METROLOGY & MEREOLOGY

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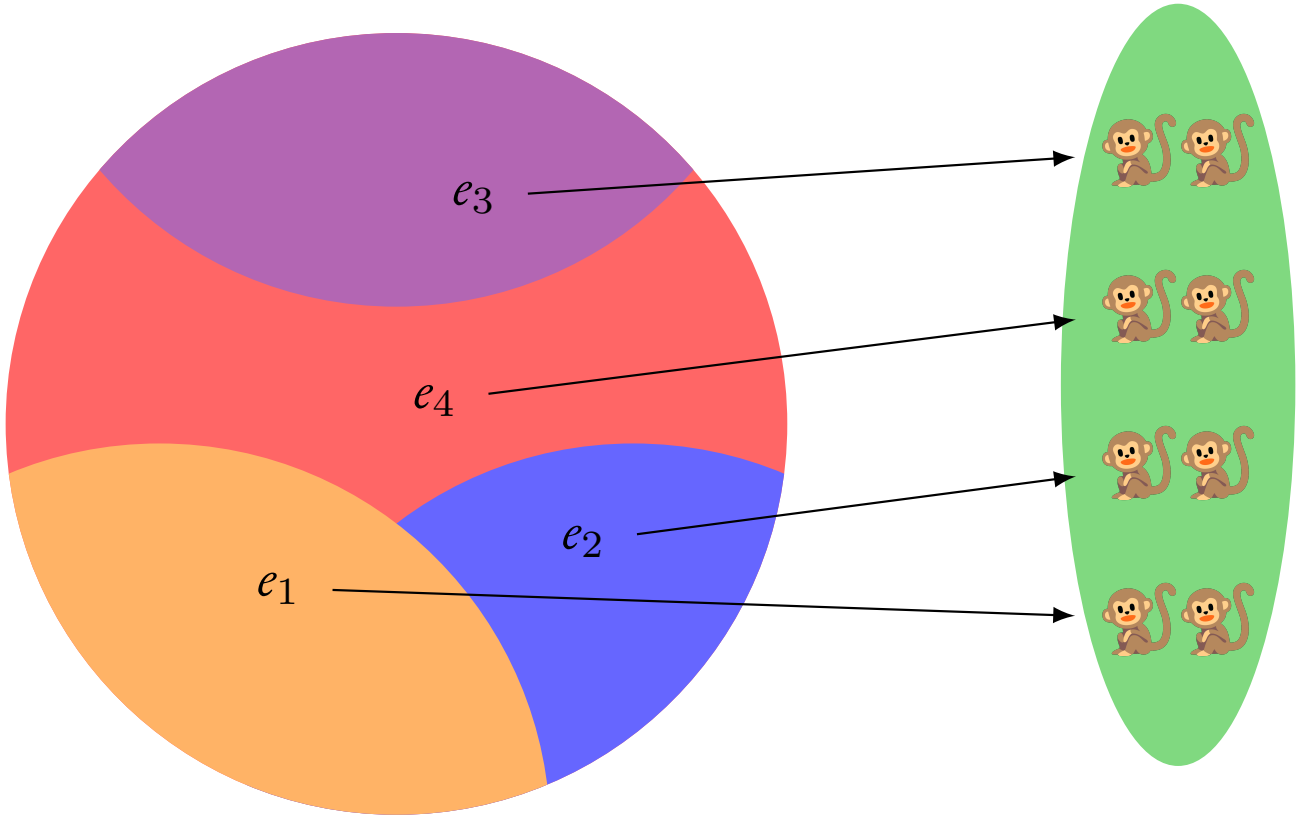
Amsterdam Colloquium · December 2024

Outline

Mereology: Useful, especially with event semantics

(1) renḍu renḍu kootu-lu egir-i-nyiyyi
2 2 monkey-PL jump-PAST-3PL
lit. '2 2 monkeys jumped' (Telugu) cf. 'I saw 2 2 monkeys'



$$\pi(e) = \{e_1, e_2, e_3, e_4\}$$

Balusu (2006). Also Krifka, Landman, Champollion, Brasoveanu, Henderson, Kuhn & Aristodemo, Pasquereau, Moltmann...

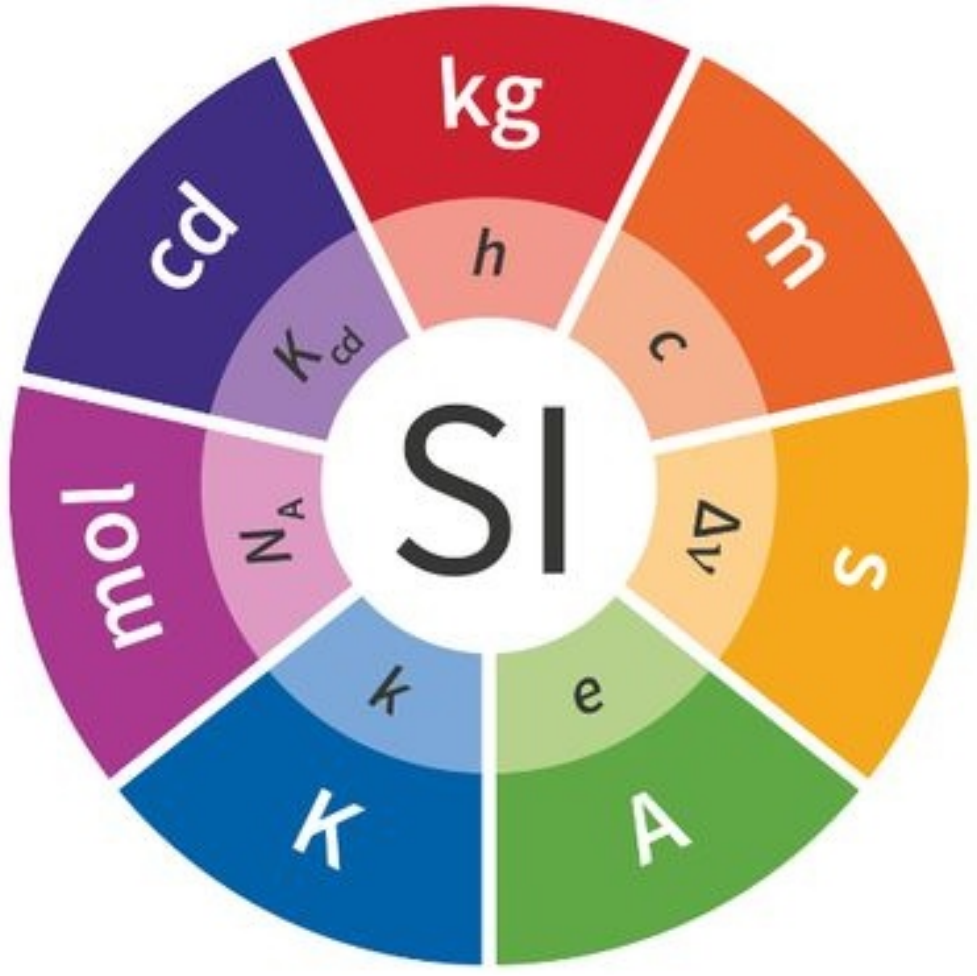
International Vocabulary of Metrology (VIM)*

Chapter 1, "Quantities and units" defines, among other terms:

1. quantity (general, e.g. *radius*, vs. individual, e.g. *radius of A*)
2. dimension (of a quantity), e.g. *length*
3. quantities of the same kind (comparable, $\Rightarrow$ same dimension)
4. quantity with dimension one (prev. 'dimensionless quantity')
5. measurement unit (a reference quantity)
6. numerical value of a quantity (ratio of a quantity and a unit)
7. system of quantities (set of equations about quantities)
8. quantity calculus (system of mathematical operations)
9. unit system (set of base units and derived units)
10. off-system measurement unit

By the Joint Committee for Guides in Metrology (JCGM) July 2023 (4th ed.).

International System of Units (SI): a unit system



time	The **second**, symbol s, is the SI unit of time. It is defined by taking the fixed numerical value of the caesium frequency $\Delta\nu_{Cs}$, the unperturbed ground-state hyperfine transition frequency of the caesium 133 atom, to be 9 192 631 770 when expressed in the unit Hz, which is equal to s^{-1}.

Quantity calculus

Three operations:

- addition of quantities of the same kind

$$q_1 + q_2$$

- product of a number times a quantity

$$n \times q$$

- product of quantities

$$q_1 \times q_2$$

History goes back to Fourier 1822 (de Boer, 1994)

Big questions

1. Would it be useful to incorporate a quantity calculus into our theories of the semantics of natural languages?
   - Yes. There are words in natural languages (really in them) that express arithmetic multiplication/division.
2. How would this system interface with the rest of semantics?
   - Cool fact:
     Mereological division and arithmetic division can co-lexify.
     Suggests they are adjacent in conceptual space.
     Can we build a bridge between them?

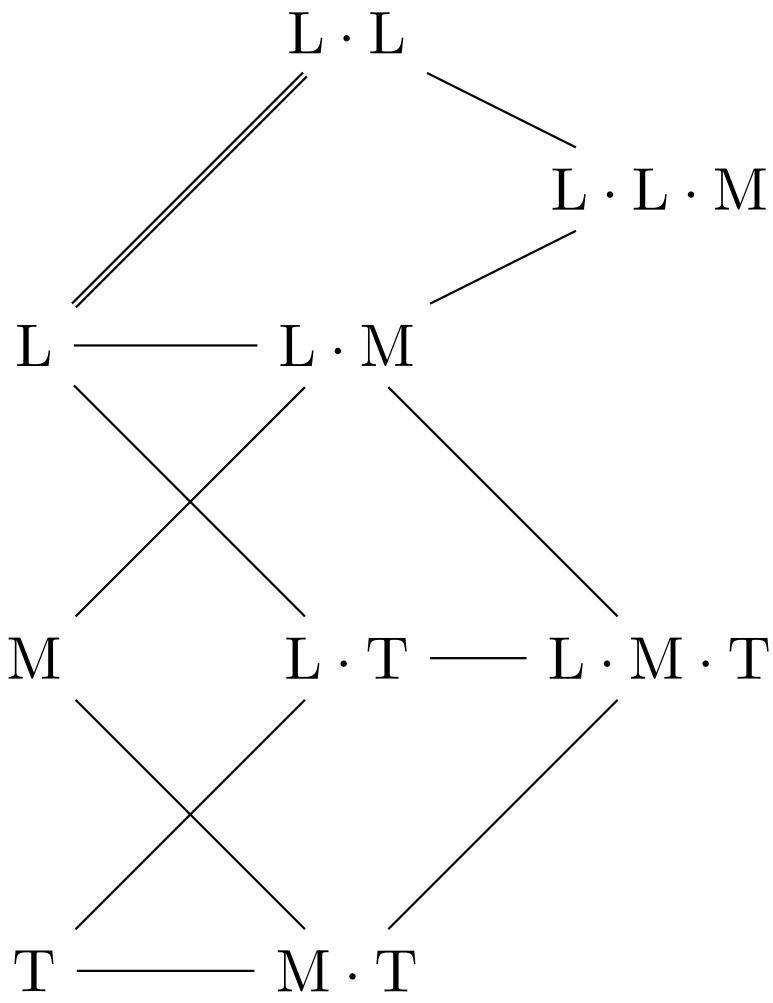
Outline

Basic dimensions ($\mathscr{B}$)

Dimension	**Base unit**
L – length	meter (m)
M – mass	kilogram (kg)
T – time	second (s)
I – electric current	ampere (A)
Θ – thermodynamic temperature	kelvin (K)
N – amount of substance	mole (mol)
J – luminous intensity	candela (cd)

(JCGM, 2012)

Basic and derived dimensions



The dimensions form a group under multiplication

$\mathcal{D}_m = \mathscr{D}$ is a group under $\cdot$, so:

- if $\mathrm{A}, \mathrm{B} \in \mathscr{D}$, then $\mathrm{A} \cdot \mathrm{B} \in \mathscr{D}$
- $\mathscr{D}$ has an identity element $\mathbf{I}_{\mathscr{D}}$, such that for every $D \in \mathscr{D}$:

$$D \cdot \mathbf{I}_{\mathscr{D}} = \mathbf{I}_{\mathscr{D}} \cdot D = D$$

- There is a multiplicative inverse D^{-1} for every $D \in \mathscr{D}$: an element such that

$$D \cdot D^{-1} = \mathbf{I}_{\mathscr{D}}$$

A dimension can be raised to any integer power

$$
\begin{aligned}
&\mathrm{D}^0 = \mathbf{1}_{\mathscr{D}} && \\
&\mathrm{D}^1 = \mathrm{D} && \mathrm{D}^1 \text{ is the multiplicative inverse of } \mathrm{D} \\
&\mathrm{D}^2 = \mathrm{D} \cdot \mathrm{D} && \mathrm{D}^{-2} = (\mathrm{D}^{-1})^2 \\
&\mathrm{D}^3 = \mathrm{D} \cdot \mathrm{D}^2 && \mathrm{D}^{-3} = (\mathrm{D}^{-1})^3 \\
&\vdots && \vdots \\
&\mathrm{D}^k = \mathrm{D} \cdot \mathrm{D}^{k-1} && \mathrm{D}^{-k} = (\mathrm{D}^{-1})^k
\end{aligned}
$$

Each dimension $\mathrm{D} \in \mathscr{D}$ has a unique expression as the product of base dimensions raised to integer powers:

$$\mathrm{D} = \mathrm{L}^{n_1} \cdot \mathrm{M}^{n_2} \cdot \mathrm{T}^{n_3} \cdot \mathrm{I}^{n_4} \cdot \Theta^{n_5} \cdot \mathrm{N}^{n_6} \cdot \mathrm{J}^{n_7}$$

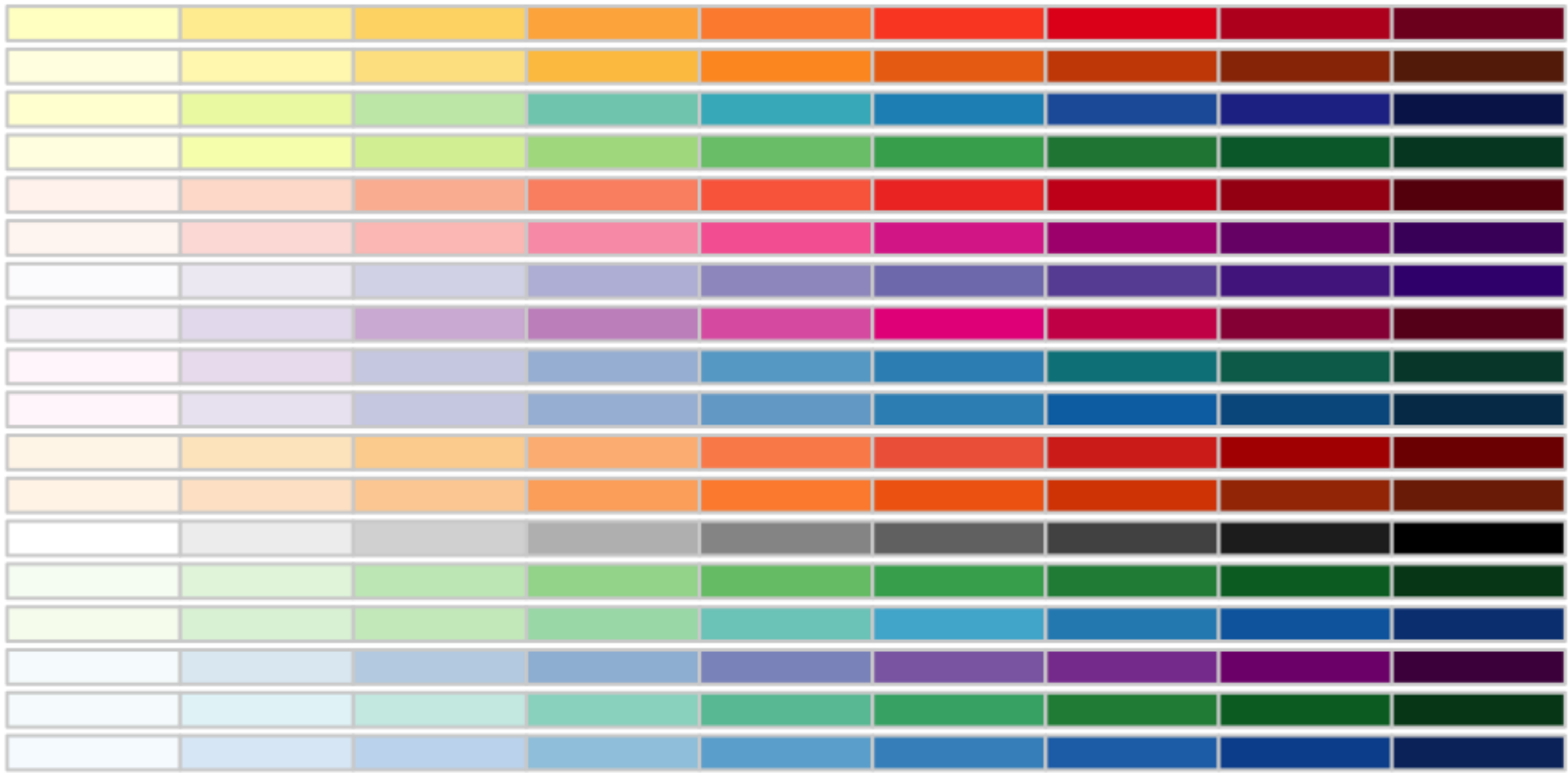
where $n_1, \ldots, n_7$ are integers

Dimension mapping

$$\mathcal{D}_d = \mathcal{Q}$$

$$\mathcal{Q} \xrightarrow{\dim} \mathcal{D}$$

The space of quantities forms a fiber bundle



Each fiber is a vector space.

Cross-dimensional multiplication

$\langle \mathscr{Q}, * \rangle$ is an **abelian monoid**, so:

- If $q_1, q_2 \in \mathscr{Q}$, then $q_1 * q_2 \in \mathscr{Q}$
- There is a **multiplicative identity** element $\mathbf{1}$ such that for all $q \in \mathscr{Q}$:

$$q * \mathbf{1} = \mathbf{1} * q = q$$

- If $q_1, q_2, q_3 \in \mathscr{Q}$ then
  $q_1 * (q_2 * q_3) = (q_1 * q_2) * q_3$ **(associativity)**
- $q_1 * q_2 = q_2 * q_1$ **(commutativity)**

Existence of inverses

Not every quantity has an inverse; you can't divide by any $\mathbf{o}_D$ ($D \in \mathscr{D}$).

But for every *non-zero* quantity $q \in \mathscr{Q}$
there is an inverse q^{-1}:

$$q * q^{-1} = \mathbf{1}$$

Or: The set of non-zero quantities forms a group under multiplication.

Unit mapping

$$\mathscr{Q} \xleftarrow{\text{unit}} \mathscr{D}$$

where $\text{unit}(D)$ picks out a $q \in \mathscr{Q}|D$
(a q such that $\dim(q) = D$)

Restrictions:

- You can't pick the zero element.
- unit must be a group homomorphism:

$$\text{unit}(\text{A} \cdot \text{B}) = \text{unit}(\text{A}) * \text{unit}(\text{B})$$

Do we have this already in degree semantics?

Arithmetic operations defined over degrees ($+, -, \times, \div$):

- Cresswell (1977): only comparison
- Klein (1991) contributes addition, through concatenation (cf. also Krifka 1998)
- Sassoon (2010b) and van Rooij (2011) explicitly discuss multiplication within a particular dimension (as in *twice as tall*), building on measurement theory (Krantz et al., 1971)

But cross-dimensional multiplication and division require more foundational changes.

Outline

Representation language

$\mathcal{L}_{\mathcal{Q}}$: a lambda calculus with quantity multiplication.

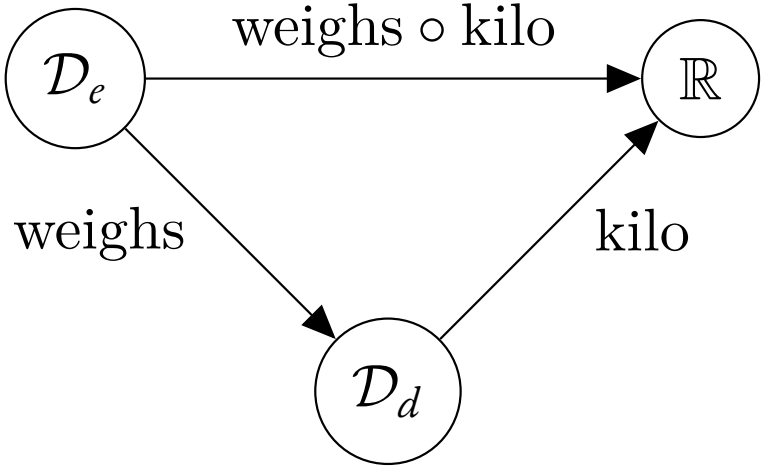
The semantic value of an expression ϕ in $\mathcal{L}_{\mathcal{Q}}$ is given by $[\![\phi]\!]^{\mathcal{M}}$, where:

$$\mathcal{M} = \langle [\langle \mathcal{D}_e, \oplus_e \rangle, \langle \mathcal{D}_v, \oplus_v \rangle, \langle \mathcal{D}_i, \oplus_i \rangle, \langle \mathcal{D}_d, +, * \rangle, \langle \mathcal{D}_m^{\mathscr{B}}, \cdot \rangle], \mathcal{I} \rangle$$

where:

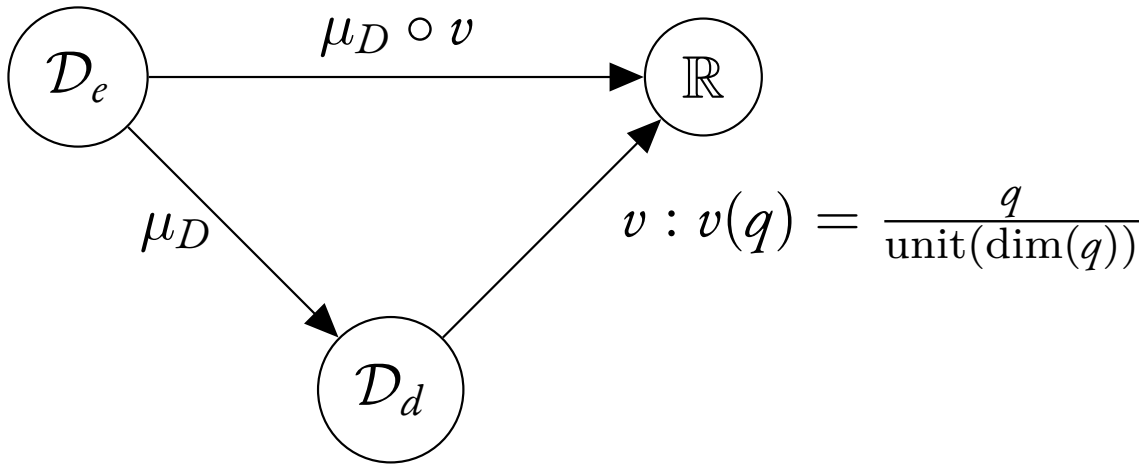
- $\mathscr{B}$ is a finite set of primitive dimensions
- $\langle \mathcal{D}_m^{\mathscr{B}}, \cdot \rangle$ is an abelian group with basis $\mathscr{B}$, a finite set of dimensions
- $\langle \mathcal{D}_d, * \rangle$ is an abelian monoid
- $\mathcal{I}(\textsc{unit})$ is a group homomorphism from $\mathscr{D}_m^{\mathscr{B}}$ to $\mathcal{D}_d$
- $\mathcal{I}(\textsc{dim})$ is a surjection map from $\mathcal{D}_d$ onto $\mathcal{D}_m^{\mathscr{B}}$
- For each $D \in \mathcal{D}_m^{\mathscr{B}}$, $\langle \mathcal{D}_d | D, +, * \rangle$ is a vector space over $\mathbb{R}$
- $\mathcal{I}$ maps each constant of type τ to an element of $\mathcal{D}_\tau$

The Lønning Triangle



(Lonning, 1987; Champollion, 2017)

The Lønning Triangle (à la metrologique)



μ_D: canonical measure function for dimension D

- For each basic dimension D there is a 'calibration object' o such that $\mu_D(o) = \textsc{unit}(D)$.
- For any individual or event o, $\mu_D(o)$ denotes the 'canonical measure' of object o (event or individual) along dimension D.

What units does natural language make use of?

Complements of *per* in EuroParl:

distance: *kilometre (of intra-Community trade)*, *100 km*
area: *hectare/decare (of arable land)*, *square metre (live weight)*
time: *annum*, *calendar year*, *day*, *24 hours*, *season*
volume: *cubic centimetre*, *hectolitre (of pure alcohol)*, *litre (of milk)*
weight: *kilo (of fertilizer)*, *reduced tonne of greenhouse gas*
power: *kilowatt (produced)*, *megajoule*
energy: *energy unit*, *kilowatt-hour (sold)*, *kW/hour*
extensive: *unit (of output/production/quantity/food)*
effort: *unit of effort*
information: *megabyte*
money: *euro (of subsidy)*, *mille of GNP*, *year of EU funding*

Cardinality denominator dimensions in EuroParl

card:human: *capita*, *head of population*, *child*, *farmer*, *taxpayer*, *pupil*
card:animate: *bird*, *fish*, *hen*, *ewe*, *million adult cattle*, *1000 animals*
card:organization: *household*, *farm*, *power station*, *country*, *NGO*
card:tangible: *car*, *cigarette*, *goods vehicle*, *olive tree*, *ship*, *dwelling*
card:intangible: *paragraph*, *policy area*, *category of cars*, *job created*
card:location: *continent*, *zone*, *region*, *port*, *lake*
card:event: *session*, *Presidency*, *accident*, *death*, *flight*, *money withdrawal*
card:human / distance: *passenger kilometer*

Cardinality dimensions (individuals)

Let us assume that for every subset of D_e $P \in D_{\langle e,t \rangle}$, there is a cardinality dimension $\#\textsc{dim}(P)$, determined by the model. If α is of type $\langle e, t \rangle$ then:

$$[\![\#\alpha]\!]^{\mathcal{M}} = \#\textsc{dim}([\![\alpha]\!]^{\mathcal{M}})$$

and the output is a basic dimension: $\#\textsc{dim}([\![\alpha]\!]^{\mathcal{M}}) \in \mathscr{B}$.

As each dimension D is associated with its own unit quantity $\textsc{unit}(D)$, we will have unit quantities for each flavor of cardinality.

$\textsc{unit}(\#\textsc{martini})$: the quantity ‘1 martini’

Sortal nouns have two denotations, e.g.:

martini $\rightsquigarrow \lambda x \,.\, \textsc{martini}(x)$ (type $\langle e, t \rangle$)

martini $\rightsquigarrow \textsc{unit}(\#\textsc{martini})$ (type d)

Outline

English *per* and *each*

Is *per* a distributivity marker?

Adnominal *each* requires a licensor, and not just any will do (cf. Safir & Stowell 1988). English *per* has similar licensing requirements.

(2) a. They ate two/several olives { each / per person }.

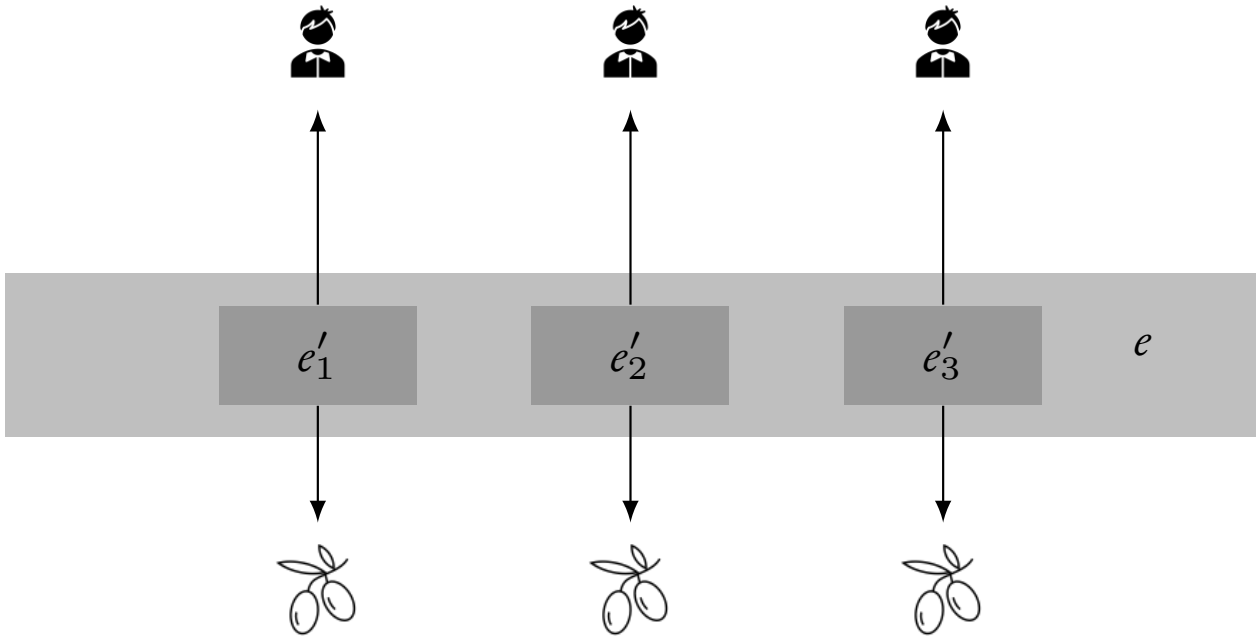
b. ??They ate those/most olives { each / per person }.

c. ??They decided to leave { each / per person }.

Gil (1995): 'share per key'

Analysis of distributivity marker *each*

They ate two olives each (Champollion 2017)



$$e \in {}^*\lambda e' \,.\, {}^*\textsc{eat}(e') \wedge {}^*\textsc{olive}({}^*\textsc{theme}(e')) \wedge \mu({}^*\textsc{theme}(e')) = 2 \wedge \textsc{atom}({}^*\textsc{agent}(e'))$$

Romanian *de* and Italian *per*

Romanian:

(3) James Bond a mâcat două măsline **de** martini
James Bond has eaten two olives DE martini
'James Bond ate two olives per martini'

Italian:

(4) James Bond ha mangiato due olive **per** martini
James Bond has eaten two olives PER martini
'James Bond ate two olives per martini'

Panaitescu & Tovena (2019, 226):
"Preposition *per*/*de* is a share-key relator."

But different events: drinking vs. eating

Panaitescu & Tovena's Inspiration

(5) James Bond ate two olives per martini.

Boolos (1981), 'For every x there is a y'

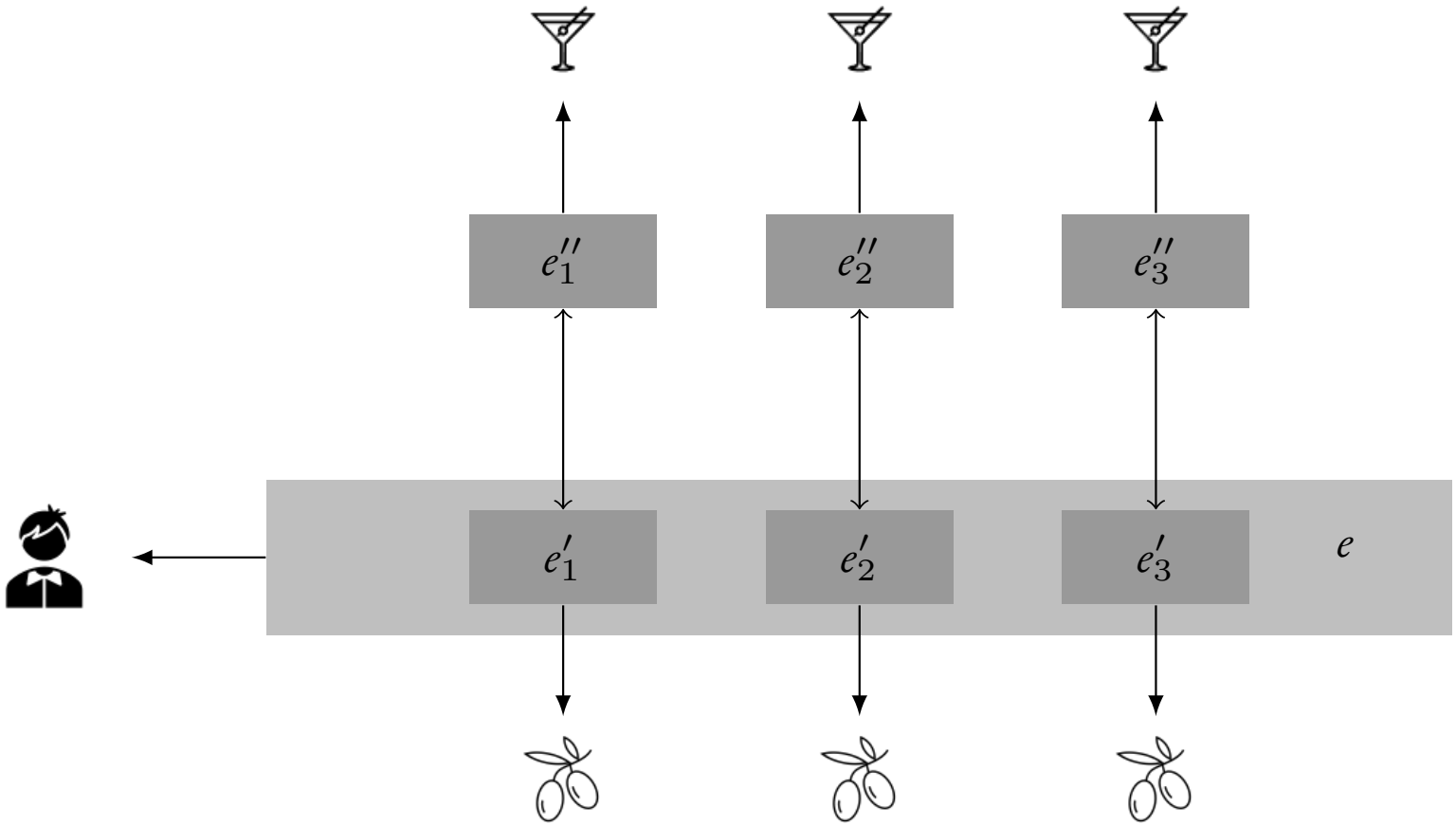
(6) For every martini, James Bond ate two olives.

Rothstein (1995), 'Adverbial quantification over events'

(7) Every time James Bond drank a martini, he ate two olives.

Distributivity marker analysis of *per*

James Bond ate two olives per martini (Panaitescu & Tovena 2019)



$$e \in {}^*\lambda e' \,.\, {}^*\textsc{eat}(e') \wedge {}^*\textsc{olive}({}^*\theta_{\text{share}}(e')) \wedge \mu({}^*\theta_{\text{share}}(e')) = 2 \wedge$$
$${}^*\textsc{drink-martini}(\textsc{match}(e'), {}^*\theta_{\text{key}}(\textsc{match}(e'))) \wedge \mu({}^*\theta_{\text{key}}(\textsc{match}(e'))) = 1$$

Ratio marker analysis of *per*

James Bond ate two olives per martini

$$\frac{\text{olives}}{\text{martinis}} = \frac{\text{olives eaten in } e}{\text{martinis drunk in } e}$$

No minimal size requirement for *per*

Unlike with *each*, the event is not always divisible into 'key'-sized chunks with *per*:

(8) James Bond drove 100 km per hour.
$\not\Rightarrow$??For each hour, James Bond drove 100 km.
(Event could last five minutes.)

(9) You can do this by taking short choppy steps and doing arm swing drills at 240 steps per minute for 20 seconds.

(10) $16,000 is invested at 12.5% per year for 6 months. How much interest is earned on the investment?

Call these 'sub-unit cases'.

Sub-unit cases with Hungarian *-nként*

Csirmaz & Szabolcsi (2012) mention *-nként* under 'rate expressions' and give the following example:

(11) Az a vonat órá-nként 400 kilométer-rel halad
that the train hour-DIST 400 km-INST advances
'That train is travelling at 400 km/hour'

The event is not necessarily composed of hour-long subevents.

Non-uniform scenarios

English:

(12) The Montreal Canadiens scored 2.82 goals per game in 2020-21.

Hungarian:

(13) a nitrát-irányelv 1,7 számosállat-egységről rendelkezik
the nitrate-directive 1.7 livestock-units provides.for
hektáronként.
hectare-DIST
'The Nitrates Directive provides for 1.7 livestock units per hectare.'

There are no subevents involving 1.7 livestock units.

Gradable predicates

(14) Fej-enként olcs-óbb is, és környezetbarát-abb.
head-DIST cheap-CMPR also, and environmentally.friendly-CMPR
'It's cheaper per person, and more environmentally friendly.'

Dimension nouns

(15) 2007 óta a hordó-nként-i ár 63%-kal növekedett
2007 since the barrel-DIST-ADJ price 63%-by increased
'The price per barrel has increased by 63% since 2007'

(16) a kilométer-enként kivetett díj csak növeked-het-ne
the km-DIST levied fee only increase-could-would
'The fee levied per km would only potentially increase.'

Term uses as differential argument of comparative

English:

(17) For every degree the body's internal temperature rises, the heart beats about 10 beats per minute **faster**.

(18) There are tables of various woods that put mahogany 200 kg per cubic meter **denser** than poplar.

Hungarian:

(19) kilométer-enként két perc-cel gyors-abb tempó-t ment.
kilometer-dist 2 minute-with fast-er tempo-acc go.3sg
'it went two minutes faster per kilometer'

(Cf. Rawlins 2013 on differential arguments)

Summary

None of these predictions of the distributivity marker analysis are met:

- **Minimal size requirement**: The eventuality described by the clause should be divisible into one or more 'key'-sized chunks.
- **Uniformity requirement**: The eventuality described by the clause should be divisible into subevents that uniformly manifest both the share and the key.
- **Indefinite share requirement**: A *per* phrase should only be able to modify (cardinal) indefinites.
- **Event predicates not terms**: A *per* phrase, together with its licensor, creates a predicate characterizing an event, and not a (degree-denoting) term.

Outline

Outline

Quotient function analysis of *per*

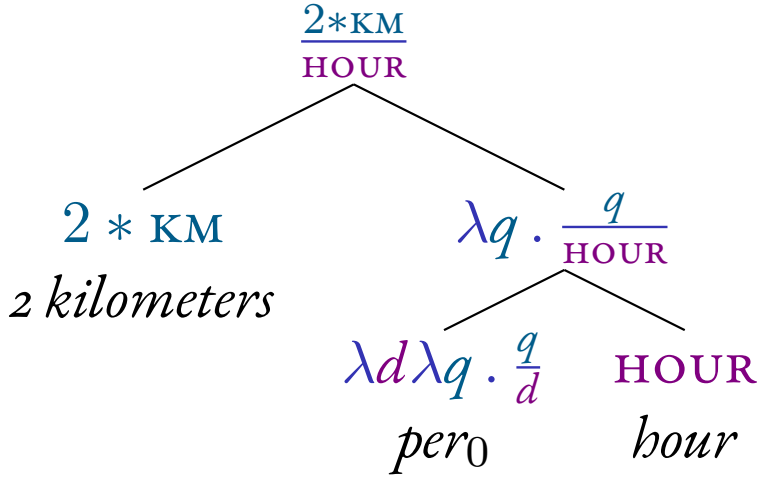
Lexical entries for English words:

(20) *kilometer(s)* $\rightsquigarrow$ KM

(21) *hour(s)* $\rightsquigarrow$ HOUR

KM, HOUR: type d

(22) $per_0 \rightsquigarrow \lambda d \lambda q \,.\, \frac{q}{d}$



(Coppock, 2021)

Outline

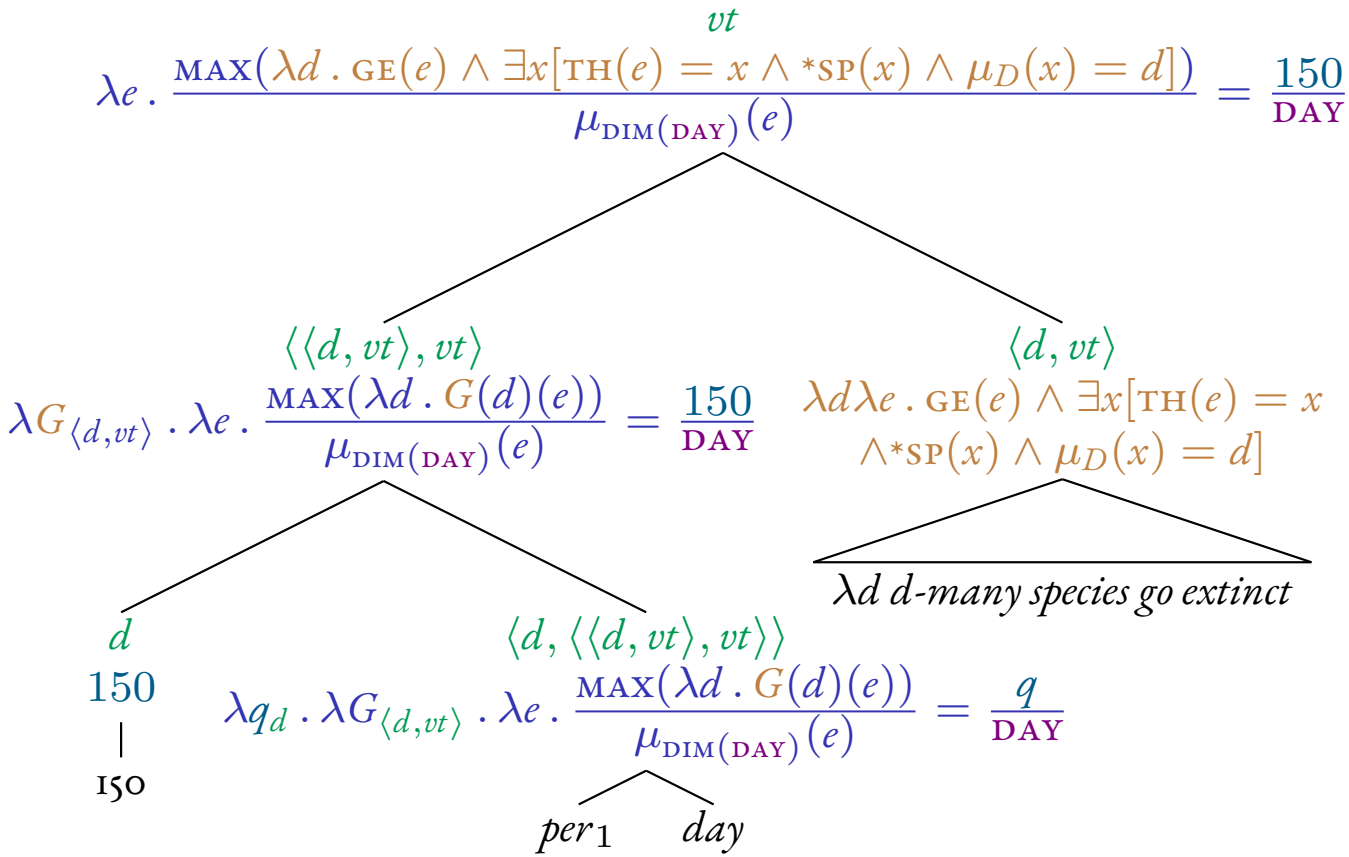
Challenge for the quotient function analysis

(23) It's estimated that 150 species per day go extinct.
150 species per day is a high rate.
#Therefore, a high rate is among those going extinct.

Target truth conditions:

$$\text{Gen } e \,.\, \frac{\text{the number of species that go extinct in } e}{\text{the duration of } e} = \frac{150}{\text{day}}$$

Compositional derivation for quotient operator use



$$per_1 \rightsquigarrow \lambda d_d\,.\,\lambda q_d\,.\,\lambda G_{\langle d,\tau t\rangle}\,.\,\lambda \alpha_\tau\,.\,\frac{\textsc{max}(\lambda d'\,.\,G(d')(\alpha))}{\mu_{\textsc{dim}(d)}(\alpha)} = \frac{q}{d}$$

(Coppock, 2022a)

Another challenge for the quotient function analysis

Measure function verbs like *weigh*:

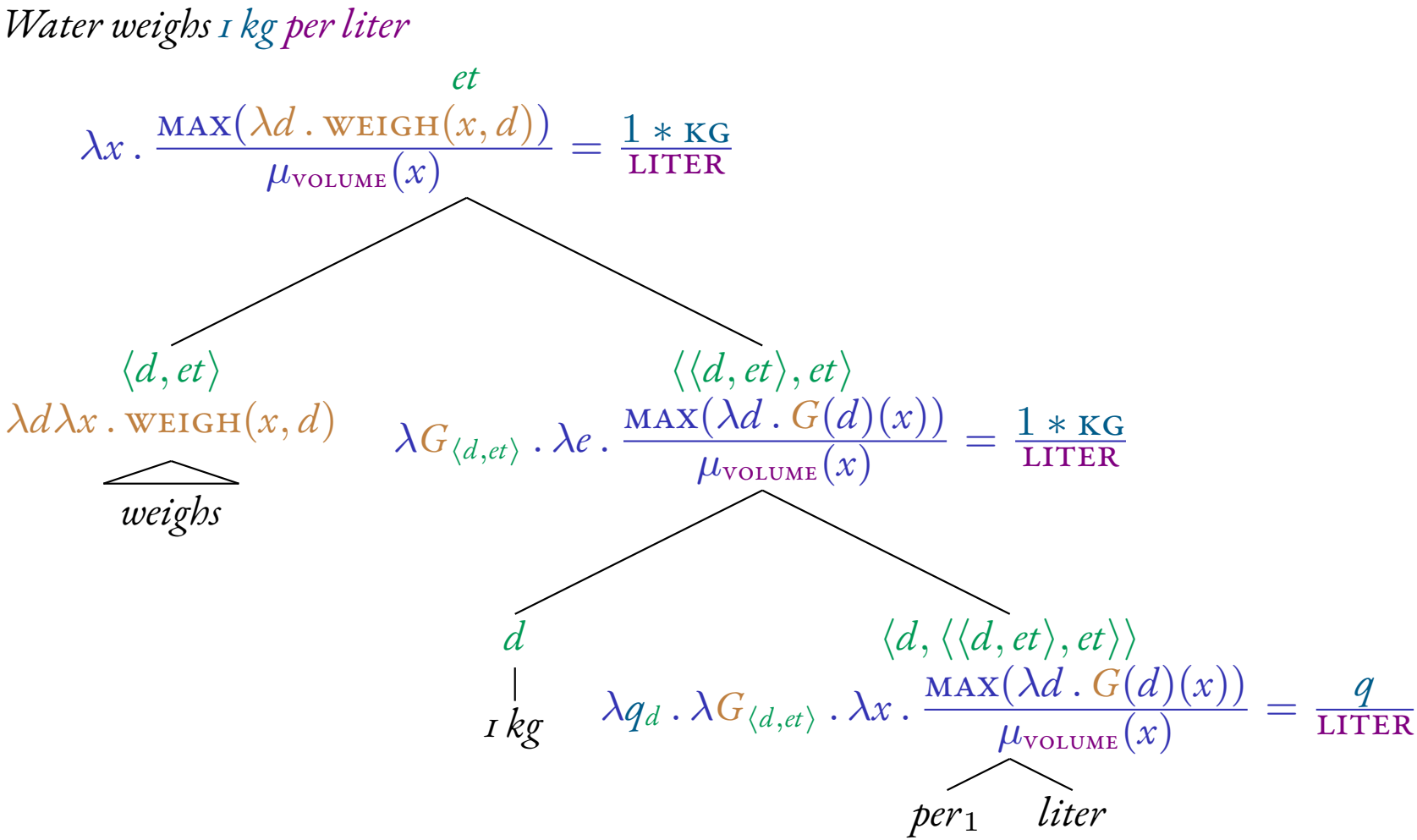
(24) Water weighs 1 kg per liter.

Schwarz & Bale (2022) point out that it does not suffice to treat '1 kg per liter' as degree-denoting term here; what water weighs is not a ratio of weight to volume.

$$\text{weight}(x) \neq \frac{1 * \text{kg}}{\text{liter}}$$

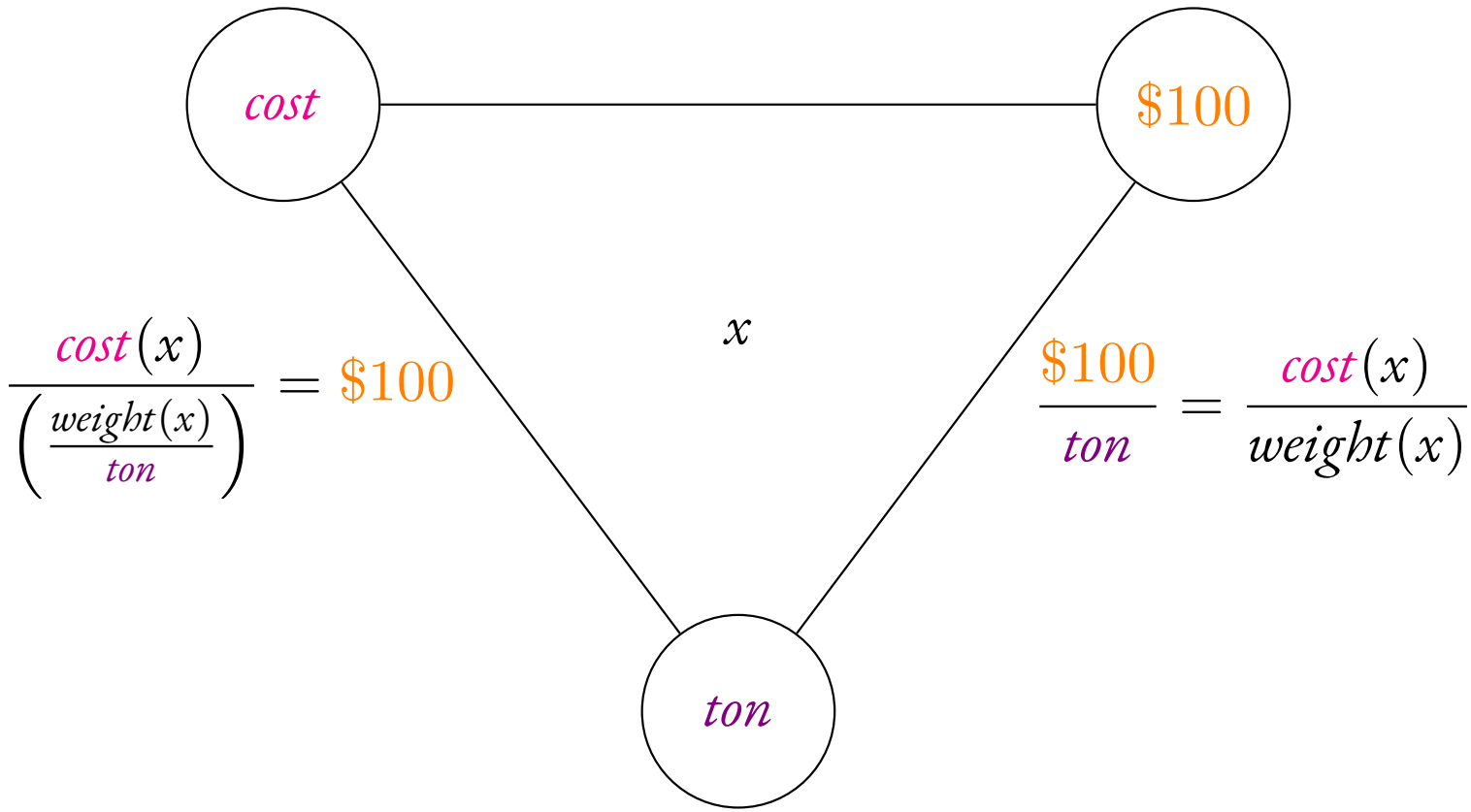
$$\dim(\text{weight}(x)) = \text{M} \qquad \dim\left(\frac{1 * \text{kg}}{\text{liter}}\right) = \frac{\text{M}}{\text{L}^3}$$

Compositional derivation for measure function verb case



Outline

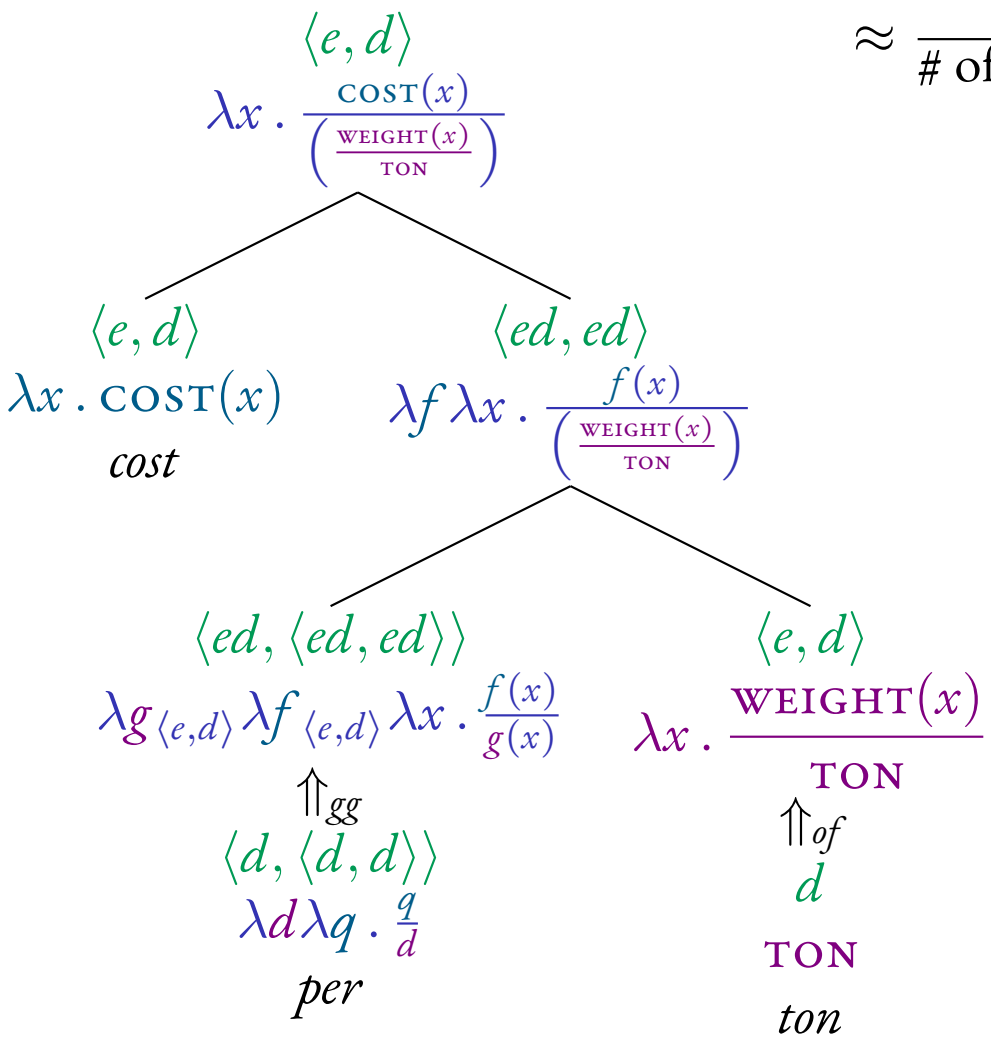
The Quotient Triangle



The cost per ton is $100

The cost is $100 per ton

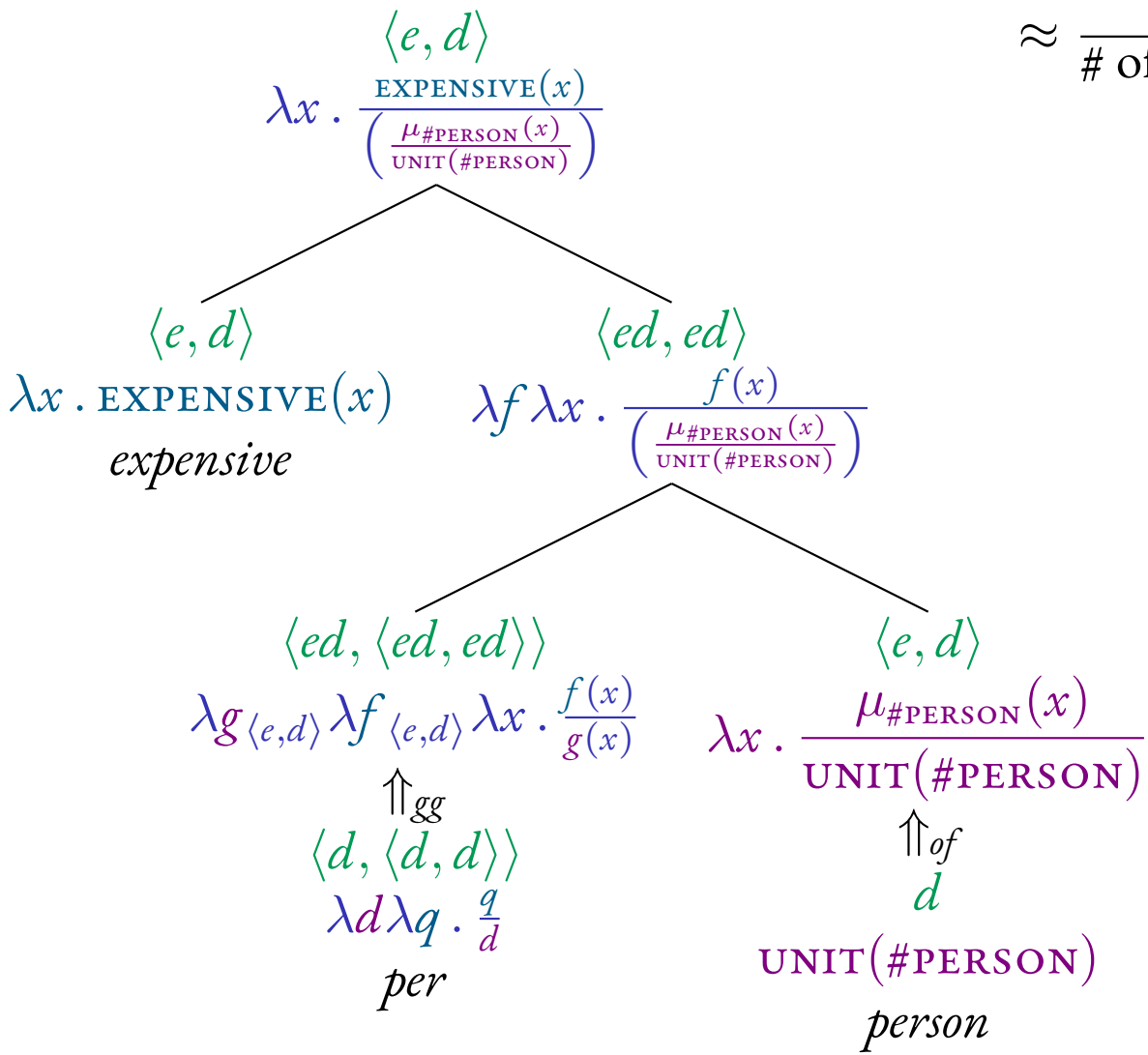
Dimension quotient analysis



$$\approx \frac{\text{cost of x}}{\text{\# of tons x weighs}}$$

Gradable predicates

(how) expensive per person



$$\approx \frac{\text{cost of x}}{\text{\# of people x measures}}$$

Lexical entries for *per*

Three ratio-marker senses of *per*:

- quotient function
  $per \rightsquigarrow \lambda d_d \lambda q_d \,.\, \frac{q}{d}$
- quotient operator
  $per \rightsquigarrow \lambda d_d \,.\, \lambda q_d \,.\, \lambda G_{\langle d,\tau t\rangle} \,.\, \lambda \alpha_\tau \,.\, \frac{\textsc{max}(\lambda d' \,.\, G(d')(\alpha))}{\mu_{\dim(d)}(\alpha)} = \frac{q}{d}$
- dimension quotient
  $per \rightsquigarrow \lambda g_{\langle e,d\rangle} \lambda f_{\langle e,d\rangle} \lambda x_e \,.\, \frac{f(x)}{g(x)}$

Conclusion

- Natural languages have vocabulary items that encode quantity definition ('ratio markers')
- Ratio markers can have scope and type-flexibility, just like other operators in the degree system.

Outline

Bridge from metrology to mereology

Why build it?
Because there seems to be one.

Empirical claims

- English *per* and Hungarian *-nként* are **ratio markers**; they express arithmetic division. (Already established.)
- **Yet,** Hungarian *-nként* has a broader distribution than *per*, encroaching into the territory of **distributive** *by*, as in *arrange by color*.
  - In fact, even English *per* ventures shyly into that territory.

These facts raise the question:

Why is it that these two concepts co-lexify in both of these languages?
Is there a common core between them that *-nként* expresses directly?

Strategy: Build a bridge.

Outline

Frequency uses of *-nként*

Originally found in temporal adverbs (Simonyi, 1888),
-nként has frequency uses:

(25) Péntek-enként / Het-enként úszni járok.
Friday-DIST / month-DIST swim.INF go.1SG
'I go swimming every Friday.'
(Dékány & Hegedűs, 2021)

unlike *per*:

(26) *I go swimming per Friday /per week.

Could be seen as:
Mereological division into equally-sized temporal units.

Pluractional uses of *-nként* (X-by-X-type)

(27) A nagymama negyed-enként vette be a nagy tablettát
the grandma quarter-DIST took the big pill.ACC
'Grandma took the big pill quarter by quarter'
(Balasz Suranyi, p.c.)

(28) *Grandma took the big pill per quarter
*... every quarter
?? ... by quarter
... quarter by quarter
... in/by quarters

Could be seen as:
Mereological division into subevents involving quarters of the pill.

Pluractional uses of Old Hungarian *-nkéd*

(29) És reggel az fráterek ajtó-nkéd kenyeret kolulának.
and morning the friars door-DIST bread begged
'And in the morning, the friars went begging for bread from door to door.'
(Bende-Farkas & Halm, 2024)

(30) *The friars went begging for bread per door
* ... every door
* ... by door
? ... door by door
* ... in/by doors
... from door to door

Could be seen as:
Mereological division into subevents, each located at a different door.

Pluractional uses of *-nként* (by-type)

(31) Mari szín-enként rendezte el a ruhákat.
Mari color-DIST arranged the clothes.ACC
'Mary arranged the clothes by color.'
(Balazs Suranyi, p.c.)

(32) *Mary arranged the clothes per color
*... every color
... by color
?? ... color by color
* ... in colors
* ... from color to color

Mereological division of the result state into substates, each corresponding to a different color of clothing.

Outline

Frequency uses

(33) Mari het-enként latogatja a nagymamát
Mary week-DIST visits the grandma.POSS.ACC
'Mary visits her grandma at least once weekly.'

How to model semantics using quantity calculus?

Maybe a silent multiplicative *egyszer* 'once'?

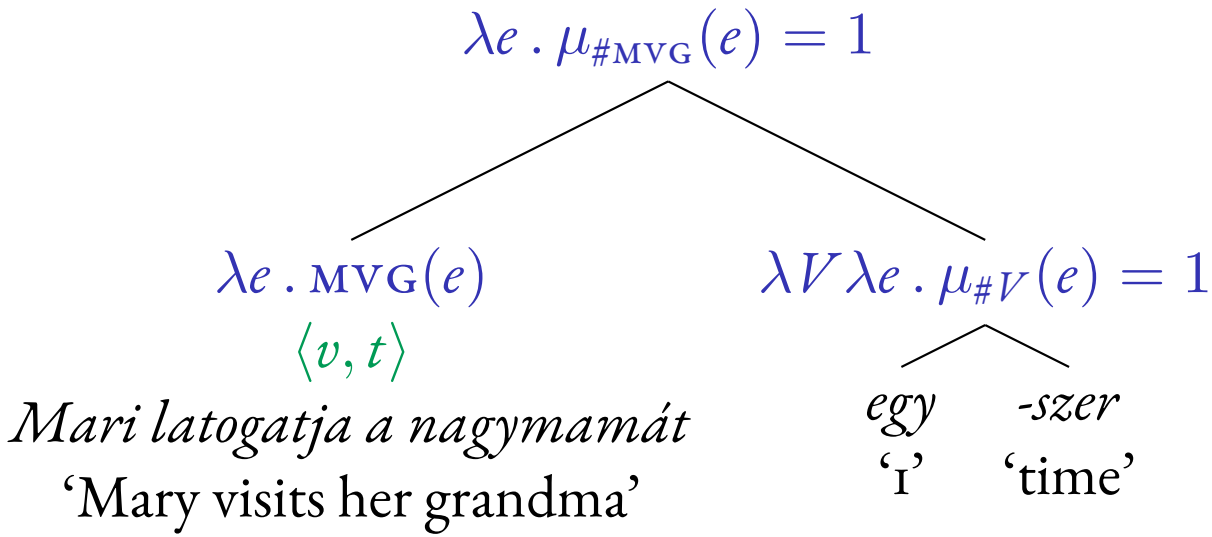
(34) Mari legalább egyszer het-enként latogatja a nagymamát
Mary at.least once week-DIST visits the grandma.POSS.ACC
'Mary visits her grandma at least once weekly.'

Visiting gramma once per week

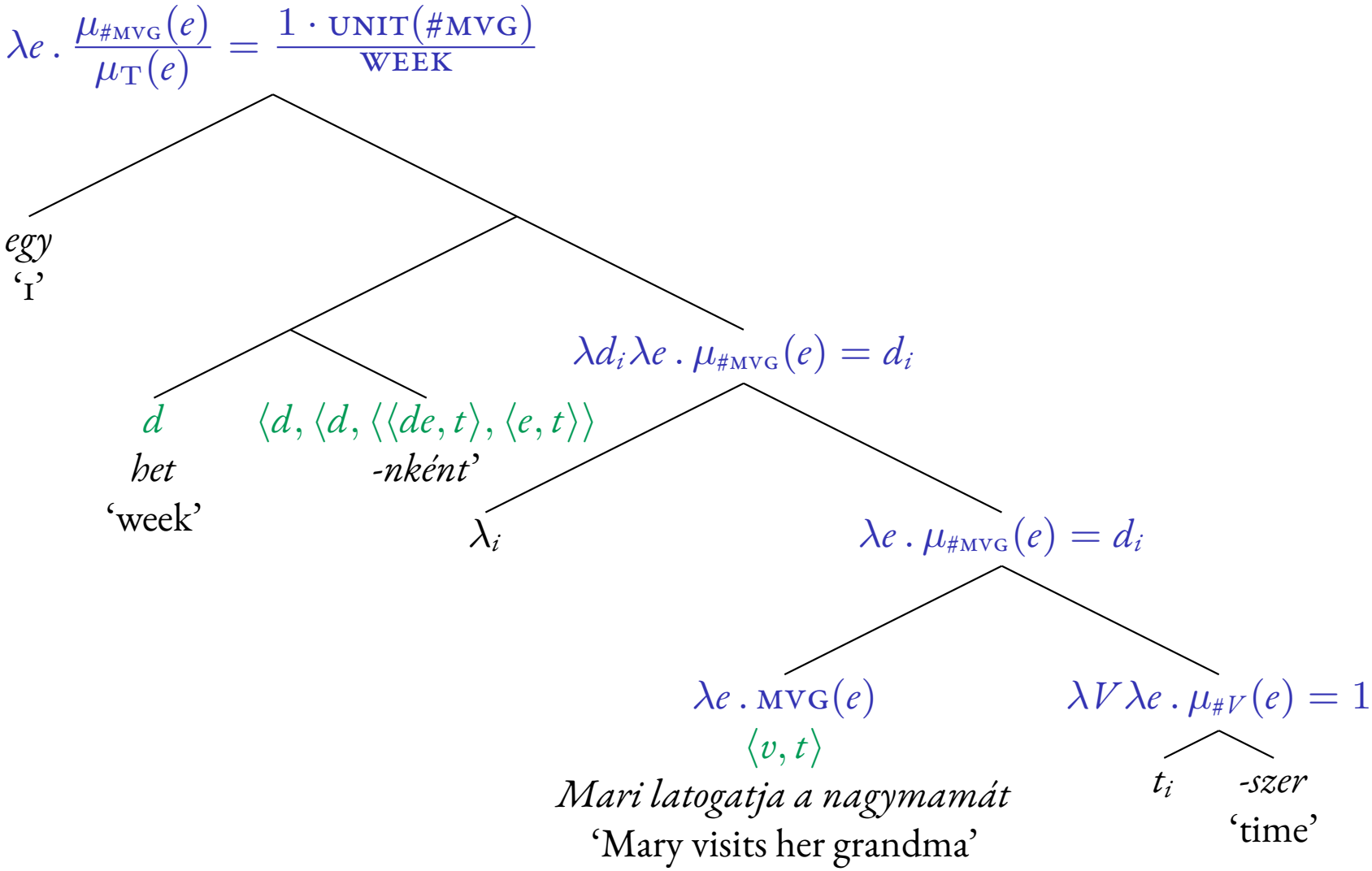
Lexical entry for the multiplicative (inspired by Wągiel 2023):
-szer $\rightsquigarrow \lambda n \lambda V_{\langle v,t \rangle} \lambda e \,.\, \mu_{\#V}(e) = n$

Assumption: For a given predicate of events V, $\mu_{\#V}(e) = n$ means that e contains as a (proper or improper) subpart exactly n instances of V.

Derivation tree:



Visiting grandma weekly



Begging from door to door

(35) És reggel az fráterek ajtó-nkéd kenyeret kolulának.
and morning the friars door-DIST bread begged
'And in the morning, the friars went begging for bread from door to door.'
(Bende-Farkas & Halm, 2024)

If we assume it's 'once per door', we can derive:

$$\lambda e \,.\, \frac{\mu_{\#\text{BEG.FOR.BREAD}}(e)}{\mu_{\#\text{DOOR}}(e)} = \frac{1 \cdot \text{UNIT}(\#\text{BEG.FOR.BREAD})}{\text{UNIT}(\#\text{DOOR})}$$

'The ratio of beggings for bread to doors in e is 1 (begging) to 1 (door).'

Towards even distributivity: Fixed ratios

An event e is **homogenous with respect to a predicate** V iff every subevent of e to which the predicate can be sensibly applied satisfies it.

Let V_R = { e | The ratio of beggings for bread to doors in e is 1 to 1.}

Can only be sensibly applied to events that can be measured in integer numbers of begging for bread and doors.

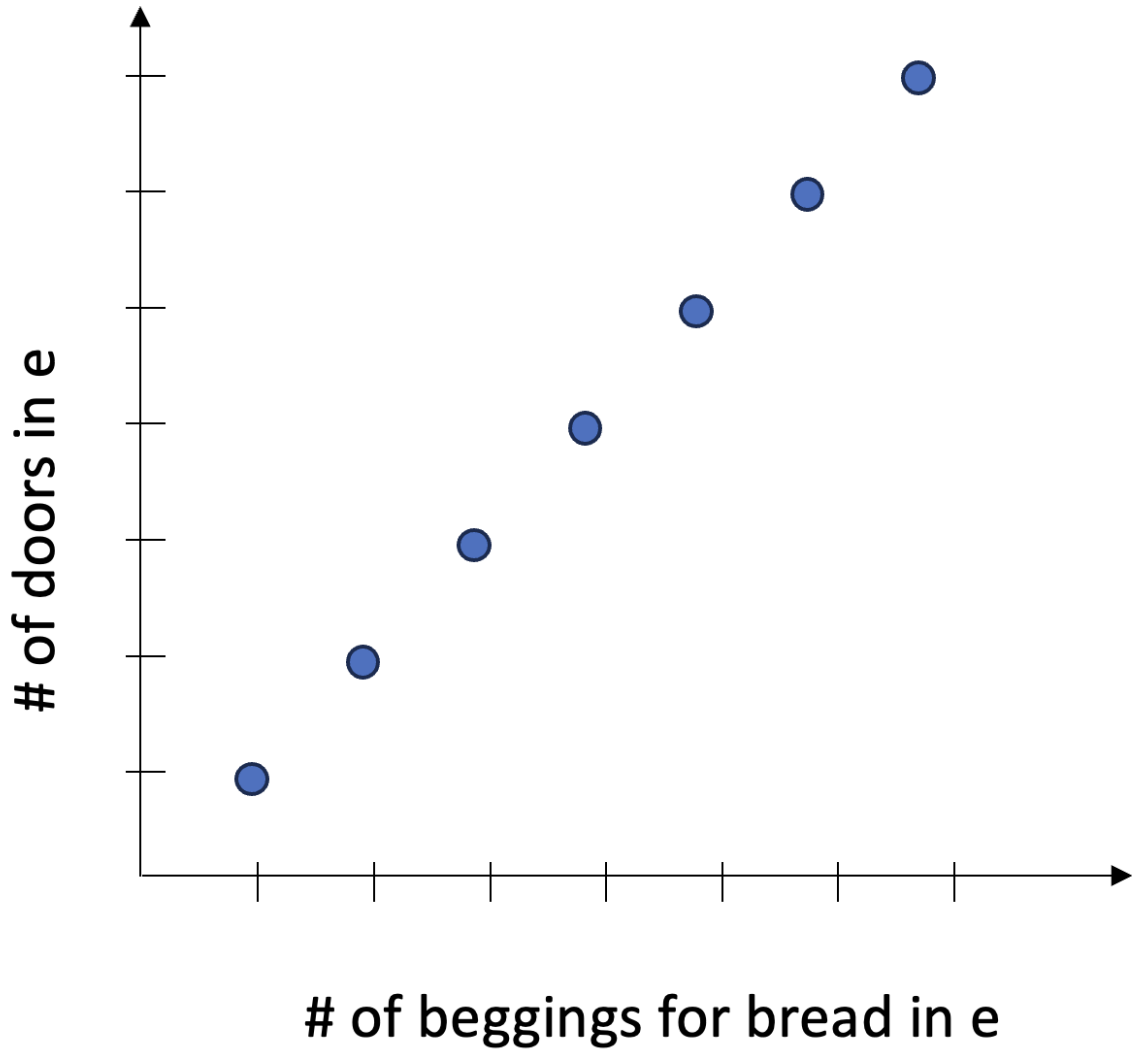
If e is homogeneous with respect to V_R, then:
For all subevents e' of e to which V_R can sensibly be applied, V_R holds.

Fixed ratio assumption (possible meaning enrichment for (35)):

e is homogenous with respect to V_R.

Cf. Champollion's 'stratified reference'

Effect of fixed ratio assumption



‘Top-down’ cases (Bende-Farkas & Halm, 2024)

Here we have only one event of taking the big pill:

(36) A nagymama negyed-enként vette be a nagy tablettát
the grandma quarter-DIST took the big pill.ACC
‘Grandma took the big pill quarter by quarter’

Here we have only one arrangement:

(37) Mari szín-enként rendezte el a ruhákat.
Mari color-DIST arranged the clothes.ACC
‘Mary arranged the clothes by color.’
(Balazs Suranyi, p.c.)

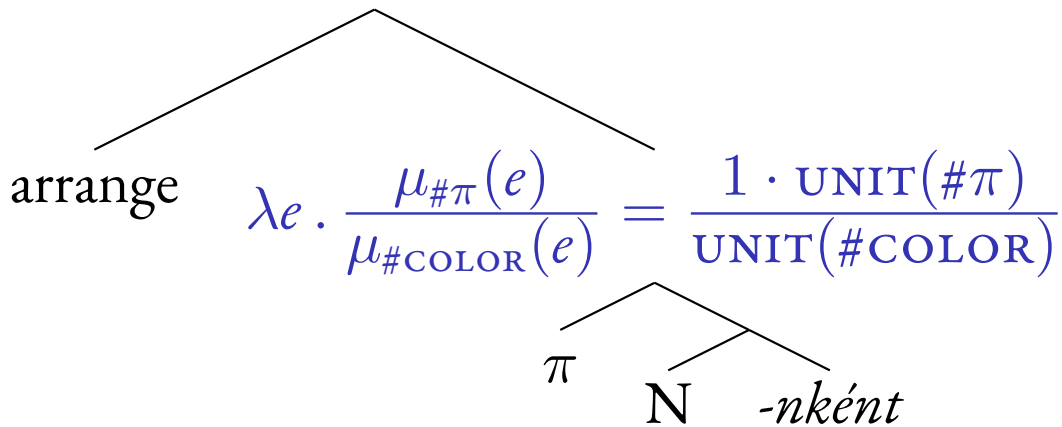
Similar to this type of use of *per* in EuroParl:

(38) The complete table with a breakdown of all applications per prior right and country of applicant can be found on the website.

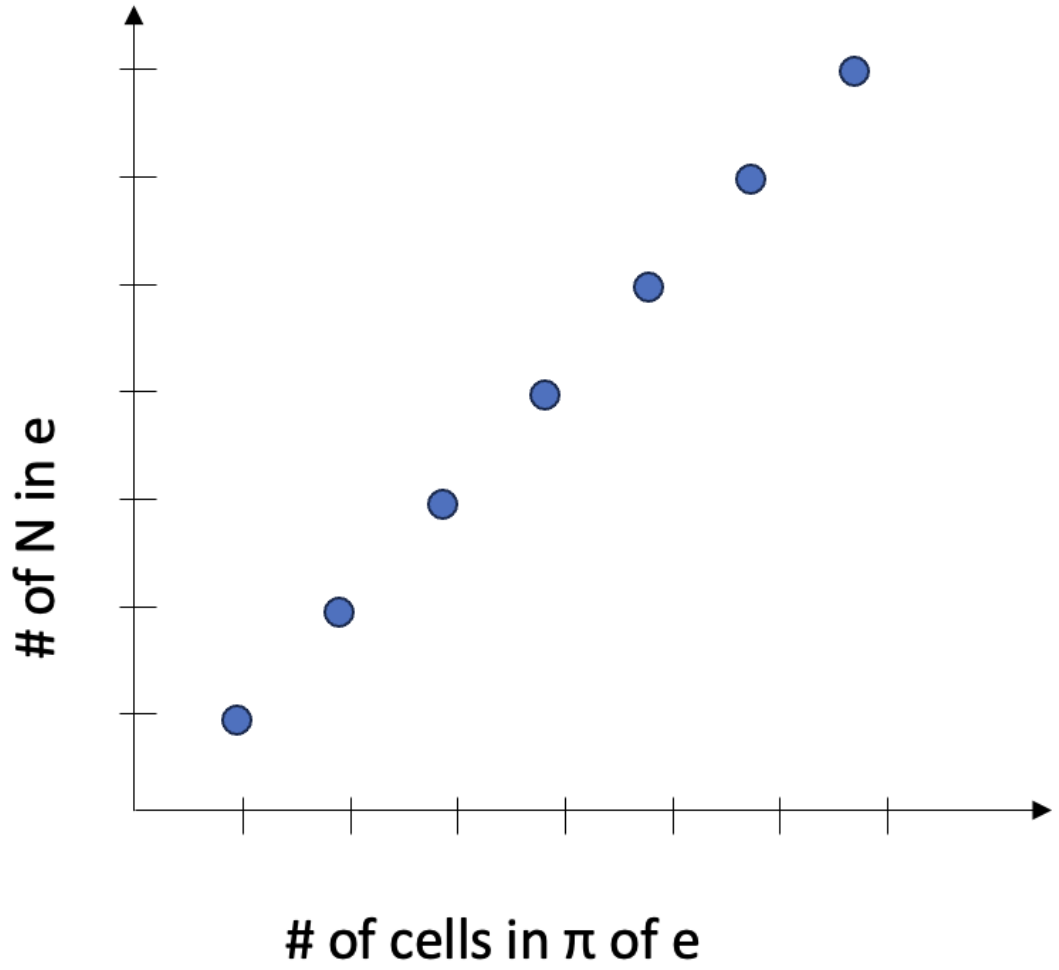
Part-based approach to distributive *-nként*

Let π be a salient partition of e (possibly a state).
Let $\mu_{\#\pi}(e')$ count the number of cells in π that e' instantiates.

Analysis:



With homogeneity assumption



Outline

Conclusions

- Quantity calculus is useful in natural language semantics
  - For example, English *per* and Hungarian *-nként* are ratio markers;
    they express arithmetic division
  - They show scope and type flexibility,
    just like other degree operators
- Arithmetic and mereological division are conceptually adjacent
  - *-nként* picks out a concept covering both of them
  - An equivalence can be obtained via cardinality dimensions, with appeal to a salient partition

Thank you!

And thanks to members of the audiences at MIT, BU, Yale, and NYU for discussions on earlier versions of some of the material.

Enormous thanks to Balazs Suranyi and the members of the audience at ELTE in Budapest for help and ideas on *-nként*.

Thanks to the research assistants in LiSLab who have been working with me to develop parallel corpora of ratio expressions.

Special thanks to undergraduate research assistant Nate Lambert, whose observations regarding the taxonomy of verbally-licensed uses of ratio markers in the EuroParl corpus helped helped me see the connection between arithmetic and mereological division.

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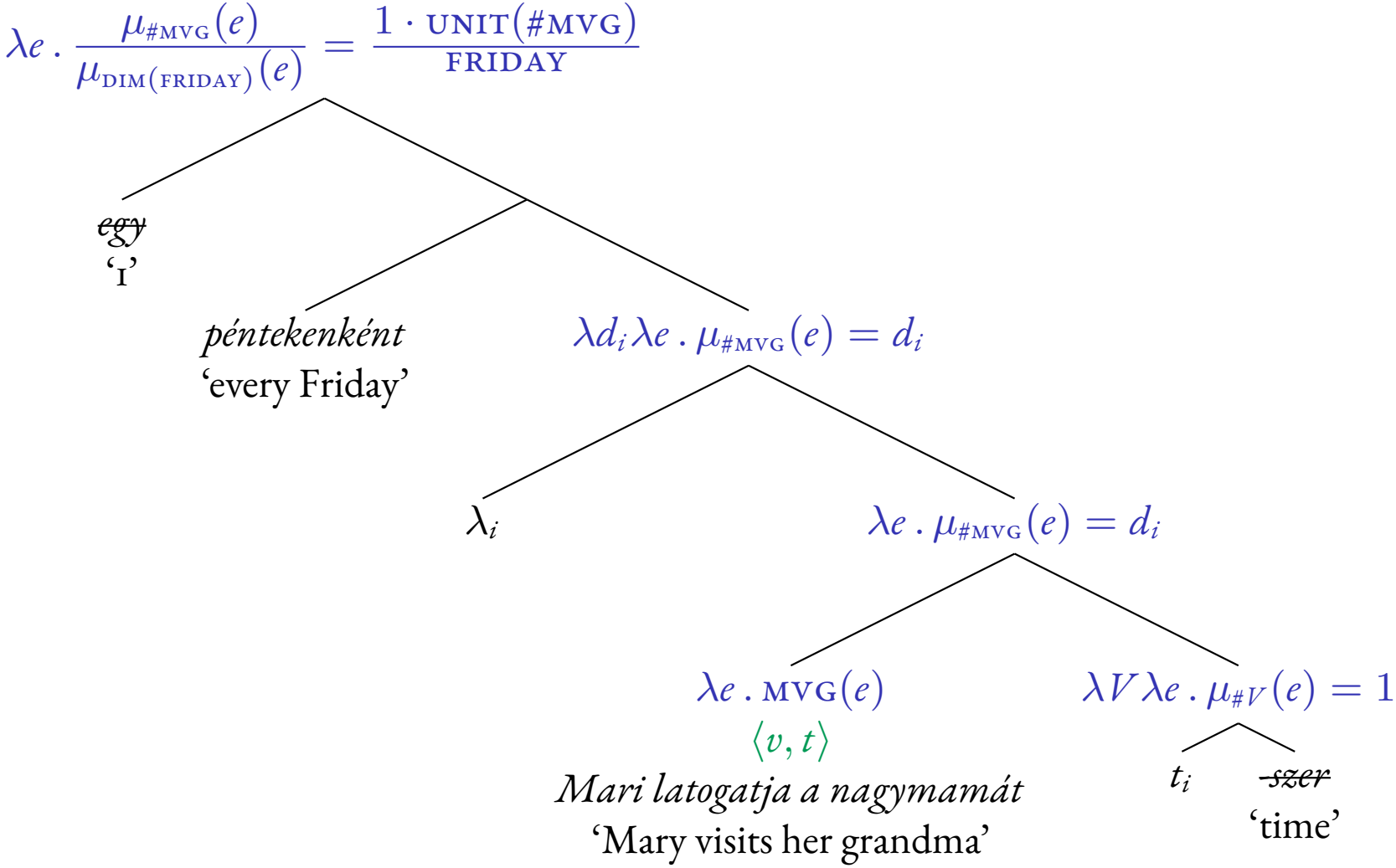
Outline

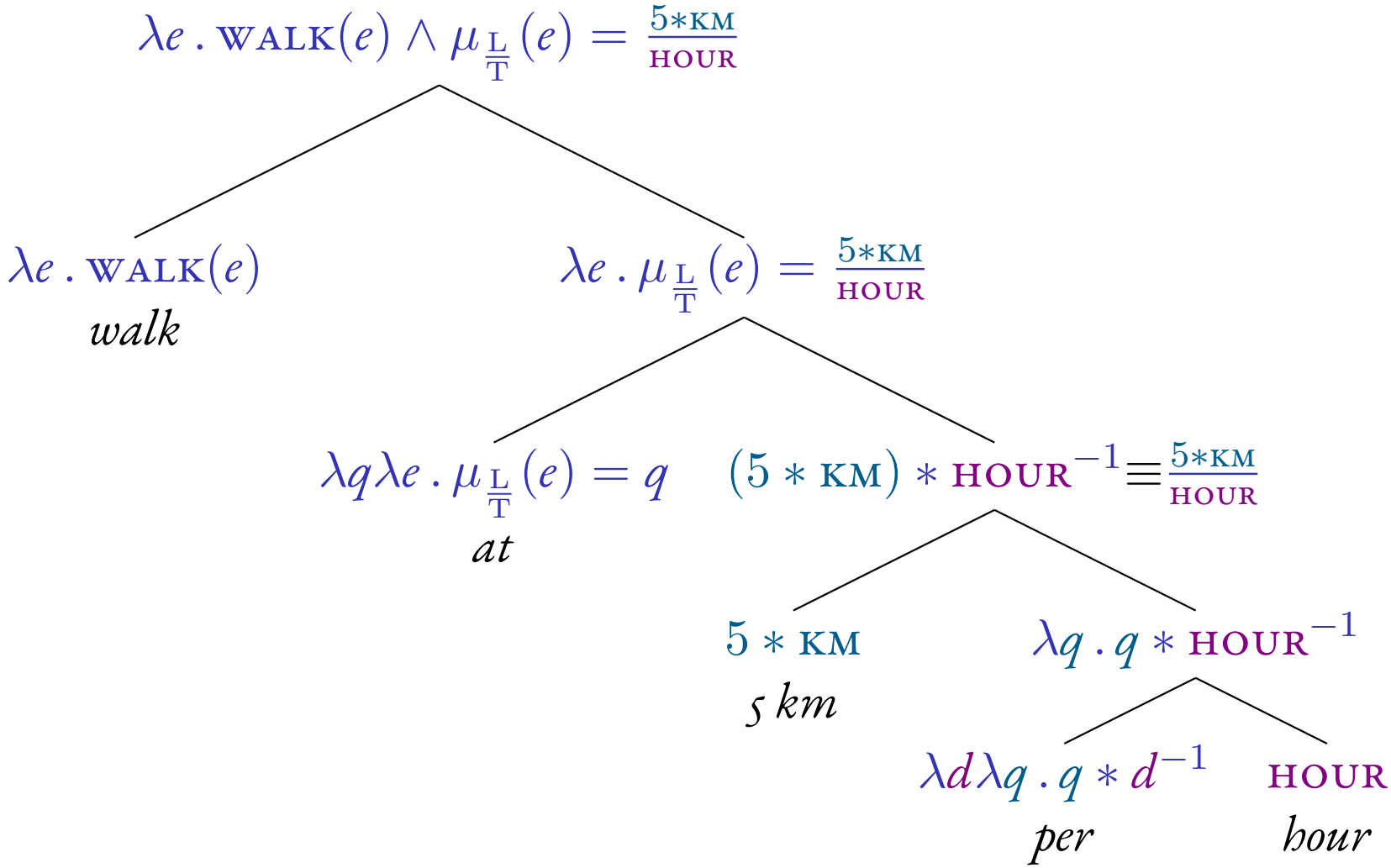
Cut slides

Triangle equivalences

Example with *péntekenként*

Assume that *péntek* ‘Friday’ can be zero-derived to an expression of type d.





$$\mu_{\frac{A}{B}}(\alpha) * \mu_B(\alpha) = \mu_A(\alpha)$$

Axiom (or 'meaning postulate')

Assumption: Speed times time equals distance

$$\mu_{\frac{\mathrm{L}}{\mathrm{T}}}(e) * \mu_{\mathrm{T}}(e) = \mu_{\mathrm{L}}(e)$$

More generally: μ-product principle:

$$\mu_A(\alpha) * \mu_B(\alpha) = \mu_{A \cdot B}(\alpha)$$

Corollary of the μ-product principle:

$$\mu_{\frac{A}{B}}(\alpha) * \mu_B(\alpha) = \mu_A(\alpha)$$

Quine on *entia non grata*

On "what to admit to the universe of values of our variables quantification" (p. 243, *Word and Object*):

"Units of measure turn out somewhat like sakes and behalves. 'Mile', 'minute', 'degree Fahrenheit', and the like resemble 'sake' and 'behalf' in being *defective* nouns: they are normally used only in a limited selection of the usual term positions. Their defectiveness... is easily exposed in absurd interrogation. **Are miles alike? If so, how can they count as many? And if they cannot, what of the two hundred between Boston and New York?** ... We can adequately accommodate these nouns as parts of the relative terms 'length in miles', 'temperature in degrees Fahrenheit'." (p. 245)

Is there multiplication and/or division in natural language?

Scalar times a quantity:

- proportional readings of *many* (Milsark, 1974; Partee, 1989; Westerståhl, 1985; Solt, 2009; Romero, 2020; Bale & Schwarz, 2020)
- *average* (Kennedy & Stanley, 2009)
- complex cardinals like *five hundred* (Ionin & Matushansky, 2006)
- *twice* (Sassoon, 2010b,a)
- *percent*, *thirds*, their translational equivalents (Ahn, 2012; Ahn & Sauerland, 2015, 2017; Coppock, 2022b; Gehrke & Wągiel, 2023)

Cross-dimensional division:

- English *per* (Coppock, 2021, 2022a; Schwarz & Bale, 2022)
- Italian *per* (Panaitescu & Tovena, 2019)
- Hungarian *-nként* (Bende-Farkas & Halm, 2024)

Ratio markers: *per* and *-nként*

Rates of pay:

(39) Kutyá-nként 5000 Ft adó-t kell fizetni
dog-DIST 5000 HUF tax-ACC must pay.INF
'One must pay 5000 Hungarian forints per dog.'
(Dékány & Hegedűs, 2021, 109)

Densities:

(40) köb-méter-enként 20 mikrogramm
cube-meter-DIST 20 mikrogram
'20 micrograms per cubic meter'

These:

- are not just distributivity markers; they express quantity division
- have scope and type-flexibility just like other degree operators

Venturing into distributive territory

The distribution of *-nként* is broader than that of *per*:

(41) Mari szín-enként rendezte el a ruhákat.
Mari color-DIST arranged the clothes.ACC
'Mary arranged the clothes by/*per color.'
(Balazs Suranyi, p.c.)

Although *per* does have uses paraphrasable with *by*:

(42) a. I shall now give a short resume of our findings per country.
(EuroParl)
b. ... by country.
c. ... country by country.

So, cool fact:
Mereological division and arithmetic division can co-lexify.

Outline

Cut slides

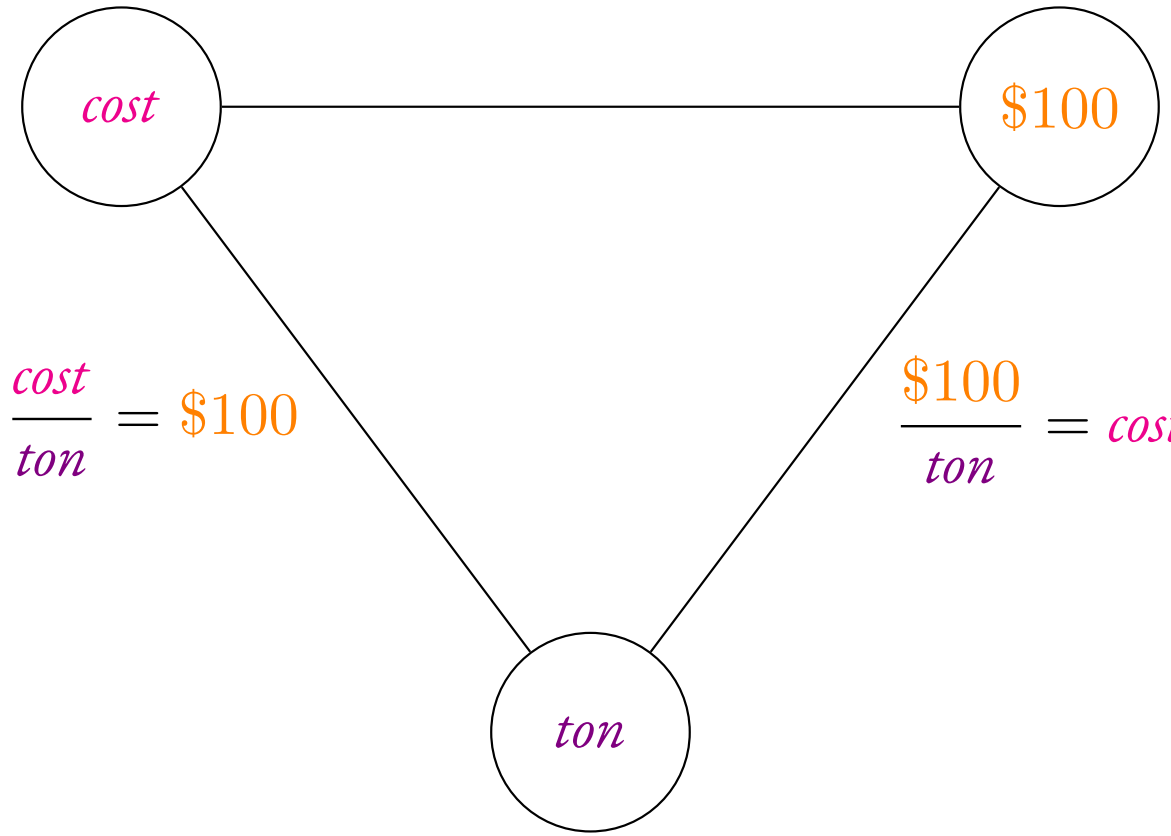
Triangle equivalences

Triangle Equivalences

(43) a. The cost of wheat is [$100 per ton].
b. The [cost of wheat per ton] is $100.

(44) a. a shortfall of [100 billion euros per annum]
b. a [per annum shortfall] of 100 billion euros

The Quotient Triangle



The cost per ton is $100

The cost is $100 per ton