lambda x . [\![\text{ negative }]\!] (x) = [\![\text{square root}]\!] ([\![\text{ of }]\!] ([\![\text{ four }]\!])) (x) = 1)$$

Now we are done breaking down the tree. No more composition rules.

[[the]](λx . [[negative]](x) = [[square root]]([[of]]([[four]]))(x) = 1)

= [[the]](λx . [[negative]](x) = [[square root]]([[of]](4))(x) = 1)

= [[the]](λx . [[negative]](x) = [[square root]]([λx . x](4))(x) = 1)

= [[the]](λx . [[negative]](x) = [[square root]](4)(x) = 1)

= [[the]](λx . [[negative]](x) = [λy . λz . z is a square root of y](4)(x) = 1)

= [[the]](λx . [[negative]](x) = [λz . z is a square root of 4](x) = 1)

= [[the]](λx . [[negative]](x) = 1 and x is a square root of 4)

= [[the]](λx . [[λz. z is negative]](x) = 1 and x is a square root of 4)

= [[the]](λx . x is negative and x is a square root of 4)

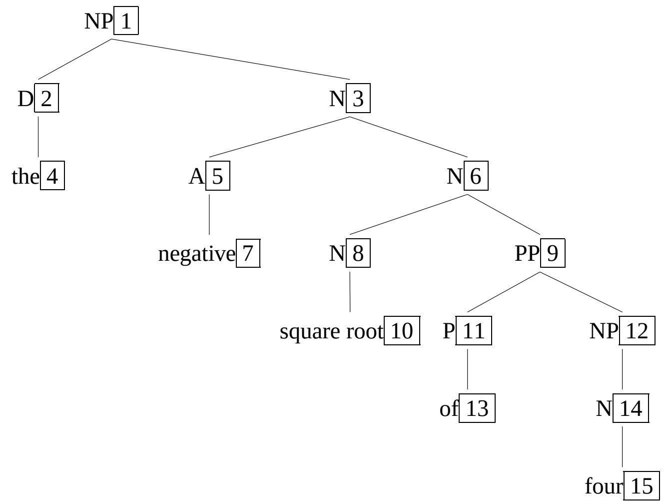
= [λf $\in D_{\langle e,t\rangle}$: there is exactly one x such that f(x) = 1 . the unique y such that f(y) = 1](λx . x is negative and x is a square root of 4)

= the unique y such that y is negative and x is a square root of 4

= -2

Exercise

Label each node of the tree with its type and the composition rule that we used to compute the value of the subtree rooted at it.



(1) *Terminal Nodes* (TN)
If α is a terminal node, $[\![\alpha]\!]$ is specified in the lexicon.

(2) *Non-Branching Nodes* (NN)
If α is a non-branching node, and β is its daughter node, then $[\![\alpha]\!] = [\![\beta]\!]$.

(3) *Functional Application* (FA)
If α is a branching node, $\{\beta, \gamma\}$ is the set of α's daughters, and $[\![\beta]\!]$ is a function whose domain contains $[\![\gamma]\!]$, then $[\![\alpha]\!] = [\![\beta]\!]([\![\gamma]\!])$.

(4) *Predicate Modification* (PM)
If α is a branching node, $\{\beta, \gamma\}$ is the set of α's daughters, and $[\![\beta]\!]$ and $[\![\gamma]\!]$ are both in $D_{\langle e,t\rangle}$, then $[\![\alpha]\!] = \lambda x \in D_e . [\![\beta]\!](x) = [\![\gamma]\!](x) = 1$

Bottom-up style

By ⟦**13**⟧, we mean "the semantic value of the subtree rooted at node 13".

We need to compute a semantic value for every subtree/node. For convenience, we can group the nodes according to the string of words that they dominate, and start from the most deeply embedded part of the tree.

four

- ⟦**15**⟧ = ⟦four⟧ [TN]
  = 4
- ⟦**14**⟧ = ⟦**15**⟧ [NN]
- ⟦**12**⟧ = ⟦**14**⟧ [NN]

of

- ⟦**13**⟧ = ⟦of⟧ [TN]
  = λx . x
- ⟦**11**⟧ = ⟦**13**⟧ [NN]

of four

- ⟦**9**⟧ = ⟦**11**⟧(⟦**12**⟧) [FA]
  = [λx . x](4)
  = 4

square root

- ⟦**10**⟧ = ⟦square root⟧ [TN]
  = λy $\in D_e$. λx $\in D_e$. x is a square root of y
- ⟦**8**⟧ = ⟦**10**⟧ [NN]

square root of four

- ⟦**6**⟧ = ⟦**8**⟧(⟦**9**⟧) [FA]
  [= λy $\in D_e$. λx $\in D_e$. x is a square root of y](4)
  = λx $\in D_e$. x is a square root of 4
  = λx $\in D_e$. x $\in \{2,-2\}$

negative

- ⟦**7**⟧ = ⟦negative⟧ [TN]
  = λx $\in D_e$. x is negative
- ⟦**5**⟧ = ⟦**7**⟧ [NN]

negative square root of four

- ⟦**3**⟧ = λx . ⟦**5**⟧(x) = ⟦**6**⟧(x)=1 [PM]
  = λx. x is a square root of 4 and x is negative
  = λx. x $\in \{-2\}$

the

- ⟦**4**⟧ = ⟦the⟧ [TN]
  = λf $\in D_{\langle e,t\rangle}$: there is exactly one x s. t. f(x) = 1 . the y such that f(y) = 1
- ⟦**2**⟧ = ⟦**4**⟧ [NN]
  = λf $\in D_{\langle e,t\rangle}$: there is exactly one x s. t. f(x) = 1 . the y such that f(y) = 1

the negative square root of four

- ⟦**1**⟧ = ⟦**2**⟧(⟦**3**⟧) [FA]
  = [λf $\in D_{\langle e,t\rangle}$: there is exactly one x such that f(x) = 1 . the unique y such that f(y) = 1](λx. x $\in \{-2\}$)
  = the unique y such that y $\in \{-2\}$
  = -2

Note that we used the same composition rules as we did using the top-down style!

This style takes up less space and requires less fancy formatting, so you might prefer it.