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Presupposition Projection in Dynamic Semantics

The best solution to the projection problem is to do away with it. The moral of this paper is: do not ask what the presupposition[s] of a complex sentence are, ask what it takes to satisfy them. –Karttunen (1974)

1 Introduction

"Instead of characterizing these contexts by compiling the presuppositions of the sentence, ask what a context would have to be like in order to satisfy those presuppositions. Of course, it is exactly the same problem but, by turning it upside down, we get a surprisingly simple answer. The reason is that we can answer the latter question directly, without having to compute what the presuppositions actually are." (Karttunen 1974)

Start by defining a notion of satisfaction of presuppositions:

- Context c **satisfies-the-presuppositions-of** ϕ just in case c entails the presuppositions of ϕ.

Instead of "satisfies-the-presuppositions-of", we can say "admits". To say "c admits ϕ" is to say that c satsifies the presuppositions of ϕ, which means that c entails the presuppositions of ϕ.

What does it mean for a context to entail a presupposition?

2 Entailment

Entailment. A sentence ϕ *entails* (or *semantically entails*) another sentence ψ (written $\phi \models \psi$) if and only if whenever ϕ is true, ψ is true.

Let us assume:

- the semantic value of a sentence is a proposition
- propositions are sets of possible worlds

It follows that the semantic value of a sentence is a set of possible worlds. Let us assume:

(1) $[\![\textbf{president}(\textbf{bart})]\!] = \{w_1, w_2, w_3\}$

Let us assume that in every world, Bart is a child. Thus, a child is a president of the United States in w_1, w_2, and w_3. But there are also other worlds where Lisa, who is also a child, is the president of the United States. Call these w_4 and w_5. Then:

(2) $[\![\exists x[\textbf{president}(x) \wedge \textbf{child}(x)]]\!] = \{w_1, w_2, w_3, w_4, w_5\}$

Now we can define entailment in terms of subset: "A sentence ϕ entails a sentence ϕ if and only if the semantic value of ψ is a subset of the semantic value of ψ." ('All of the ϕ worlds are ψ worlds.')

(3) **Entailment**
$\phi \vDash \psi$ if and only if $[\![\phi]\!] \subseteq [\![\psi]\!]$

Given (1) and (2), and since

$$\{w_1, w_2, w_3\} \subseteq \{w_1, w_2, w_3, w_4, w_5\}$$

It follows from the definition of entailment that $\textbf{president}(\textbf{bart})$ entails $\exists x[\textbf{president}(x) \wedge \textbf{child}(x)]$. Written differently:

$$\textbf{president}(\textbf{bart}) \vDash \exists x[\textbf{president}(x) \wedge \textbf{child}(x)]$$

Entailment by contexts. Karttunen's idea: *in order for a context to admit a (surface) sentence, the context must entail the presuppositions of the sentence.* Let us assume that presuppositions are propositions, i.e. sets of possible worlds.

For Karttunen (1974), contexts are *sets of logical forms,* where logical forms are sentences of a formal language such as first order predicate calculus. For example:

(4) $X = \{\ulcorner\textbf{president}(\textbf{bart})\urcorner, \ulcorner\exists x[\textbf{president}(x) \wedge \textbf{child}(x)]\urcorner\}$

Logical forms determine propositions (i.e., for every logical form, there is a unique corresponding proposition).

Instead of thinking of contexts as sets of *logical forms,* we might think of contexts as sets of *propositions* (and since propositions are sets of possible worlds, we are then thinking of contexts as *sets of sets* of possible worlds).

What does it mean for a *context* to entail a proposition? We know what it means for a *proposition* to entail another proposition, but what does it mean for a *set of propositions* to entail a proposition?

We can say that a context c entails a proposition P if the conjunction of all of the propositions in c entails P. In fact, we can just think of the context as the conjunction itself, as Heim suggests:

> A context is here construed more or less... as a set of propositions, or more simply, as a proposition, namely that proposition which is the conjunction of all the elements of the set. (Heim 1983, p. 399)

Following Heim, let's treat the context as a single proposition, the conjunction of all of the propositions that are mutually believed (Stalnaker's *context set*).

If propositions are sets of possible worlds, then what is the conjunction of a set of propositions? Example:

$P = \{w_1, w_2, w_3\}$	[‘Bart is president’]
$Q = \{w_1, w_2, w_3, w_4, w_5\}$	[‘A child is president’]
$R = \{w_2, w_3\}$	[‘Bart has a girlfriend’]
$S = \{w_1, w_4\}$	[‘Santa’s little helper is sick’]
$W = \{w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_8, w_9, w_{10}\}$	[the set of all worlds]

What is the conjunction of P and S, the conjunction of the proposition that Bart is president and the proposition that Santa’s little helper is sick? It is the set of worlds where both propositions are true. That is the *intersection* (not the union).

$$\begin{aligned} & P \cap S \\ = \; & \{w_1, w_2, w_3\} \cap \{w_1, w_4\} \\ = \; & \{w_1\} \end{aligned}$$

3 Admittance

Admittance: A context c admits ϕ if and only if c satisfies the presuppositions of ϕ.

(5) **Satifaction of presuppositions**
Let P_ϕ be the set of worlds where the presuppositions of ϕ are satisfied. A context c satisfies the presuppositions of ϕ if $c \subseteq P_\phi$.

(6) **Admittance conditions for non-compound sentences**
If ϕ is a simple, non-compound sentence, then A context c admits ϕ if and only if c satisfies the basic presuppositions of ϕ. (Karttunen 1974, p. 184)

Example from Karttunen (1974): Context c admits “a is P too” only if either (i) X entails “b is P” for some b distinct from a, or (ii) X entails “a is Q” for some Q distinct from P (p. 184).

Suppose that the surface sentence *All of Homer’s children are bald* presupposes that Homer has at least 3 children. Suppose that in worlds $w_1 ... w_8$, Bart, Lisa and Maggie are all children of Homer, but in w_9 and w_{10}, Maggie is actually adopted, and therefore strictly speaking not Homer’s child. So the proposition that Homer has at least three children (call it K) is the set of worlds $w_1 ... w_8$:

(7) $K = \{w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_8\}$

K is a basic presupposition of ϕ; let us pretend that it is the only basic presupposition of ϕ. So $P_\phi = K = \{w_1, w_2, w_3, w_4, w_5, w_6, w_7, w_8\}$.

Since our sentence ϕ = *All of Homer’s children are bald* is a simple, non-compound sentence, it is admitted in contexts c that entail K. Suppose that $c = P \cap S$. Does c admit ϕ?

Suppose instead that $c = W$. Does c admit ϕ?

Local contexts. Karttunen (1974) proposes to think of presupposition projection in terms of admittance conditions. Here is Karttunen's insight, with Heim's way of putting it:

(8) **Admittance conditions for a conditional sentence**
Context c admits "If ϕ then ψ" just in case (i) c admits ϕ, and (ii) $c + \phi$ admits ψ.

Karttunen (1974): "We could look at satisfaction of presuppositions in an even more general way. [...] by our definition a given initial context satisfies-the-presuppositions-of a complex sentence just in case the presuppositions of each of the constituent sentences are satisfied by a certain specific extension of that initial context. [...] In compound sentences, the initial context is incremented in a left-to-right fashion giving for each constituent sentence a local context that must satisfy its presuppositions. We could easlily define a notion of local context separately and give the following general definition of satisfaction for all compound sentences."

(9) Context c satisfies-the-presuppositions-of ϕ just in case the presuppositions of each of the constituent sentences in ϕ are satisfied by the corresponding local context.

For a conditional, the global context incremented by the antecedent is the *local context* for the consequent. We can identify a range of local contexts (c here stands for the global context):

- the consequent of a conditional → c+ the antecedent
- the second conjunct in a conjunction → c+ the negation of the first conjunct
- the second disjunct in a disjunction → c+ the negation of the first disjunct
- the complement of a propositional attitude verb → the beliefs of the holder of the propositional attitude (e.g. *Hans wants the ghost in his attic to be quiet tonight* presupposes that Hans believes that there is a ghost in his attic)

Exercise. Refer to Figure 1. Does C admit *If the king has a son, then the king's son is bald*? Why or why not? Does K admit it? Why or not? Explain using Heim's definition of admittance, and assume that the result of updating a context with *The king has a son* is the intersection of A with the context *if the presuppositions of 'The king has a son' are satisfied*; undefined otherwise.

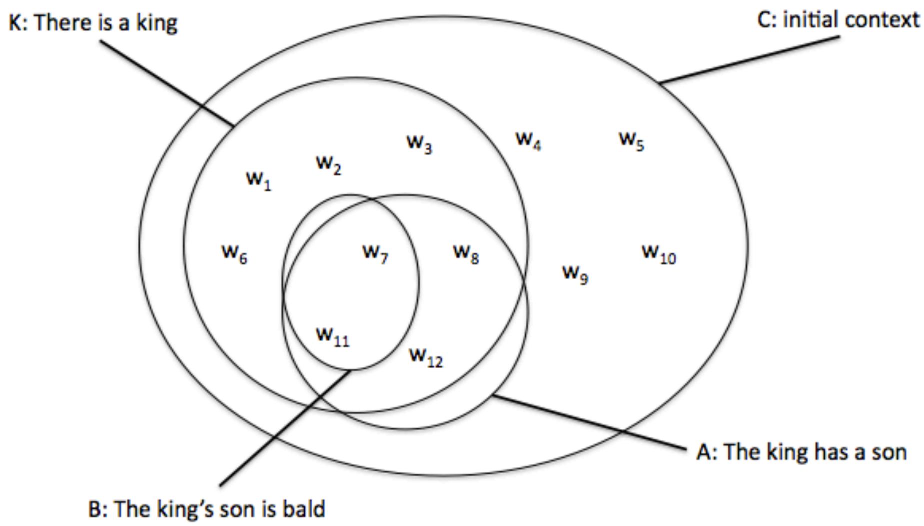

Figure 1: Example propositions

4 Gazdar (1979)

4.1 Gazdar's theory

Why doesn't (10a) presuppose (10b)?

(10) a. If Jack has children, then all of Jack's children are bald.
b. Jack has children.

Gazdar: Because whenever you utter a sentence of the form *If P then Q,* then you are conversationally implicating that *P*, *not P, Q,* and *not Q* are all possible.

This is how Heim describes it:

> [Gazdar] invokes a general and quite simple theory of how utterances change the context in which they occur. In the case of (3) [*If the king has a son, the king's son is bald*], for instance, G. assumes that one of the existence presuppositions of the consequent gets cancelled by a conflicting conversational implicature of (3): (3) implicates that, for all the speaker knows, the king may not have a son, which is not consistent with a presupposition to the efect that the king must have a son.

In this case, *P* = 'Jack has children'. Because the speaker conversationally implicates that Jack might not have children, and this is inconsistent with the 'potential presupposition' triggered by *all,* the latter does not become an actual presupposition of the sentence.

This theory is completely general; it also applies to disjunctions. Whenever you utter *P or Q,* you conversationally implicate *maybe P, maybe not P, maybe Q,* and *maybe not Q*. So it correctly predicts, like Karttunen's theory, that (11a) does not presuppose (11b).

(11) a. Either Bill has no friends or all of Bill's friends are keeping very quiet.

b. Bill has friends.

4.2 Advantages of Gazdar's account over Karttunen's account

Two main advantages:

1. It makes use of very general principles; it is not *ad hoc,* stipulating for each item whether it is a plug, a hole, or a filter.
2. It can explain certain cases where presuppositions disappear.

For each of the following examples, Karttunen's theory predicts that (a) presupposes (b), while Gazdar's theory correctly predicts that it does not.

(12) a. John doesn't regret killing his mother because he didn't kill her.

b. John killed his mother.

(13) a. Nobody has yet discovered that protons are (in any way) influenced by the CIA.

b. Protons are influenced by the CIA.

(14) a. Either all Bill's friends are keeping very quiet or he has no friends.

b. Bill has friends.

(15) a. If Nixon knows the war is over, the war is over.

b. The war is over.

(16) a. Either John has stopped beating his wife or he hasn't begun beating her.

b. John has / has not beaten his wife.

(17) a. Maybe John used to beat his wife but has now stopped doing so.

b. John used to beat his wife.

(18) a. Perhaps John has no children, but perhaps his children are away on vacation.

b. John has children.

5 Heim (1983)

5.1 Heim's rebuttal of Gazdar

Karttunen's theory can be made less ad hoc. What it means to be *ad hoc:* specifying three pieces of information about a word separately.

ITEM	CONTENT	PRESUPPOSITION	HERITAGE
if A then B	A → B	-	P_A, A → P_B
A regrets that P	A considers P bad	P	P_P

We want a theory on which the heritage property *follows* from the content + presupposition. Heim attempts this within Karttunen's general approach.

Disappearing presuppositions: Explainable? Heim seems to think that these can be explained away:

> The literature also contains a battery of examples designed to show that G.'s predictions are superior to those of [Karttunen and Peters]. One group of such examples is supposed to discredit K.&P.'s assumption that conditionals presuppose the conditional p → q' [p = the antecedent; q' = the presupposition of the consequent] rather than q' simpliciter. I agree with Soames (1982) that none of these examples are convincing. The remaining groups of genuine counterexamples to K.&P. are disjunctions whose disjuncts carry contradictory presuppositions (e.g., "He either jsut stopped or just started smoking.") and conditionals in which a presupposition of the antecendent fails to survive (e.g., "If I later realize I haven't told the truth, I will tell you.")

Problem #1 for Gazdar: When antecedent entails but is not equal to the presupposition of the consequent:

(19) a. If John has twins, then Mary will not like his children.

b. John has children.

Intuitively, this sentence presupposes nothing (or, does not presuppose anything about any children that John may have) and that is what Karttunen's theory predicts.

Gazdar would say that this sentence implicates that John might or might not have twins. The potential presupposition in question is that John has children. The potential presupposition is consistent with the idea that John might not have twins, so it is predicted to project. So Gazdar's theory predicts that (19a) presupposes (19b).

Problem #2 for Gadzar: Presuppositions with free variables.

(20) Every nation$_i$ cherishes its$_i$ king.

Contains these parts:

(21) a. every x_i

b. x_i is a nation

c. x_i cherishes x_i's king ← presupposes (22)

(22) x_i has a king

> But whatever [(22)] expresses is not a proposition: the free variable in it makes it incomplete. Would G. say that [(22)] expresses a potential presupposition of a part of [(20)] and hence of [(20)] as a whole? If so, what would it mean for this presupposition to get added to the context?

5.2 Getting content and heritage from CCPs

With Context Change Potentials, we can get the content property and the heritage property in one go. "A two-fold lexical specification of each item, in terms of CCP and presupposition property, can replace the three-fold specification that appeared to be needed in the K.&P.-theory" (p. 400).

Context Change Potentials. What Karttunen basically said is this:

(23) If *If A, B* is uttered in context c, then c is the local context for *A*, and c+*A* (read: "c incremented by *A*") is the local context for *B*.

(24) A context c admits a sentence S just in case each of the constituent sentences of S is admitted by the corresponding local context.

So the global context c must admit *A*, and c+*A* must admit *B*.

(25) S presupposes p iff all contexts that admit S entail P.

The "heritage property" of *if* follows from these three assumptions.

"c + S" designates the result of executing the *context change potential* (CCP) of sentence S on context c.

> What are CCPs? Intuitively, they are instructions specifying certain operations of context change. The CCP of "It is raining," for instance, is the instruction to conjoin the current context with the proposition that it is raining. (If we construe propositions as sets of possible worlds, as we will here, "conjoin" means "intersect.")

Heim's point: (23) *almost* tells you what the word *if* means. If we just specify it a little bit further by giving a CCP for *if*, we can say what *if* means (get the content property) and derive the heritage property at the same time.

CCP for *if*:

(26) c + If A, B = c \ ((c + A) \ (c + A + B))

How does (26) get us the content property?

- There is an intimate connection between CCPs and truth:

  (27) Suppose c is true (in w) and c admits S. Then S is true (in w) with respect to c iff c + S is true (in w).

  In other words, to be a true CCP is to keep the context true.

How does (26) get us the heritage property?

- Assumption: *Admittance conditions are conditions on the definedness of the CCP*, i.e. *c + S is defined iff c admits S.*

This works with negation, too.

(28) c + Not S = c \ (c + S)

5.3 Presuppositions with free variables

Remaining problem: How to account for (20), repeated here:

(29) Every nation$_i$ cherishes its$_i$ king.

Heim's solution: Contexts are sets of pairs $\langle g, w \rangle$, where g is an assignment function (a sequence of individuals, or a function from integers to individuals) and w is a world. This allows her to assign CCPs to sentences with free variables. For example:

(30) $c + x_i$ has a king =
$c \cup \{\langle g, w \rangle : g(i)$ has a king in $w\}$

Suppose that the context is the following set of assignment/world pairs:

$$
(31)\quad c = \left\{
\begin{array}{l}
\left\langle \begin{bmatrix} 1\rightarrow\text{Sweden} \\ 2\rightarrow\text{Sweden} \\ 3\rightarrow\text{Sweden} \end{bmatrix}, w_1 \right\rangle \\
\left\langle \begin{bmatrix} 1\rightarrow\text{Sweden} \\ 2\rightarrow\text{Sweden} \\ 3\rightarrow\text{Sweden} \end{bmatrix}, w_2 \right\rangle \\
\left\langle \begin{bmatrix} 1\rightarrow\text{Sweden} \\ 2\rightarrow\text{Sweden} \\ 3\rightarrow\text{Sweden} \end{bmatrix}, w_3 \right\rangle \\
\left\langle \begin{bmatrix} 1\rightarrow\text{America} \\ 2\rightarrow\text{Sweden} \\ 3\rightarrow\text{America} \end{bmatrix}, w_1 \right\rangle \\
\left\langle \begin{bmatrix} 1\rightarrow\text{America} \\ 2\rightarrow\text{Sweden} \\ 3\rightarrow\text{America} \end{bmatrix}, w_2 \right\rangle \\
\left\langle \begin{bmatrix} 1\rightarrow\text{America} \\ 2\rightarrow\text{Sweden} \\ 3\rightarrow\text{America} \end{bmatrix}, w_3 \right\rangle
\end{array}
\right\}
$$

And assume that in both w_1 and w_2, Sweden has a king but America does not, but in w_3, both Sweden and America have a king. What would the update give us? The set of world-assignment pairs $\langle g, w \rangle$ that are in c and are such that $g(3)$ has a king in w.

To analyze (20), we also need to be able to test whether a context admits a sentence containing a free variable, and we need a CCP for *every*.

The CCP that Heim gives for *every*:

(32) **CCP for *every***
c + Every x$_i$ A, B =
$\{\langle g, w \rangle \in C$: for every a, if $\langle g^{i/a}, w \rangle \in$ c+A, then $\langle g^{i/a}, w \rangle \in$ c + A + B$\}$

where $g^{i/a}$ is the assignment that is the same as g except that the individual a is assigned to the number i, and x_i must be a "new" variable in the following sense:

(33) x_i **is a new variable iff:**
For any two sequences g and g' that differ at most in their i-th member, and for any world w: $\langle g, w \rangle \in c$ iff $\langle g', w \rangle \in c$.

Suppose:

- $i = 3$

- A = x_3 is a nation
- B = x_3 cherishes x_3's king
- Sweden and America are both nations in all of our worlds (w_1, w_2, and w_3)
- As before, in w_1 and w_2, Sweden has a king but America does not, but in w_3, both have kings.
- In w_1, Sweden cherishes its king; in w_2, Sweden does not cherish its king; in w_3, both Sweden and America cherish their kings.

What is c + 'Every x_3, x_3 is a nation, x_3 cherishes x_3's king'? Is it defined? (I.e. does c admit this sentence?)

Hint: In order to admit (satisfy-the-presuppositions-of) 'x_i cherishes x_i's king', a context must entail 'x_i has a king'. By this Heim means that "it has to be a context c such that, for every $\langle g, w\rangle \in c,\ g(i)$ has a king in w'.