UNIFYING MEREOLOGICAL AND ARITHMETIC DIVISION

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Outline

Hungarian *-nként* and English *per*

Today's talk will center around Hungarian *-nként*:

(1) ... hogy hajó-nként egy tudós-t alkalmazzunk,
... that boat-___ one scientist-ACC employ.SBJ.1PL
'[It's not realistic] that we employ one scientist per vessel'
(EuroParl corpus)

How to gloss *-nként*?

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- DIST 'distributive'
(Tompa, 1968; Kenesei et al., 1998; Csirmaz & Szabolcsi, 2012; Dékány & Hegedűs, 2021)

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- DIST 'distributive'
(Tompa, 1968; Kenesei et al., 1998; Csirmaz & Szabolcsi, 2012; Dékány & Hegedűs, 2021)
- I gloss it as DIV for 'division' (ambiguity intended).
Division can be either arithmetic or mereological.

Event-mereology analysis of binominal *each*

They ate two olives each$_{\theta_{\text{agent}}}$ (Champollion 2017)

They = key; two olives = share

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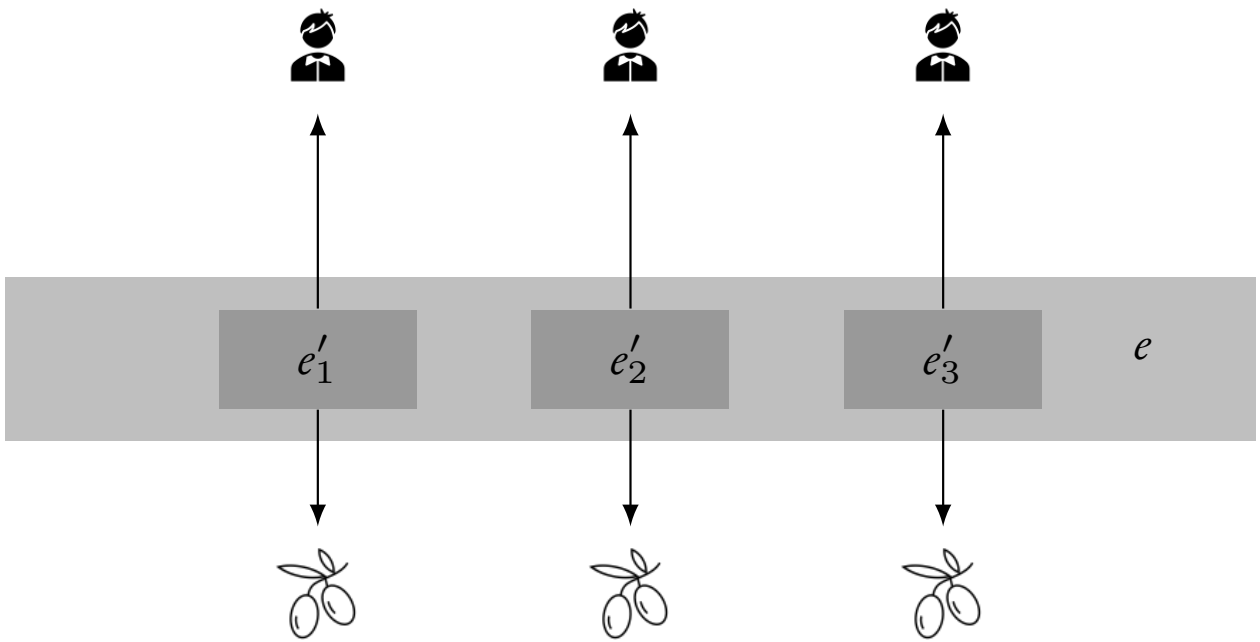

Distributivity marker analysis of *per* (or *-nként*)

James Bond ate two olives per martini (Panaitescu & Tovena 2019)

martini = key; two olives = share

'Match' function inspired by Boolos (1981), Rothstein (1995).

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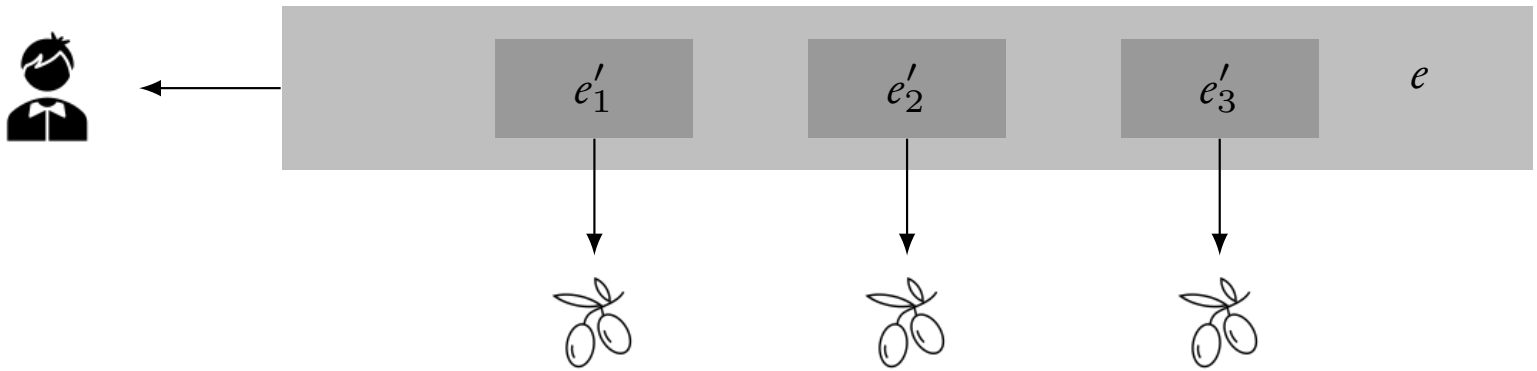

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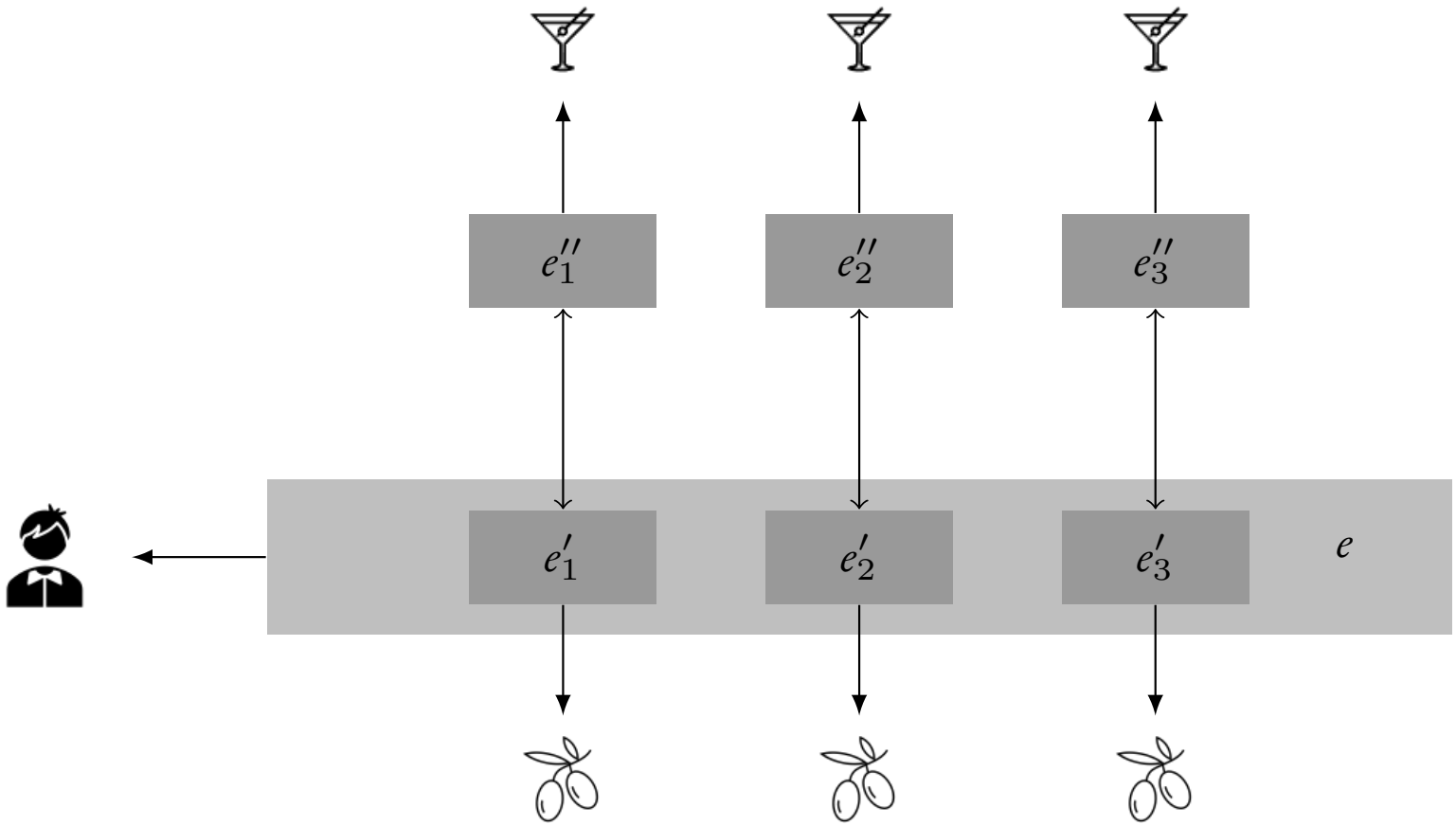

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Ratio marker analysis of *per* (or *-nként*)

James Bond ate two olives per martini

$$\frac{\text{olives}}{\text{martinis}} = \frac{\text{olives eaten in } e}{\text{martinis drunk in } e}$$

What I hope to convince you of

- A distributivity marker suffices neither for *per* nor for *-nként*; they (at least sometimes) express arithmetic division.
- But *-nként* also expresses mereological division of eventualities.
- Hence mereological division co-lexifies with arithmetic division.
- So they are adjacent in conceptual space and form a natural class.
- A unified analysis can be obtained via some tricks involving measurement and partitions.

Outline

Distributivity marker analysis (again)

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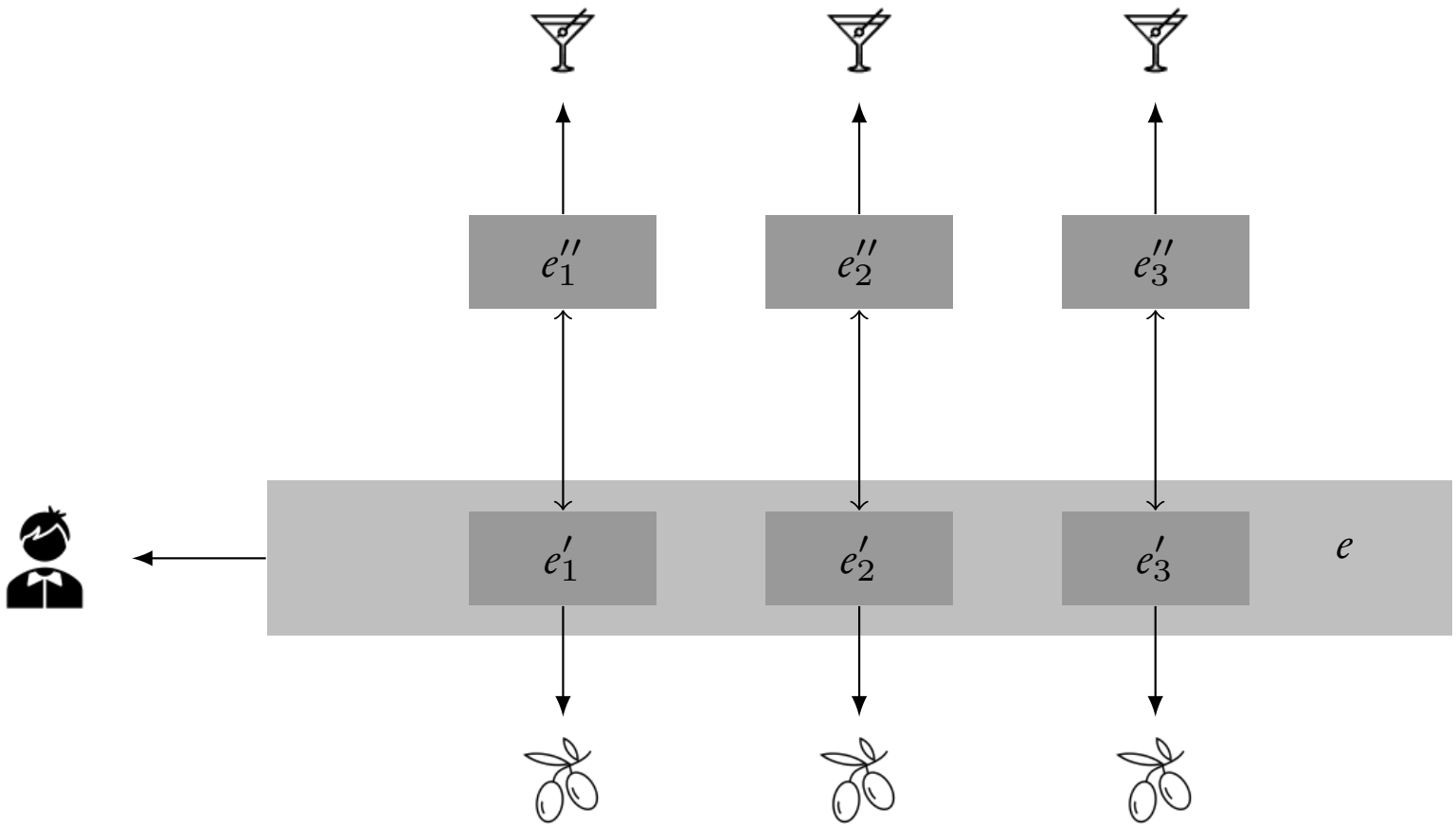

Outline

Predictions of distributivity marker analysis

- **Minimal size requirement**: The eventuality described by the clause should be divisible into one or more 'key'-sized chunks.
- **Uniformity requirement**: The eventuality described by the clause should be divisible into subevents that uniformly manifest both the share and the key.
- **Indefinite share requirement**: A *per* phrase should only be able to modify (cardinal) indefinites.
- **Event predicates not terms**: A *per* phrase, together with its licensor, creates a predicate characterizing an event, and not a (degree-denoting) term.

No minimal size requirement for *per*

Unlike with *each*, the event is not always divisible into 'key'-sized chunks with *per*:

(2) James Bond drove 100 km per hour.
$\not\Rightarrow$??For each hour, James Bond drove 100 km.
(Event could last five minutes.)

(3) Do arm swing drills at 240 steps per minute for 20 seconds.

Call these 'sub-unit cases'.

Sub-unit cases with Hungarian *-nként*

Csirmaz & Szabolcsi (2012) mention *-nként* under 'rate expressions' and give the following example:

(4) Az a vonat óra-nként 400 kilométer-rel halad
that the train hour-DIV 400 km-INST advances
'That train is travelling at 400 km/hour'

The event is not necessarily composed of hour-long subevents.

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Non-uniform scenarios

English:

(5) The Montreal Canadiens scored 2.82 goals per game in 2020-21.

Hungarian:

(6) a nitrát-irányelv 1,7 számosállat-egységről rendelkezik
the nitrate-directive 1.7 livestock-units provides.for
hektáronként.
hectare-DIV
'The Nitrates Directive provides for 1.7 livestock units per hectare.'

There are no subevents involving 1.7 livestock units.

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Gradable predicates

English *per* allows gradable predicate hosts, unlike adnominal *each*:

(7) a. It's $2 {per person, each}.
b. It's cheaper {per person, *each}.

Hungarian *-nként* is like *per* in this respect:

(8) Fej-enként olcs-óbb is, és környezetbarát-abb.
head-DIV cheap-CMPR also, and environmentally.friendly-CMPR
'It's cheaper per person, and more environmentally friendly.'

Dimension nouns

English *per* can be hosted by measure function-denoting nouns, unlike adnominal *each*:

(9) a. The price is $2 {per person, each}.
b. The price {per person, *each} is $2.

(10) a kilométer-enként kivetett díj csak növeked-het-ne
the km-DIV levied fee only increase-could-would
'The fee levied per km would only potentially increase.'

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Term uses as differential argument of comparative

English:

(11) There are tables of various woods that put mahogany 200 kg per cubic meter **denser** than poplar.

Hungarian:

(12) kilométer-enként két perc-cel **gyors-abb** tempó-t ment.
kilometer-dist 2 minute-with fast-er tempo-acc go.3sg
'it went two minutes faster per kilometer'

(Cf. Rawlins 2013 on differential arguments)

Inference patterns with term uses

(13) a. Az orvos nap-onként egy beteget kezelt.
the doctor day-DIV a patient-ACC treats
'The doctor treats one patient per day.'

b. ⇒ Az orvos tegnap kezelt egy beteget.
the doctor yesterday treats one patient-acc
'The doctor treated one patient yesterday.'

(14) a. Az orvos felírt naponként egy tablettát.
the doctor prescribed day-DIV a pill-ACC
'The doctor prescribed one pill per day.'

b. ⇏ Tegnap az orvos felírt egy tablettát.
yesterday the doctor prescribed a pill-ACC
'The doctor prescribed a pill yesterday.'

(Balazs Suranyi, p.c.)

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Outline

Representation language

$\mathcal{L}_{\mathcal{Q}}$: a λ-calculus with **dimension-centric quantity calculus**

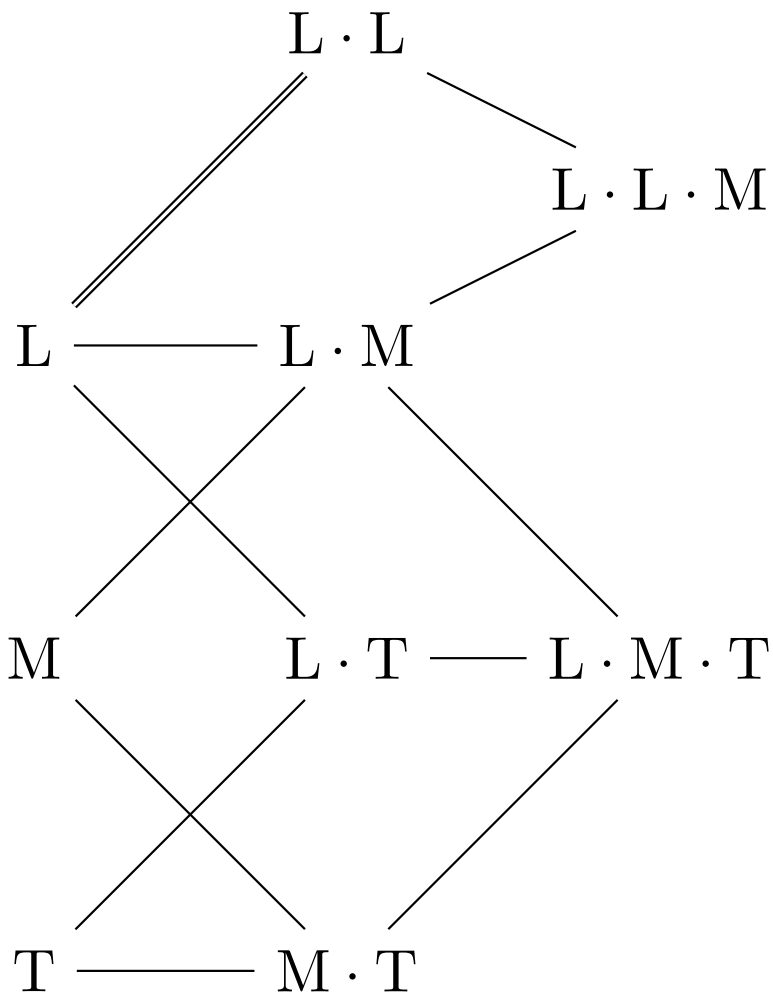
The semantic value of an expression ϕ in $\mathcal{L}_{\mathcal{Q}}$ is given by $[\![\phi]\!]^{\mathcal{M}}$, where:

$$\mathcal{M} = \langle [\langle \mathcal{D}_e, \oplus_e \rangle, \langle \mathcal{D}_v, \oplus_v \rangle, \langle \mathcal{D}_i, \oplus_i \rangle, \langle \mathcal{D}_d, +, * \rangle, \langle \mathcal{D}_m^{\mathscr{B}}, \cdot \rangle], \mathcal{I} \rangle$$

The history of quantity calculus goes back to Fourier 1822 (de Boer, 1994) and is studied in the field of metrology; see for example the International Vocabulary of Metrology (VIM). Here I borrow a dimension-centric approach from (Raposo, 2018).

International System of Units (SI)

Basic and derived dimensions



The dimensions form a group under multiplication

$\mathcal{D}_m = \mathscr{D}$ is a group under $\cdot$, so:

- if $\mathrm{A}, \mathrm{B} \in \mathscr{D}$, then $\mathrm{A} \cdot \mathrm{B} \in \mathscr{D}$
- $\mathscr{D}$ has an identity element $\mathbf{I}_{\mathscr{D}}$, such that for every $D \in \mathscr{D}$:

$$D \cdot \mathbf{I}_{\mathscr{D}} = \mathbf{I}_{\mathscr{D}} \cdot D = D$$

- There is a multiplicative inverse D^{-1} for every $D \in \mathscr{D}$: an element such that

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$$D \cdot D^{-1} = \mathbf{1}_{\mathscr{D}}$$

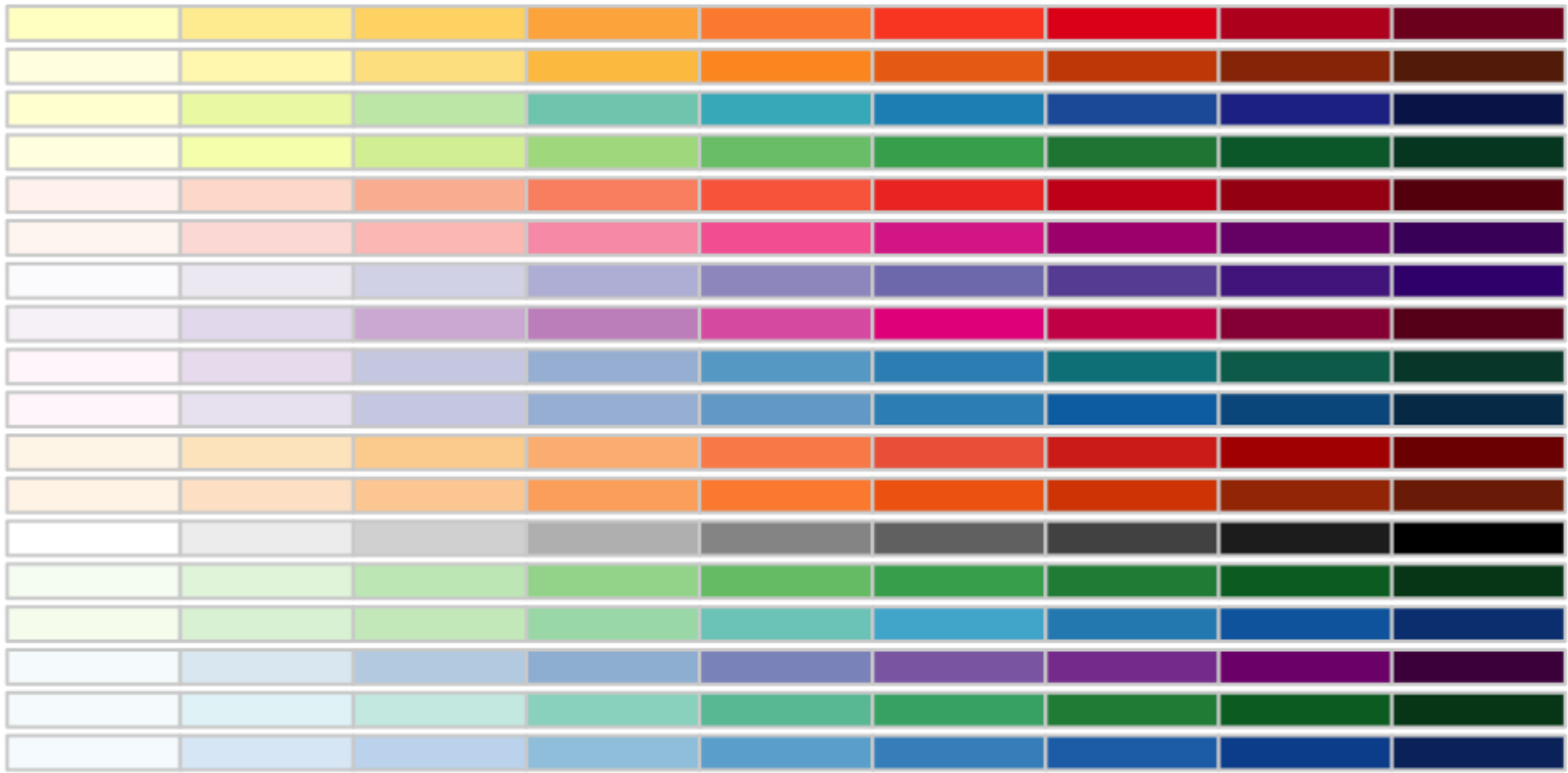
Example:

$\frac{\mathrm{M}}{\mathrm{L}^3}$ is the dimension 'weight per volume'.

Mapping from quantities to dimensions

$$\mathscr{Q} \xrightarrow{\dim} \mathscr{D}$$

The space of quantities forms a fiber bundle



Each fiber is a vector space, with its own additive identity (zero) element.

Cross-dimensional multiplication

$\langle \mathscr{Q}, * \rangle$ is an **abelian monoid**, so:

- If $q_1, q_2 \in \mathscr{Q}$, then $q_1 * q_2 \in \mathscr{Q}$
- There is a **multiplicative identity** element $\mathbf{1}$ such that for all $q \in \mathscr{Q}$:

$$q * \mathbf{1} = \mathbf{1} * q = q$$

- If $q_1, q_2, q_3 \in \mathscr{Q}$ then
  $q_1 * (q_2 * q_3) = (q_1 * q_2) * q_3$ **(associativity)**
- $q_1 * q_2 = q_2 * q_1$ **(commutativity)**

Existence of inverses

$\langle \mathscr{Q}, * \rangle$ is an **abelian monoid**, not a group.

Not every quantity has an inverse; you can't divide by any $\mathbf{o}_D$ ($D \in \mathscr{D}$).

But for every *non-zero* quantity $q \in \mathscr{Q}$
there is an inverse q^{-1}:

$$q * q^{-1} = \mathbf{1}$$

Or: The set of non-zero quantities forms a group under multiplication.

Unit mapping

$$\mathscr{Q} \xleftarrow{\text{unit}} \mathscr{D}$$

where $\text{unit}(D)$ picks out a q such that $\dim(q) = D$

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Restrictions:

- You can't pick the zero element (the additive identity).
- unit must be a group homomorphism:

$$\text{unit}(\text{A} \cdot \text{B}) = \text{unit}(\text{A}) * \text{unit}(\text{B})$$

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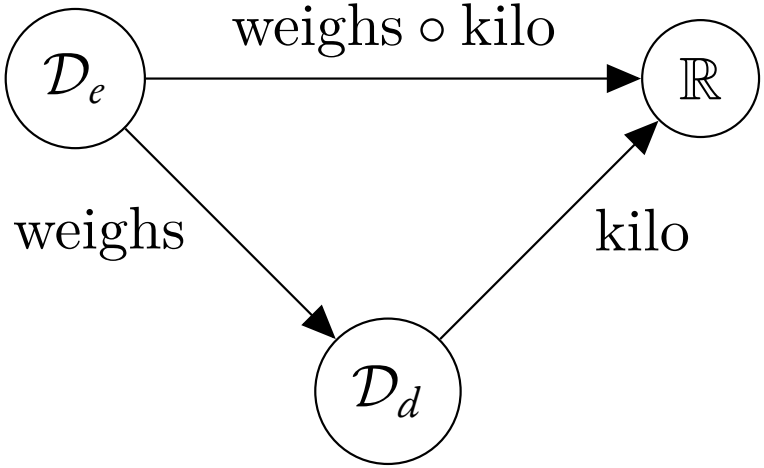
where:

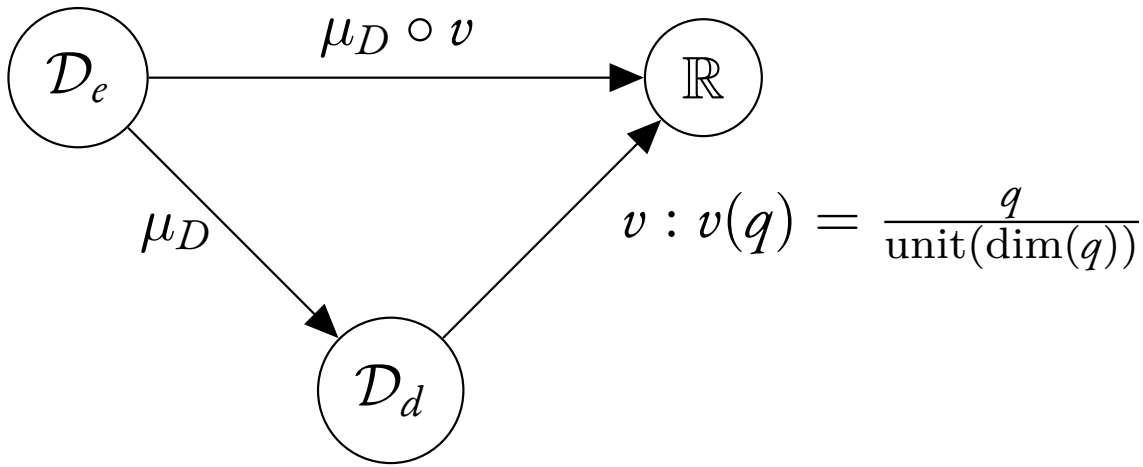
- $\mathscr{B}$ is a finite set of primitive dimensions
- $\langle \mathcal{D}_m^{\mathscr{B}}, \cdot \rangle$ is an abelian group with basis $\mathscr{B}$, a finite set of dimensions
- $\langle \mathcal{D}_d, * \rangle$ is an abelian monoid
- $\mathcal{I}(\textsc{unit})$ is a group homomorphism from $\mathscr{D}_m^{\mathscr{B}}$ to $\mathcal{D}_d$
- $\mathcal{I}(\textsc{dim})$ is a surjection map from $\mathcal{D}_d$ onto $\mathcal{D}_m^{\mathscr{B}}$
- For each $D \in \mathcal{D}_m^{\mathscr{B}}$, $\langle \mathcal{D}_d | D, +, * \rangle$ is a vector space over $\mathbb{R}$
- $\mathcal{I}$ maps each constant of type τ to an element of $\mathcal{D}_\tau$

The Lønning Triangle



(Lonning, 1987; Champollion, 2017)

The Lønning Triangle (à la metrologique)



μ_D: canonical measure function for dimension D

What dimensions does natural language make use of?

Complements of *per* in EuroParl:

distance: *kilometre (of intra-Community trade)*, *100 km*
area: *hectare/decare (of arable land)*, *square metre (live weight)*
time: *annum*, *calendar year*, *day*, *24 hours*, *season*
volume: *cubic centimetre*, *hectolitre (of pure alcohol)*, *litre (of milk)*
weight: *kilo (of fertilizer)*, *reduced tonne of greenhouse gas*
power: *kilowatt (produced)*, *megajoule*
energy: *energy unit*, *kilowatt-hour (sold)*, *kW/hour*
extensive: *unit (of output/production/quantity/food)*
effort: *unit of effort*
information: *megabyte*
money: *euro (of subsidy)*, *mille of GNP*, *year of EU funding*

Cardinality denominator dimensions in EuroParl

card:human: *capita*, *head of population*, *child*, *farmer*, *taxpayer*, *pupil*
card:animate: *bird*, *fish*, *hen*, *ewe*, *million adult cattle*, *1000 animals*
card:organization: *household*, *farm*, *power station*, *country*, *NGO*
card:tangible: *car*, *cigarette*, *goods vehicle*, *olive tree*, *ship*, *dwelling*
card:intangible: *paragraph*, *policy area*, *category of cars*, *job created*
card:location: *continent*, *zone*, *region*, *port*, *lake*
card:event: *session*, *Presidency*, *accident*, *death*, *flight*, *money withdrawal*
card:human / distance: *passenger kilometer*

Cardinality dimensions (individuals)

Let us assume that for every subset of D_e $P \in D_{\langle e,t \rangle}$, there is a basic dimension $\#\textsc{dim}(P)$. I call these 'cardinality dimensions'.

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Sortal nouns are ambiguous, e.g.:

$martini \rightsquigarrow \lambda x \,.\, \textsc{martini}(x)$ (type $\langle e, t \rangle$)

$martini \rightsquigarrow \textsc{unit}(\#\textsc{martini})$ (type d)

Outline

Lexical entries for *per*

Three ratio-marker senses of *per*:

- quotient function
  $per \rightsquigarrow \lambda d_d \lambda q_d \,.\, \frac{q}{d}$
- quotient operator
  $per \rightsquigarrow \lambda d_d \,.\, \lambda q_d \,.\, \lambda G_{\langle d,\tau t\rangle} \,.\, \lambda \alpha_\tau \,.\, \frac{\textsc{max}(\lambda d' \,.\, G(d')(\alpha))}{\mu_{\dim(d)}(\alpha)} = \frac{q}{d}$
- dimension quotient
  $per \rightsquigarrow \lambda g_{\langle e,d\rangle} \lambda f_{\langle e,d\rangle} \lambda x_e \,.\, \frac{f(x)}{g(x)}$

Challenge for the quotient function analysis

(15) It's estimated that 150 species per day go extinct.

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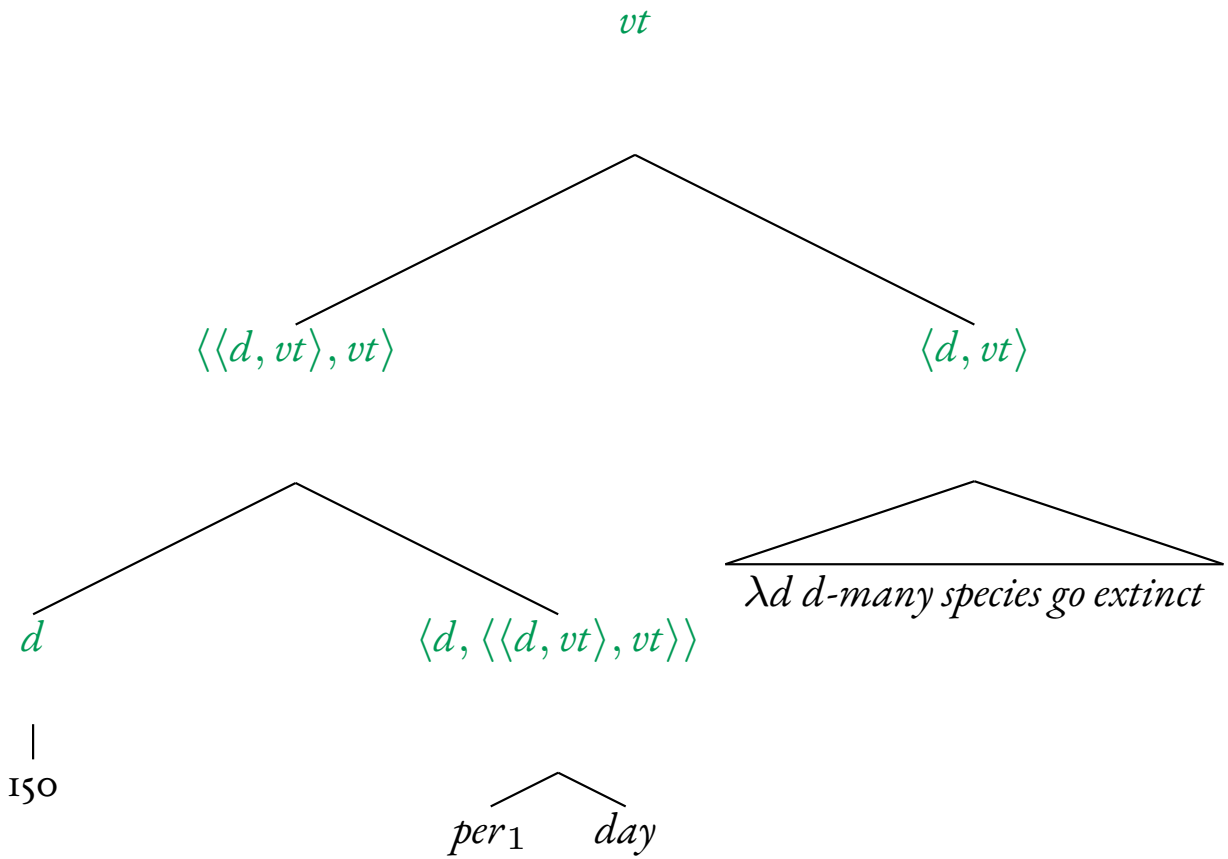
150 species per day is a high rate.
#Therefore, a high rate is among those going extinct.

Target truth conditions:

$$\text{Gen } e \,.\, \frac{\text{the number of species that go extinct in } e}{\text{the duration of } e} = \frac{150}{\text{day}}$$

Compositional derivation for quotient operator use



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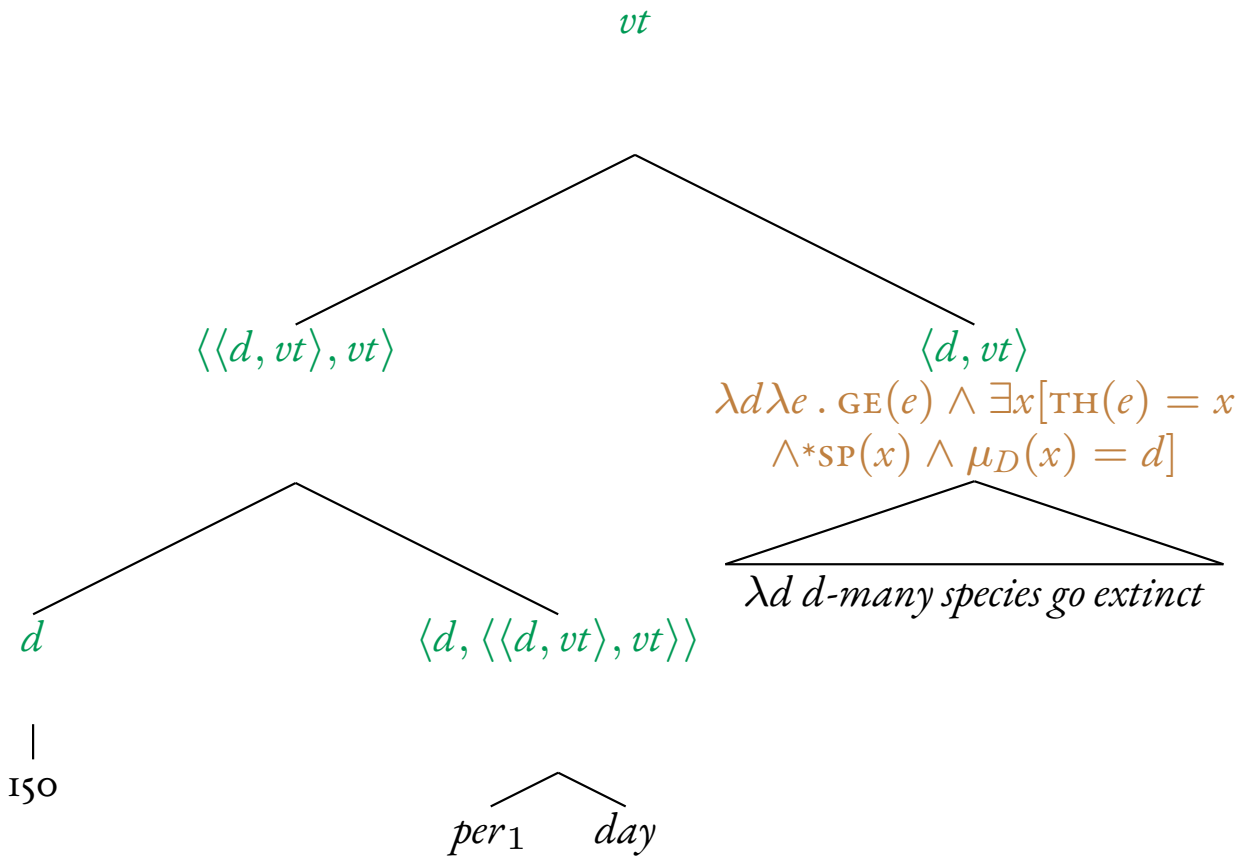

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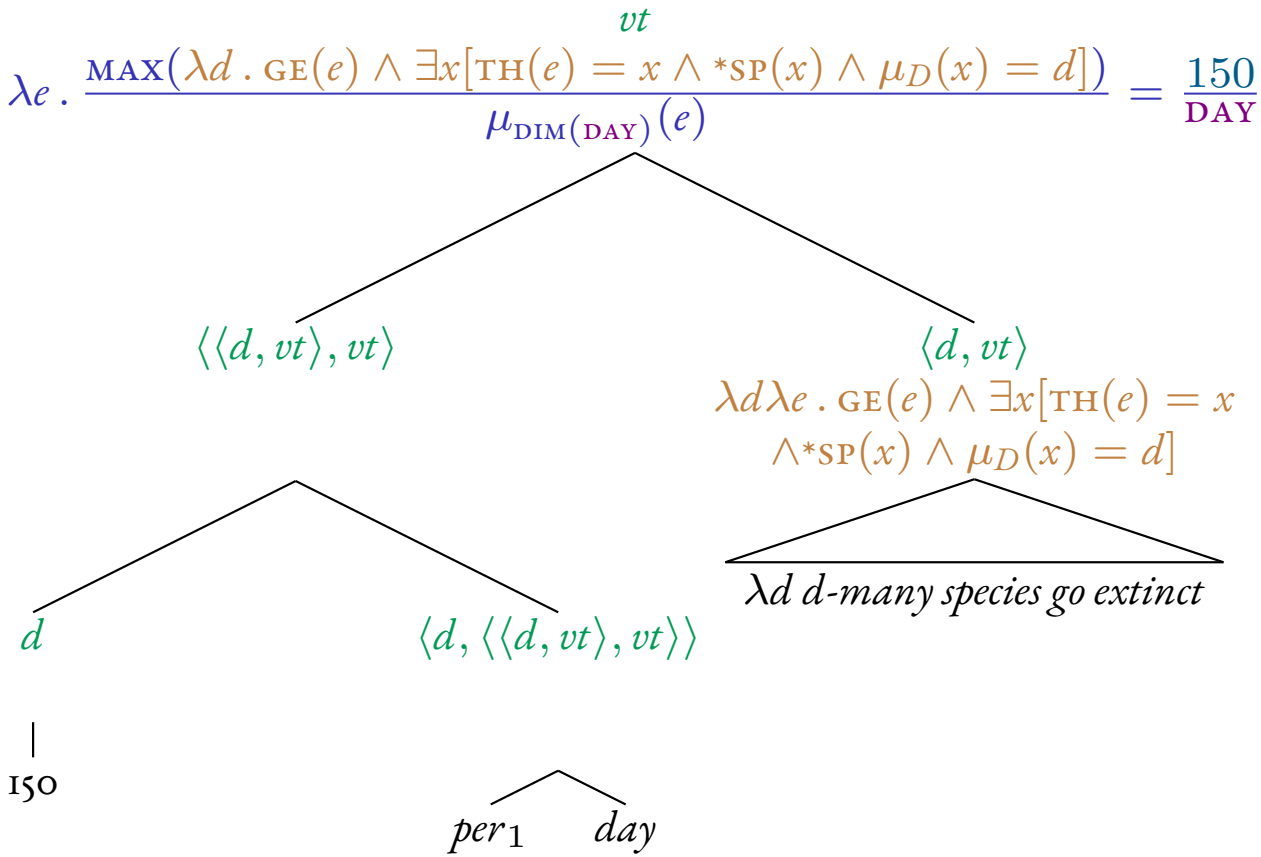

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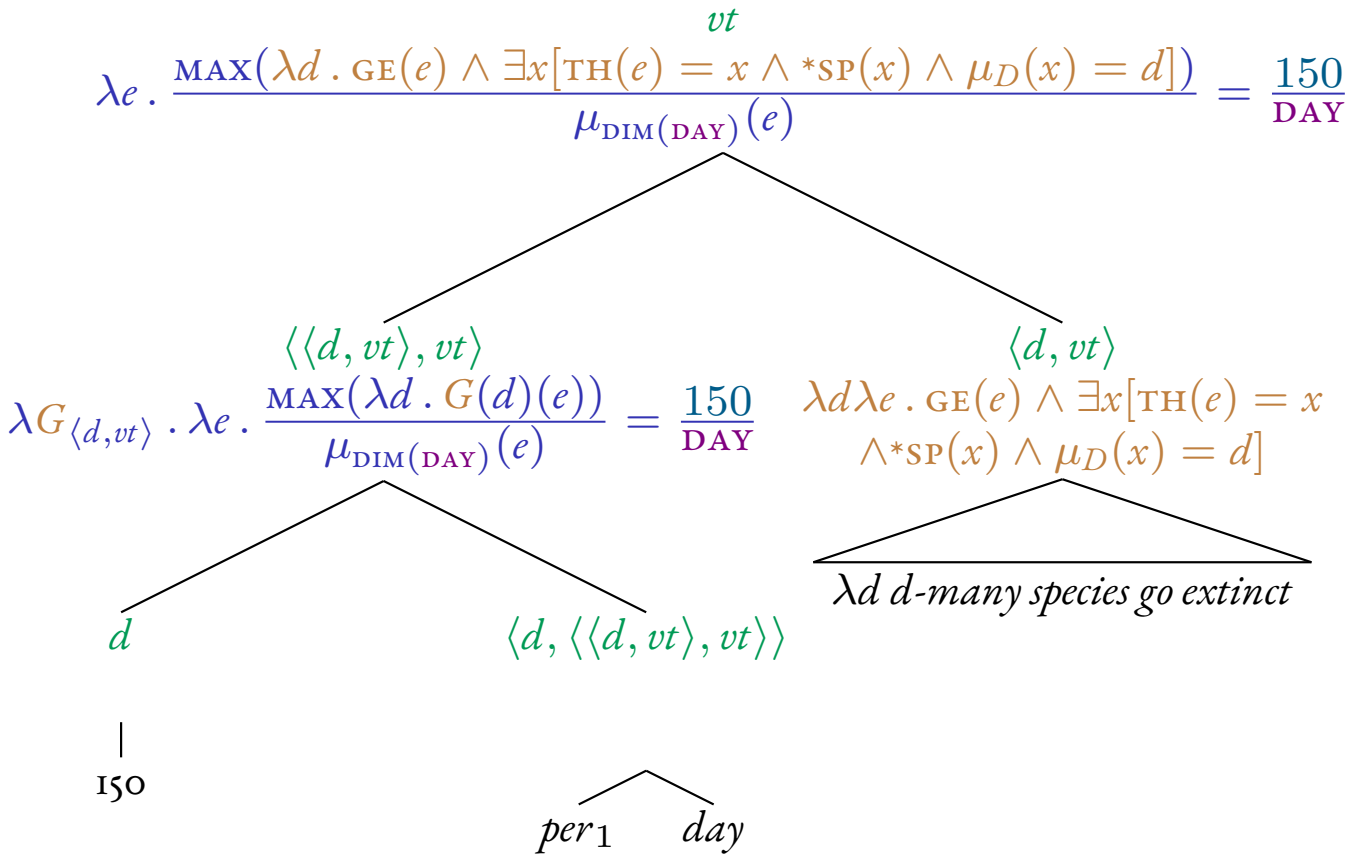

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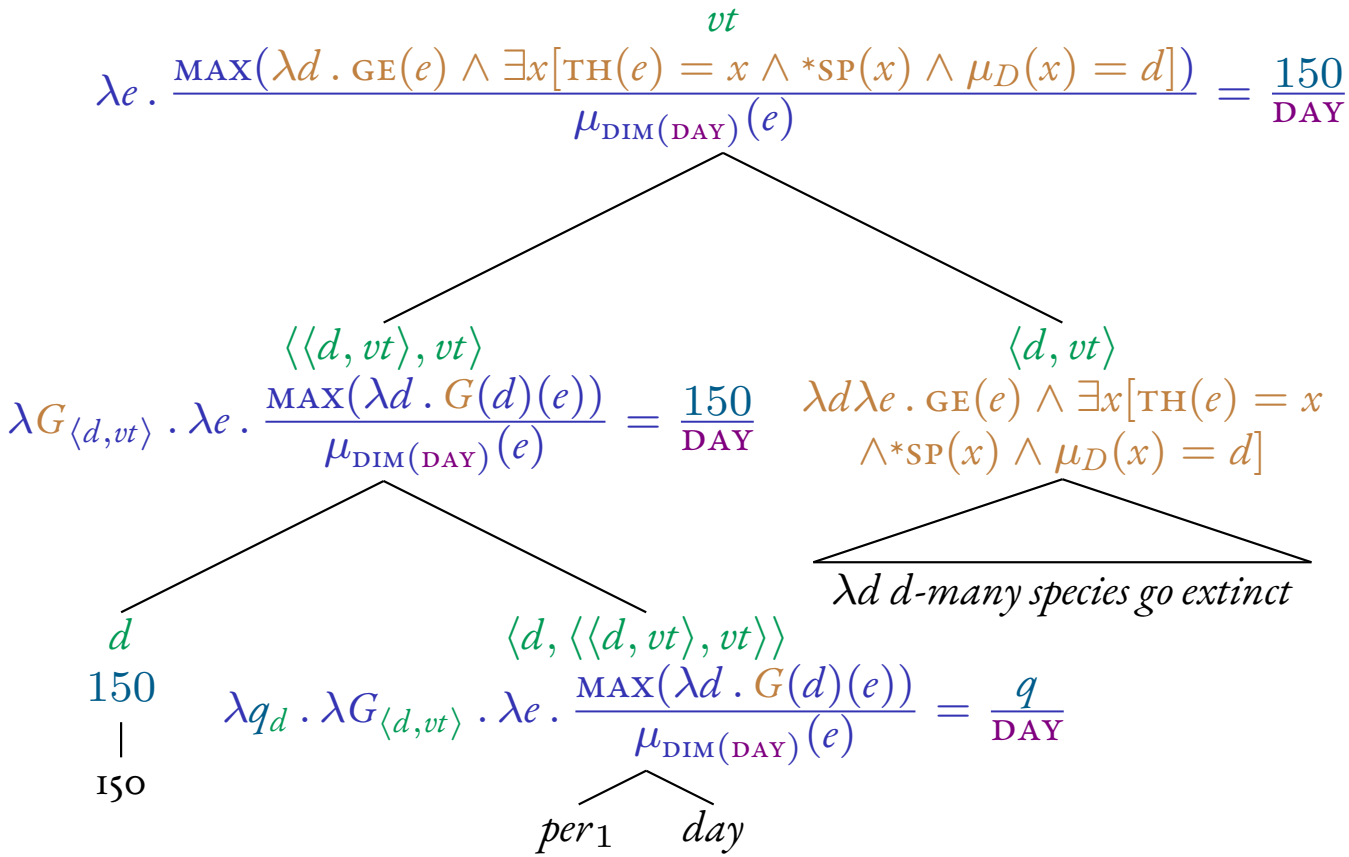

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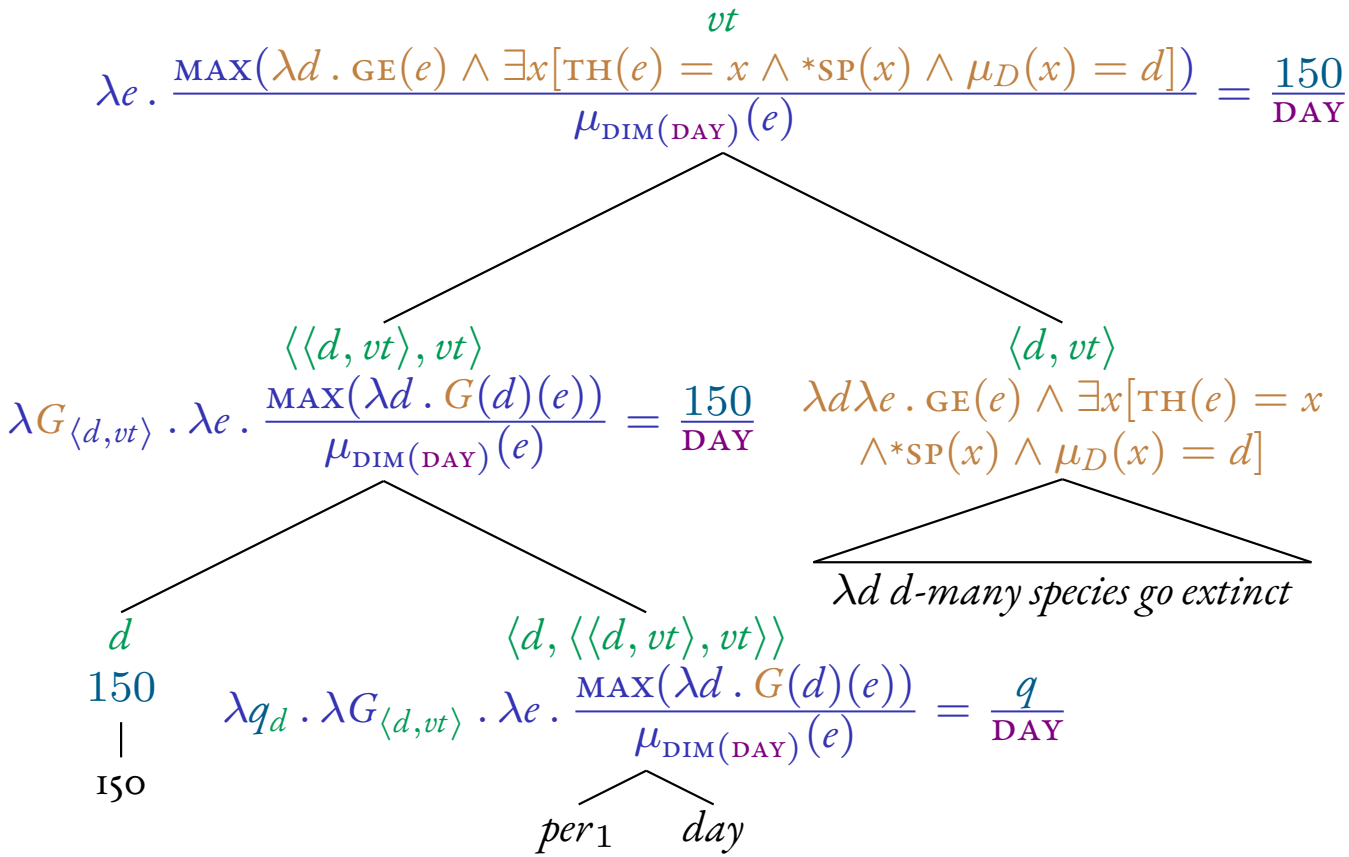
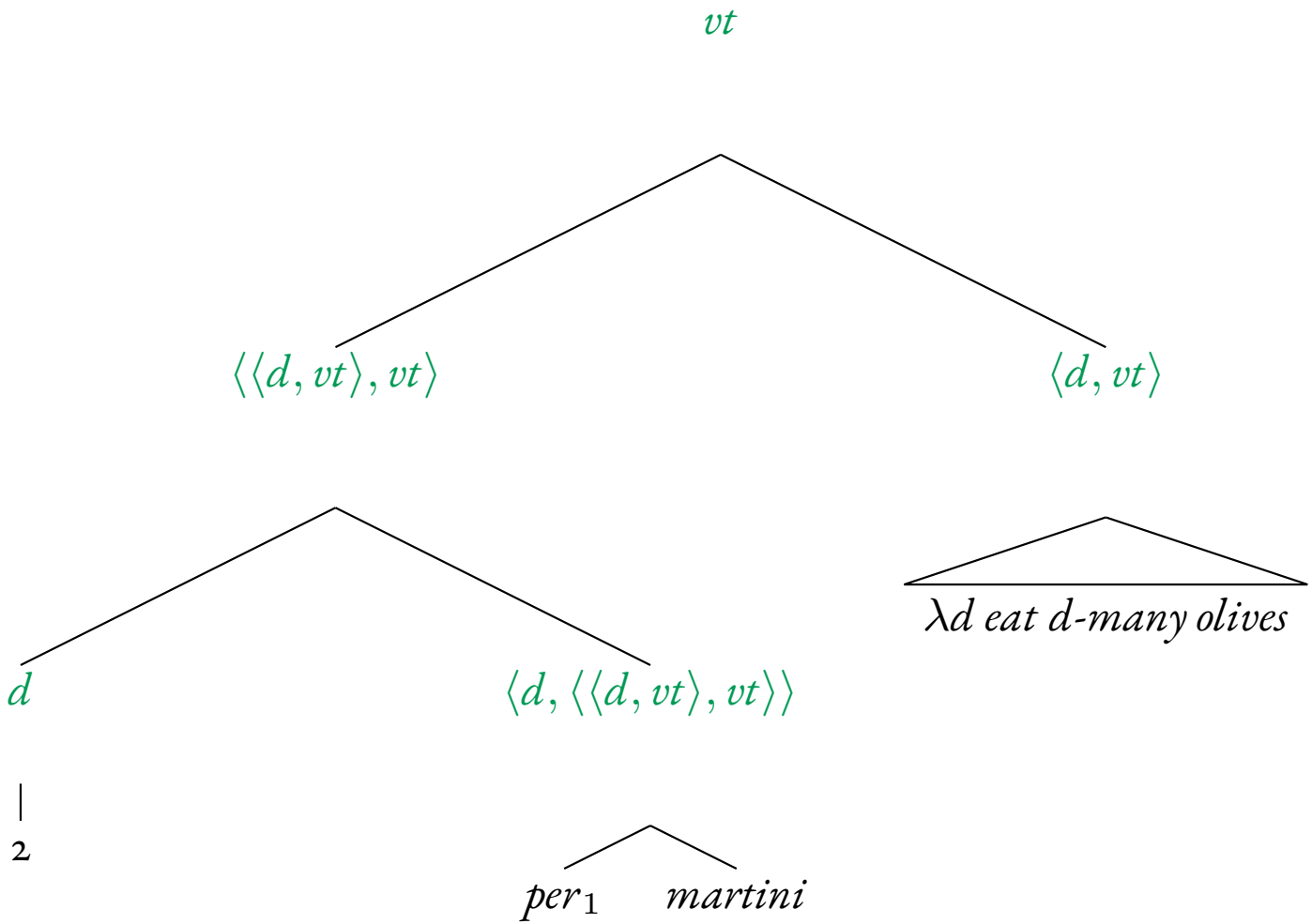


$$per_1 \rightsquigarrow \lambda d_d\,.\,\lambda q_d\,.\,\lambda G_{\langle d,\tau t\rangle}\,.\,\lambda \alpha_\tau\,.\,\frac{\textsc{max}(\lambda d'\,.\,G(d')(\alpha))}{\mu_{\textsc{dim}(d)}(\alpha)} = \frac{q}{d}$$

(Coppock, 2022)

James Bond ate two olives per martini



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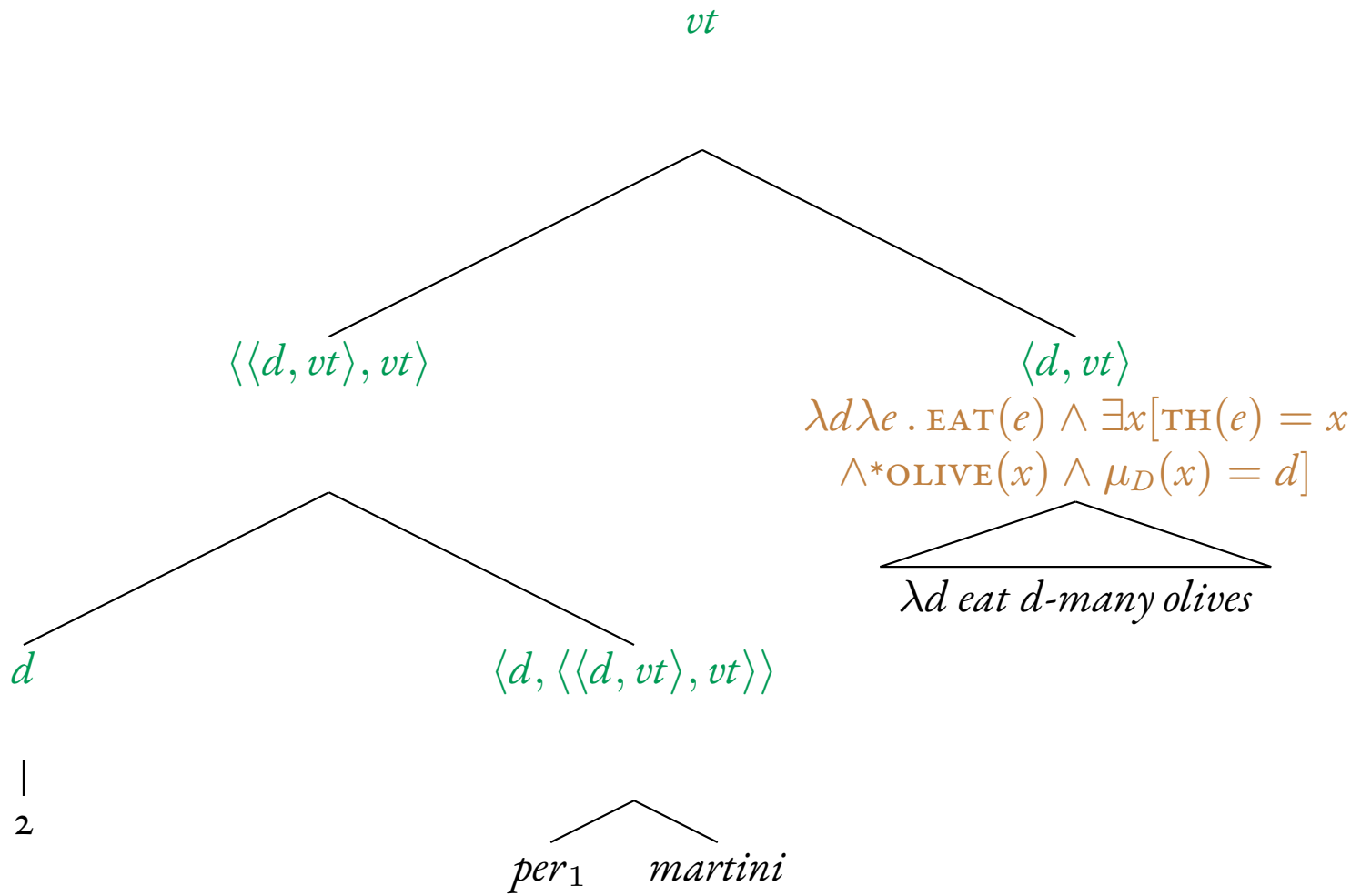

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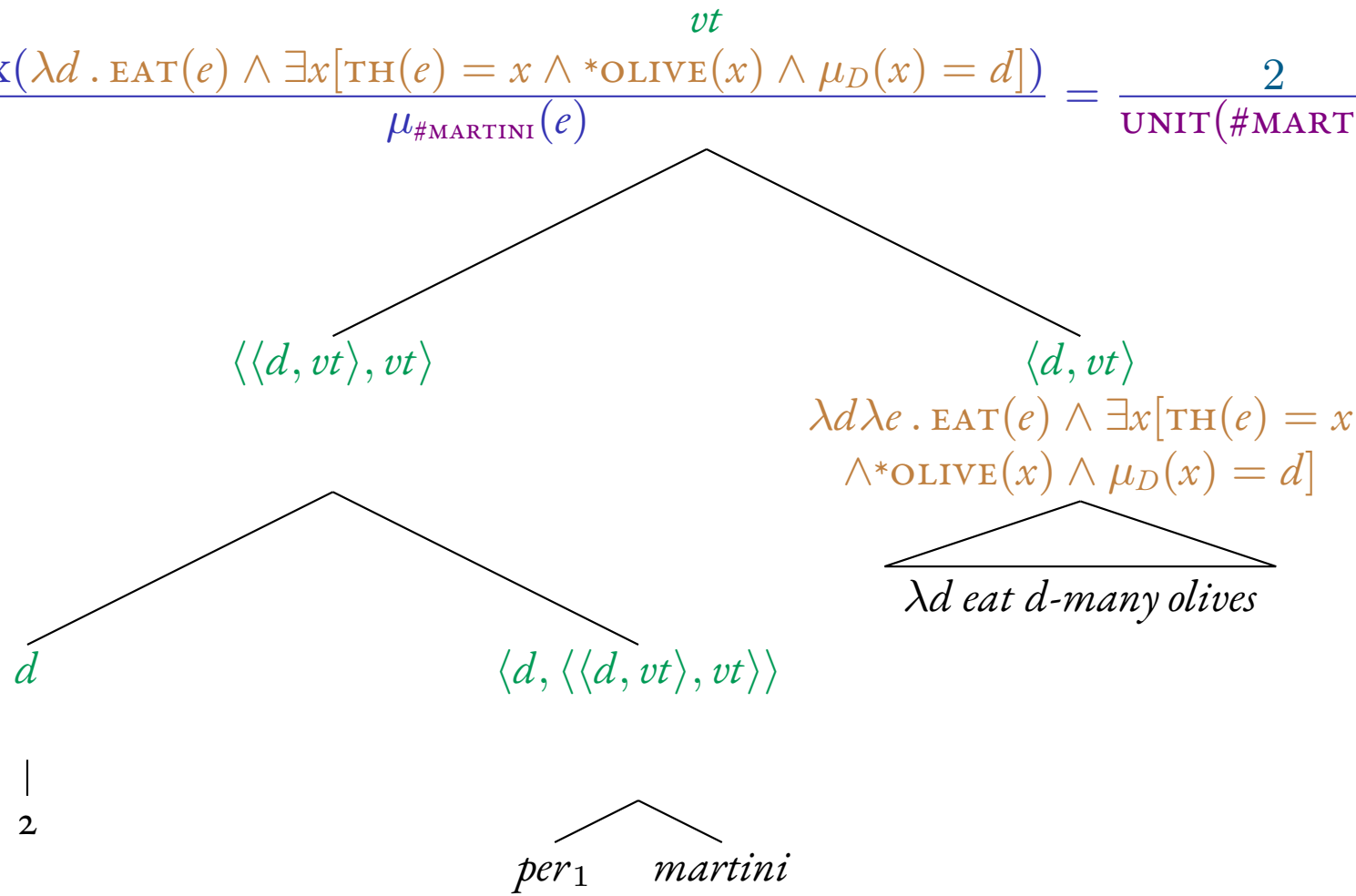
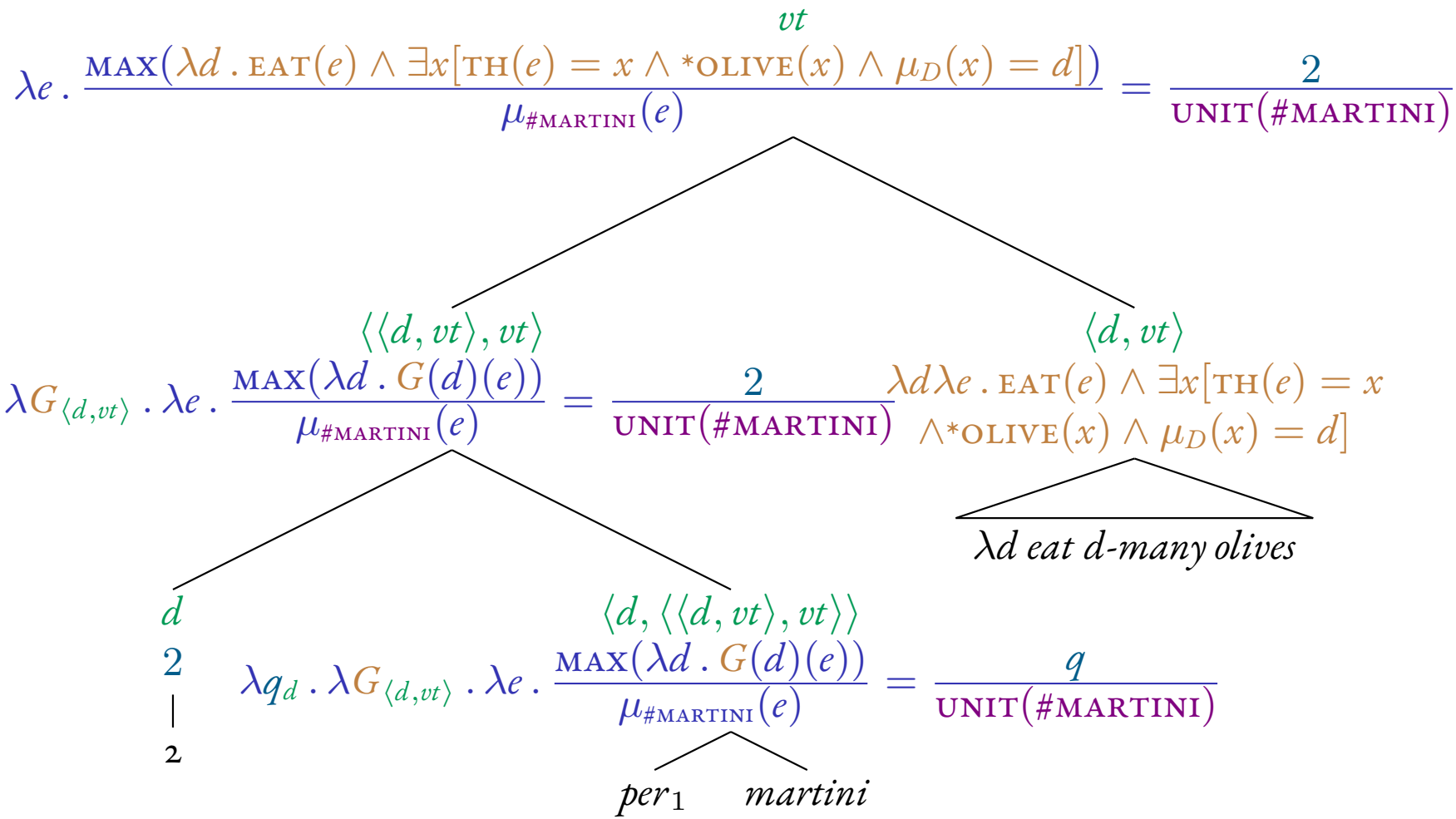
$$\lambda e\,.\ \frac{\textsc{max}(\lambda d\,.\,\textsc{eat}(e) \wedge \exists x[\textsc{th}(e) = x \wedge {*}\textsc{olive}(x) \wedge \mu_D(x) = d])}{\mu_{\#\textsc{martini}}(e)} = \frac{2}{\textsc{unit}(\#\textsc{martini})}$$



'The ratio of how many olives are eaten in e to the measure of e along the number-of-martinis dimension is equal to 2 divided by one martini.'

James Bond ate two olives per martini



'The ratio of how many olives are eaten in e to the measure of e along the number-of-martinis dimension is equal to 2 divided by one martini.'

Another challenge for the quotient function analysis

Measure function verbs like *weigh*:

(16) Water weighs 1 kg per liter.

Schwarz & Bale (2022) point out that it does not suffice to treat '1 kg per liter' as degree-denoting term here; what water weighs is not a ratio of weight to volume.

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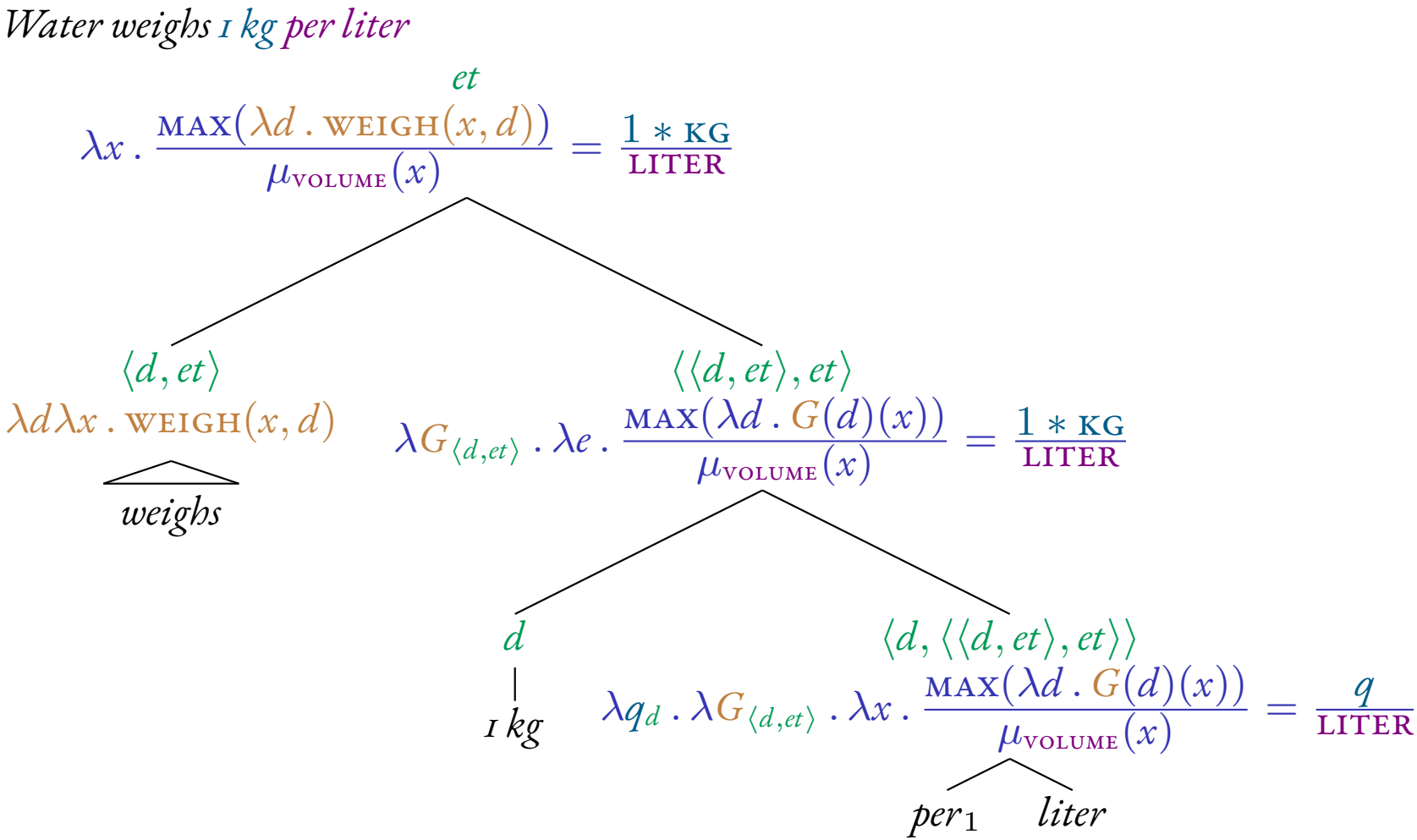
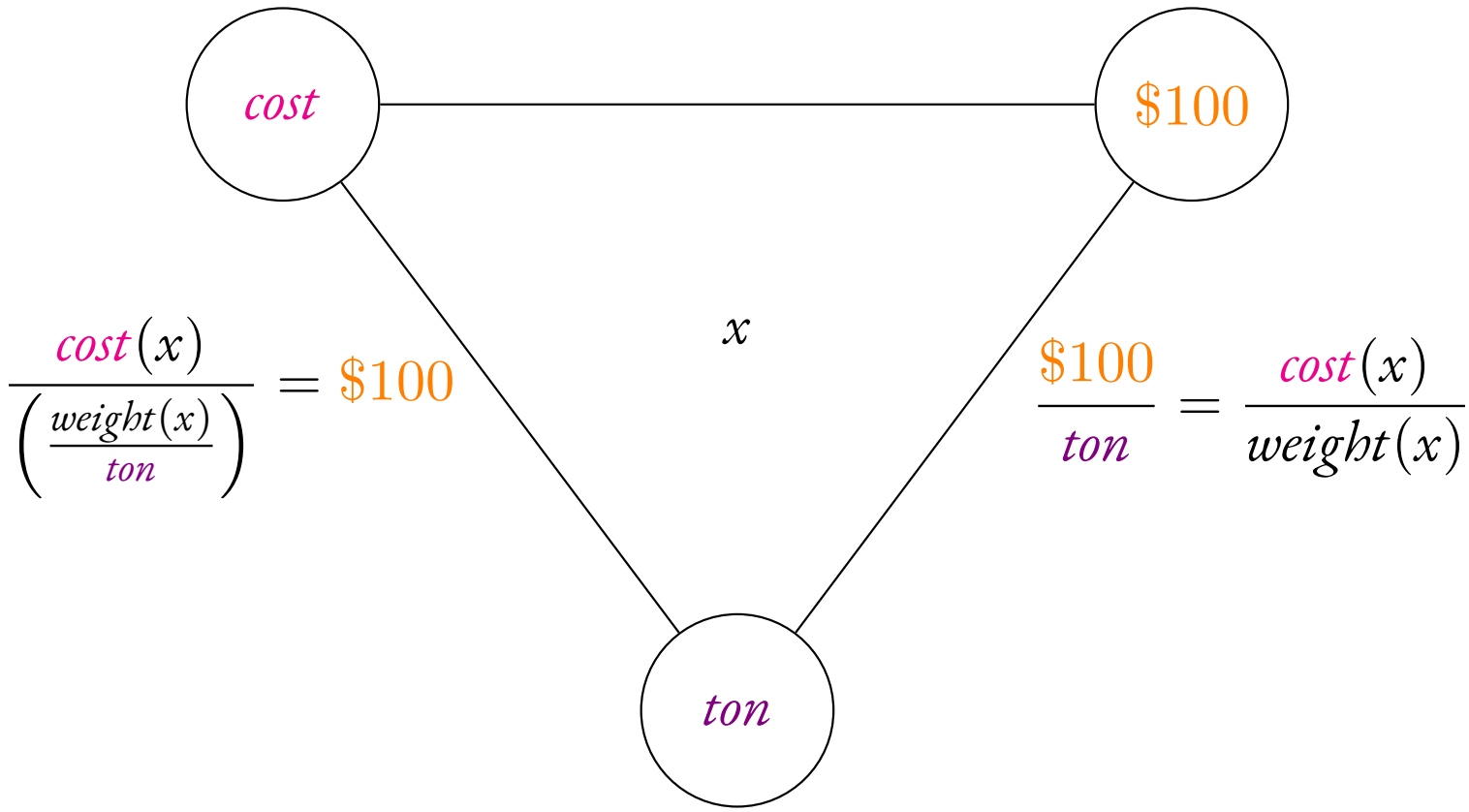
$$\text{weight}(x) \neq \frac{1 * \text{kg}}{\text{liter}}$$

$$\dim(\text{weight}(x)) = \text{M} \qquad \dim\left(\frac{1 * \text{kg}}{\text{liter}}\right) = \frac{\text{M}}{\text{L}^3}$$

Compositional derivation for measure function verb case



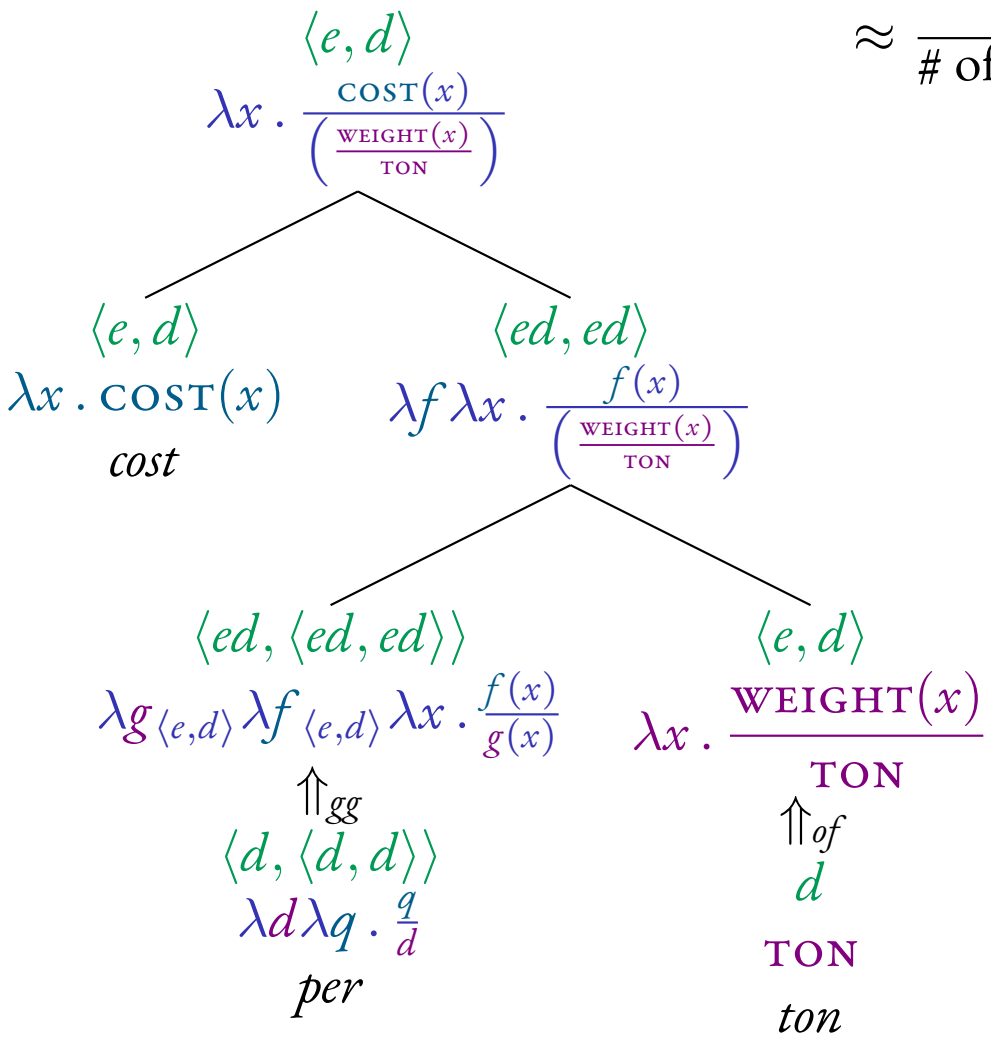
The Quotient Triangle



The cost per ton is $100

The cost is $100 per ton

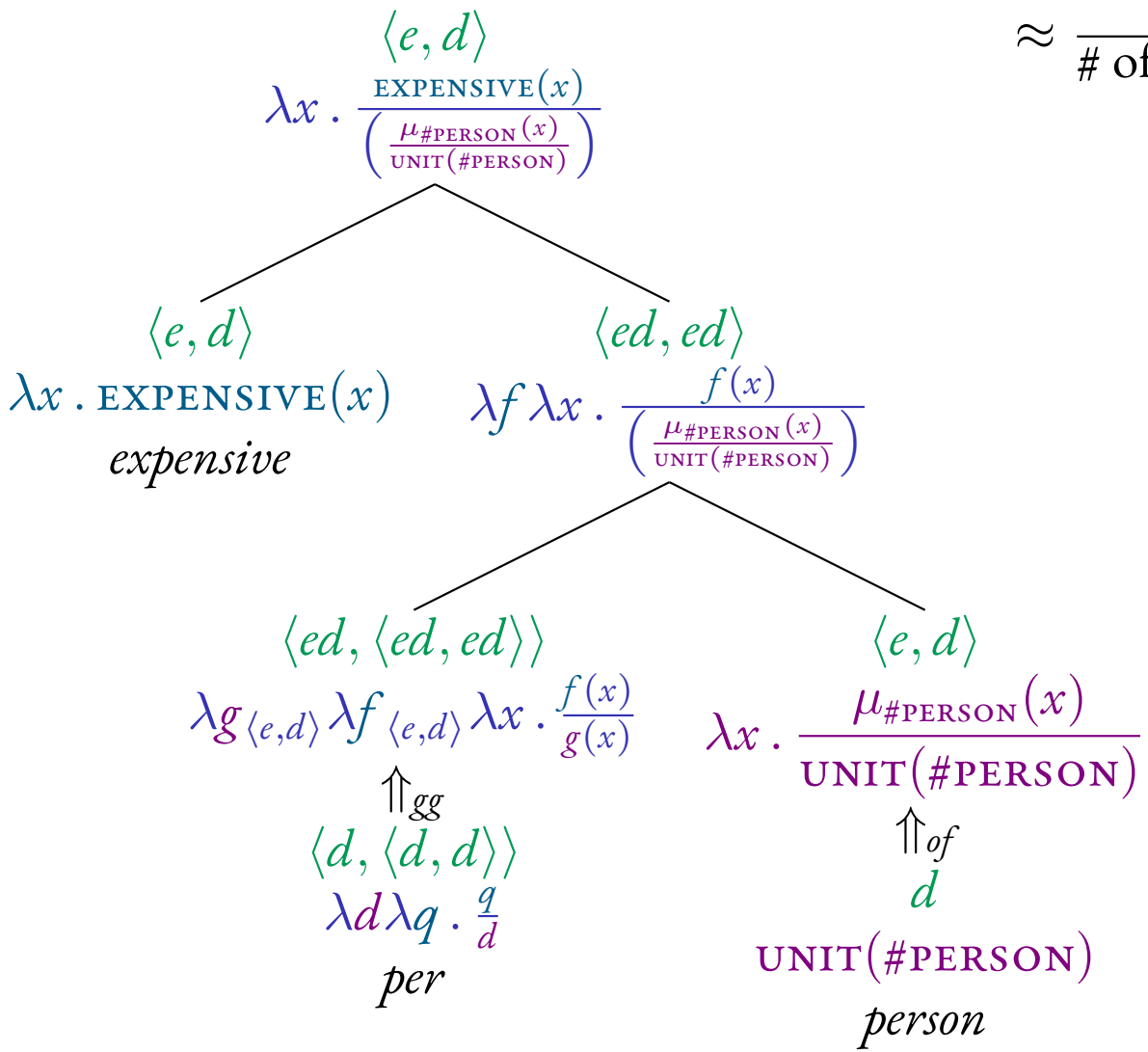
Dimension quotient analysis



$$\approx \frac{\text{cost of x}}{\text{\# of tons x weighs}}$$

Gradable predicates

(how) expensive per person



$$\approx \frac{\text{cost of x}}{\text{\# of people x measures}}$$

Lexical entries for *per*

Three ratio-marker senses of *per*:

- quotient function
  $per \rightsquigarrow \lambda d_d \lambda q_d \,.\, \frac{q}{d}$
- quotient operator
  $$per \rightsquigarrow \lambda d_d \,.\, \lambda q_d \,.\, \lambda G_{\langle d,\tau t\rangle} \,.\, \lambda \alpha_\tau \,.\, \frac{\textsc{max}(\lambda d' \,.\, G(d')(\alpha))}{\mu_{\dim(d)}(\alpha)} = \frac{q}{d}$$
- dimension quotient
  $per \rightsquigarrow \lambda g_{\langle e,d\rangle} \lambda f_{\langle e,d\rangle} \lambda x_e \,.\, \frac{f(x)}{g(x)}$

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  $per \rightsquigarrow \lambda g_{\langle e,d\rangle} \lambda f_{\langle e,d\rangle} \lambda x_e \,.\, \frac{f(x)}{g(x)}$

I assume that *-nként* has all these uses, too.

Outline

Possible hosts for *-nként*

- Nominal, as in ‘martini-*nként* two olives’

Possible hosts for *-nként*

- Nominal, as in ‘martini-*nként* two olives’
- Adjectival, as in ‘person-*nként* more expensive’

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- Verbal

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  - ‘bottom-up’ (repetition of VP)
    as in ‘week-*nként* visit Grandma’

Possible hosts for *-nként*

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- Verbal
  - ‘bottom-up’ (repetition of VP)
    as in ‘week-*nként* visit Grandma’
  - ‘top-down’ (subdivision of VP)
    as in ‘quarter-*nként* eat the pill’

Outline

Frequency uses (X-ly, every X)

(17) Mari het-enként latogatja a nagymamát
Mary week-DIV visits the grandma.POSS.ACC
'Mary visits her grandma at least once weekly.'

Compare:

(18) Mary visits her grandmother...
#...per week.
weekly.
every week.
#week by week.
#by week.
#by weeks.
#in/on weeks.
#from week to week.

More frequency uses

(19) Péntek-enként úszni járok.
Friday-DIV swim.INF go.1SG
'I go swimming every Friday.'

(20) terrorizmusellenes üléseket kell tartani
counter-terrorism meetings must happen
elnökség-enként
presidency-DIV/presidency
'Counter-terrorism meetings must be held once per Presidency.'

Note: these are bad with the iterative suffix *-ente* (unlike 'day', etc.).

Outline

Silent multiplicative?

(21) Mari legalább egyszer het-enként latogatja a nagymamát
Mary at.least once week-DIV visits the grandma.POSS.ACC
'Mary visits her grandma at least once weekly.'

Visiting gramma once

Lexical entry for the multiplicative (inspired by Wągiel 2023):
-szer $\rightsquigarrow \lambda n \lambda V_{\langle v,t \rangle} \lambda e \,.\, \mu_{\#V}(e) = n$

Visiting gramma once

Lexical entry for the multiplicative (inspired by Wągiel 2023):
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Assumption: For a given predicate of events V, $\mu_{\#V}(e) = n$ means that e contains as a (proper or improper) subpart exactly n instances of V.

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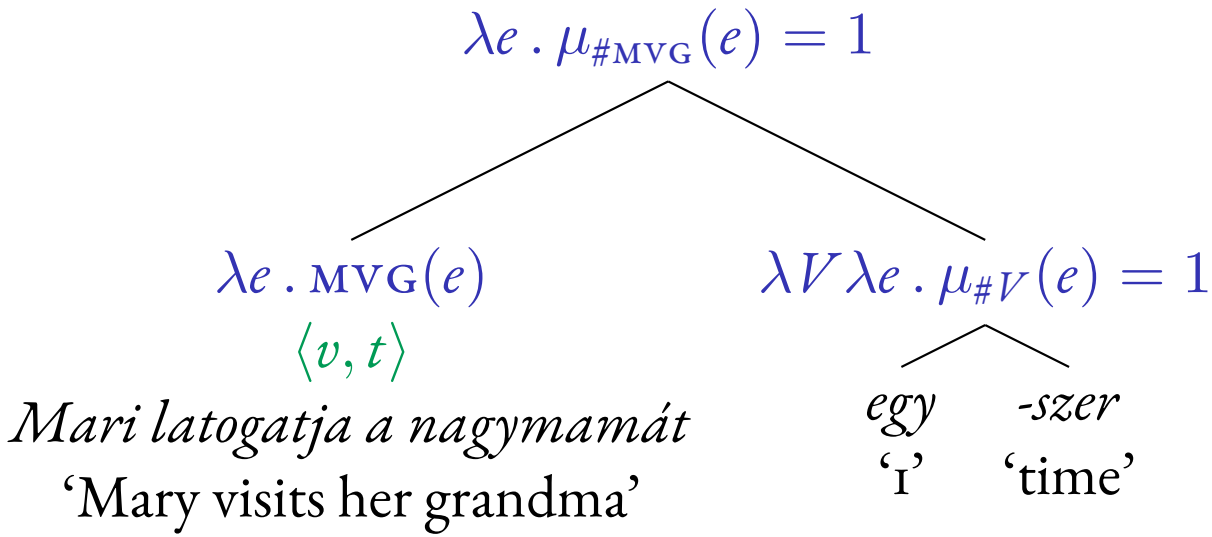
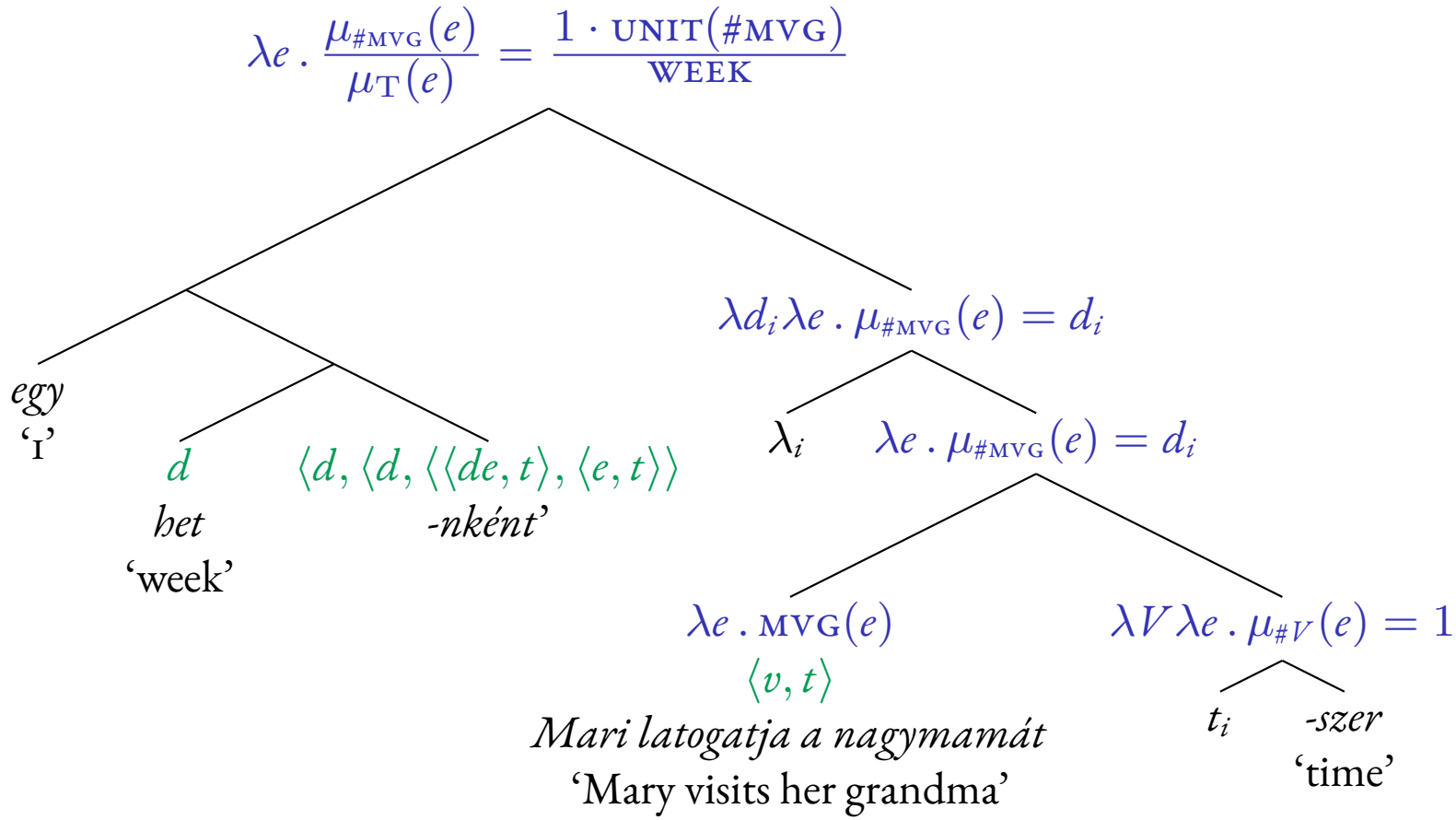
Derivation tree:



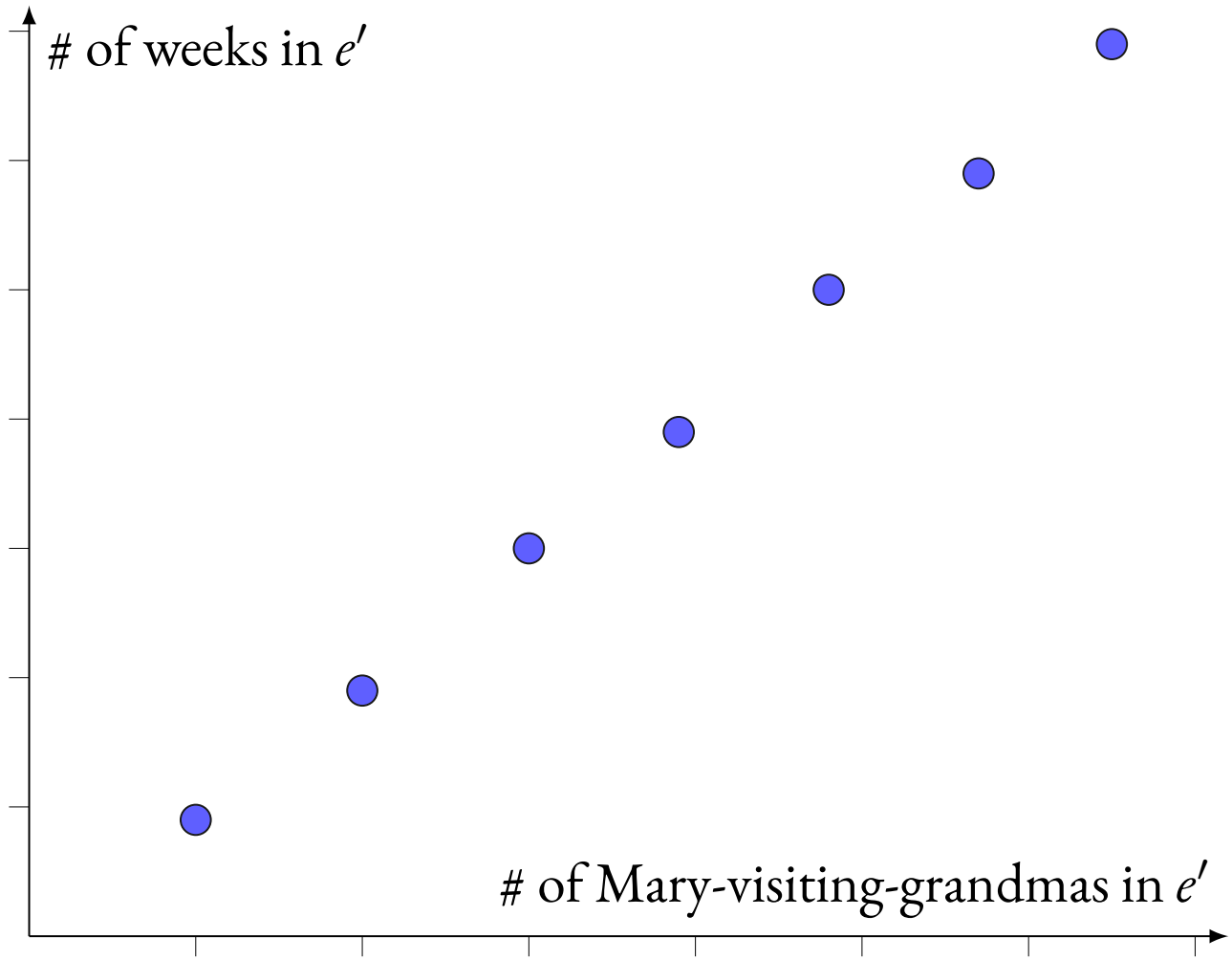
Visiting grandma once per week



Outline

Getting even ratios

Let e' be a subevent of e.



Champollion's 'stratified reference'

Accoridng to Champollion, 'waltz for an hour' presupposes that *waltz* has stratified reference with respect to the dimension 'runtime' and the granularity 'one hour':

$$\forall e[\textsc{waltz}(e) \rightarrow e \in {}^*\lambda e'[\textsc{waltz}(e') \wedge \epsilon(1 \cdot \textsc{hour})(\tau(e'))]]$$

'Every waltzing event can be divided into one or more parts, each of which is a waltzing event whose runtime is very short compared with one hour.'

In general, V has **stratified reference** with respect to dimension θ and granularity $\epsilon(K)$, $SR_{\theta,\epsilon(K)}(V)$, iff:

$$\forall e[V(e) \rightarrow e \in {}^*\lambda e'[V(e') \wedge \epsilon(K)(\theta(e'))]]$$

Defining homogeneity

Let us say that e is **homogeneous** with respect to dimension D, granularity ϵ, and predicate V iff:

$$e \in {}^*\lambda e' [V(e') \wedge \mu_D(e') = \epsilon]$$

Proposal: Quotient-operator *-nként* has can be strengthened with the inference that e is homogenous with respect to the dimension and granularity of its complement, and the ratio-predicate that it builds using its surrounding syntactic context.

Strengthening *per* with homogeneity

$$per_1^{\mathcal{H}} \rightsquigarrow$$

$$\lambda d_d \,.\, \lambda q_d \,.\, \lambda G_{\langle d,\tau t\rangle} \,.\, \lambda \alpha_\tau \,.\, \mathcal{H}(\alpha)(\lambda \alpha' \,.\, \frac{\textsc{max}(\lambda d' \,.\, G(d')(\alpha'))}{\mu_{\textsc{dim}(d)}(\alpha'))} = \frac{q}{d})(d)$$

where

$$\mathcal{H}(\alpha)(V)(d) \equiv V(\alpha) \wedge \alpha \in {}^*\lambda \alpha' [V(\alpha') \wedge \mu_{\dim(d)}(\alpha') = d]$$

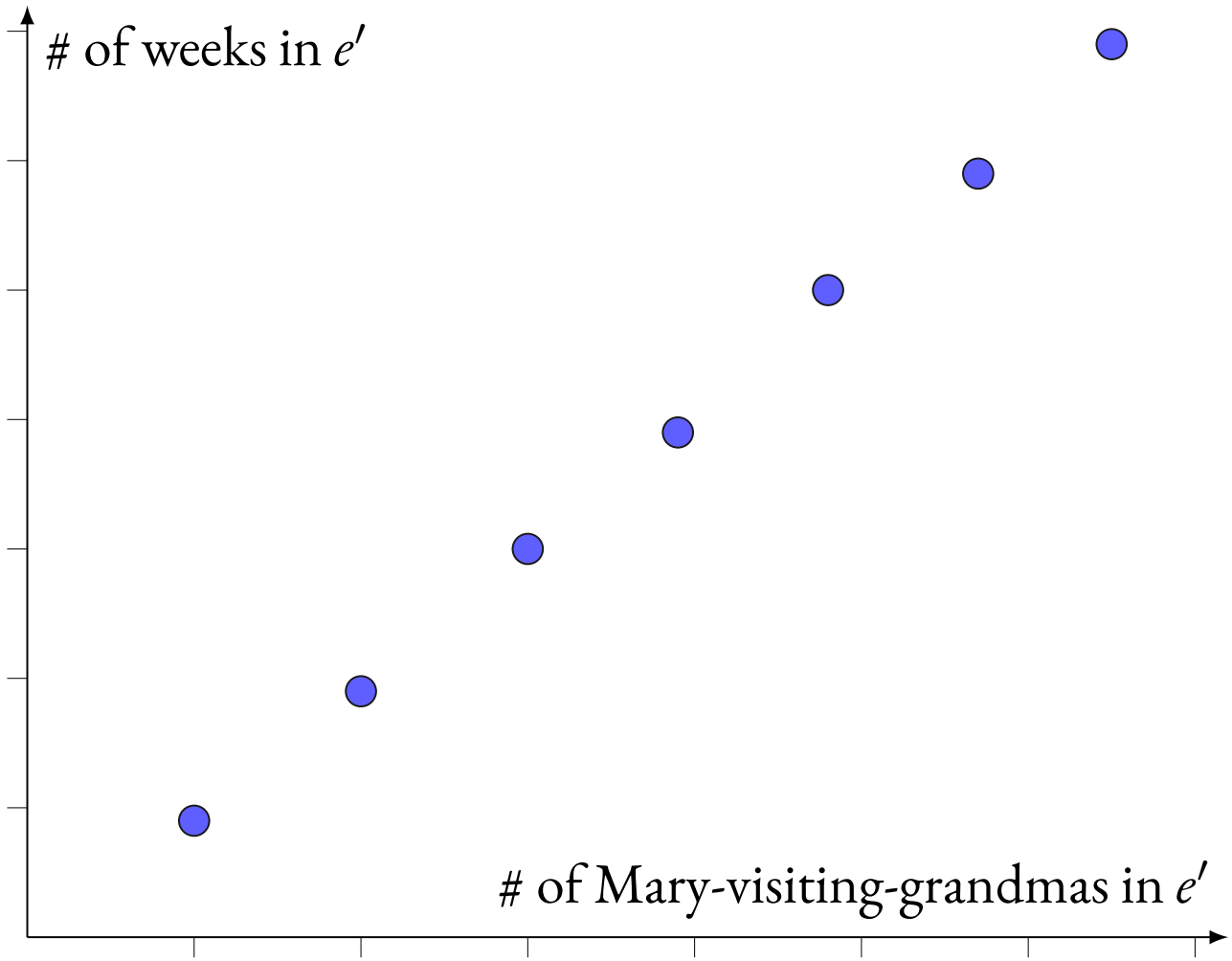
Example:

$$\lambda e \,.\, \mathcal{H}(e)(\lambda e' \,.\, \frac{\mu_{\#\textsc{mvg}}(e')}{\mu_{\mathrm{T}}(e')} = \frac{1 \cdot \textsc{unit}(\#\textsc{mvg})}{\textsc{week}})(\textsc{week})$$

'The ratio of Mary-visiting-grandmas to weeks in e is 1:1, and e is composed of week-long subevents exhibiting the same ratio.'

Getting even ratios

Let e' be a subevent of e.



Outline

Top-down: taking a pill in quarters

A 'top-down' case (one event of taking the big pill):

(22) A nagymama negyed-enként vette be a nagy tablettát
the grandma quarter-DIV took the big pill.ACC
'Grandma took the big pill quarter by quarter'

Compare:

(23) Grandma took the big pill...
#per quarter.
#quarterly/??quarter-wise.
#every quarter.
quarter by quarter.
#by quarter
?by quarters
in/#on quarters
#from quarter to quarter

More ‘X by X’ cases

Similar examples can be constructed with

felenként	half by half (not: **per half*)
adagonként	portion by portion
darabonként	piece by piece
lépésenként	step by step
téglánként	brick by brick
hármanként	three by three

Top-down: arrange by color

(24) Mari szín-enként rendezte el a ruhákat.
Mari color-DIV arranged the clothes.ACC
'Mary arranged the clothes by color.'
(Balazs Suranyi, p.c.)

(25) Mary arranged the clothes...
#per color.
#colorly / ?color-wise.
#every color.
#color by color.
by color
?by colors
#in/on colors
#from color to color
according to color

Encroachment of *per* into *by*-territory

(26) I shall now give a short resume of our findings per country.

(27) The complete table with a breakdown of all applications per prior right and country of applicant can be found on the website.

(28) They broke down the applications...
?per country.
#countryly / ??country-wise.
#every country.
?country by country.
by country
?by countries
#in countries
#from country to country
according to country

Hidden *once* won't work for these

(29) #A nagymama negyed-enkent egyszer vette be a tablettakat.
the grandma quarter-DIV once took the pill
'Grandma took the pill once per quarter.'

(30) #Mari szín-enként egyszer rendezte el a ruhákat.
Mari color-DIV once arranged the clothes.ACC
'Mary arranged the clothes once per color.'

Outline

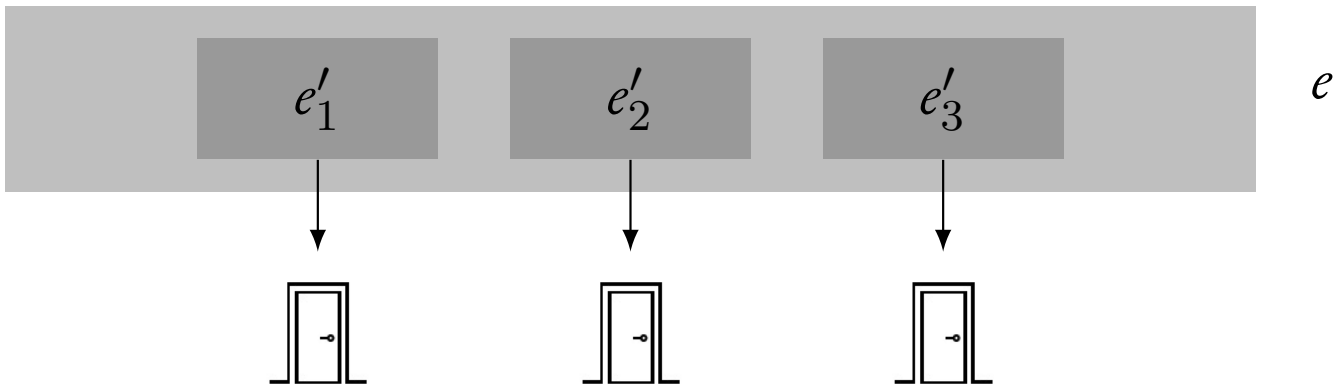
Old Hungarian *-nkéd*

(31) És reggel az fráterek ajtó-nkéd kenyeret kolulának.
and morning the friars door-DIV bread begged
'And in the morning, the friars went begging for bread from door to door.'

Halm & Bende-Farkas (2024): Old Hungarian *-nkéd* is a pluractional marker à la Beck & von Stechow 2007.

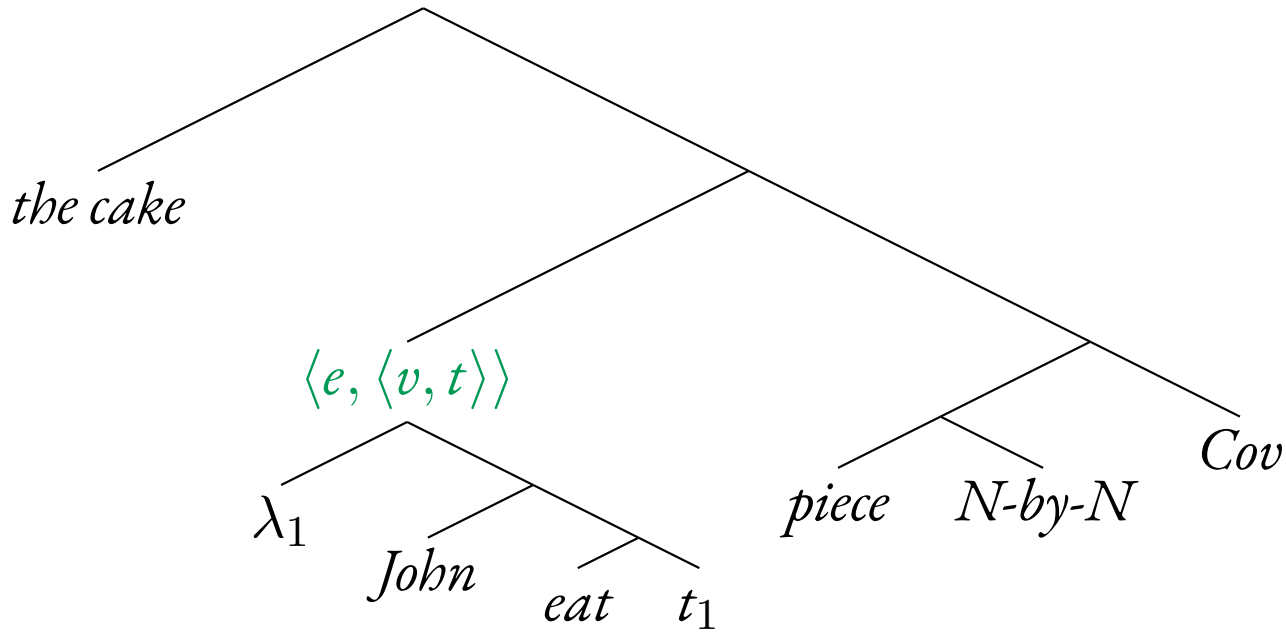
- The eventuality is partitioned into subeventualities
- Some parameter of the eventuality (theme, location or time) is partitioned in terms of the N-denotation
- There is a 1:1 relation between the two partitions.

Visualization: door-*nkeed* (à la B&vS)



Beck and von Stechow's analysis

$$\lambda e\,.\,\partial(\textsc{part}(\mathbf{Cov}, e \oplus y)) \wedge$$
$$\langle e, \textsc{thecake}\rangle \in {}^{**}[\lambda y' \lambda e'.\mathbf{Cov}(y') \wedge \mathbf{Cov}(e') \wedge$$
$$\textsc{piece}(y') \wedge \textsc{eat}(e') \wedge \textsc{ag}(e') = \textsc{j} \wedge \textsc{th}(e') = y']$$



Cov is a free variable (hence bold) introduced by *Cov*.

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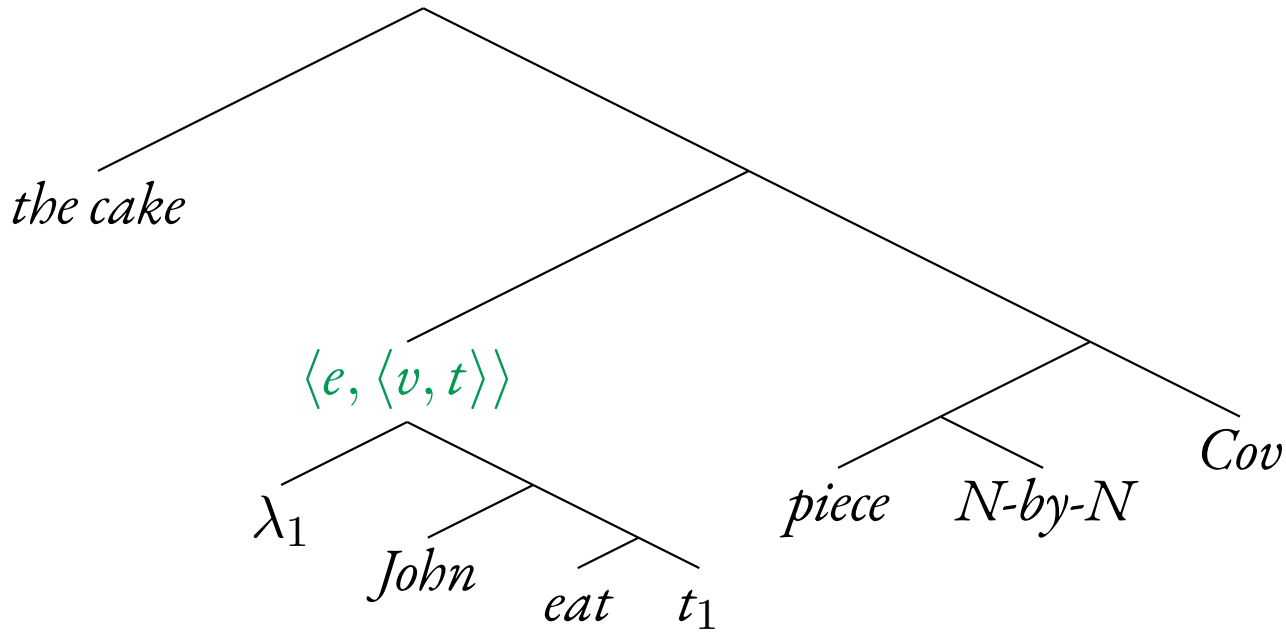
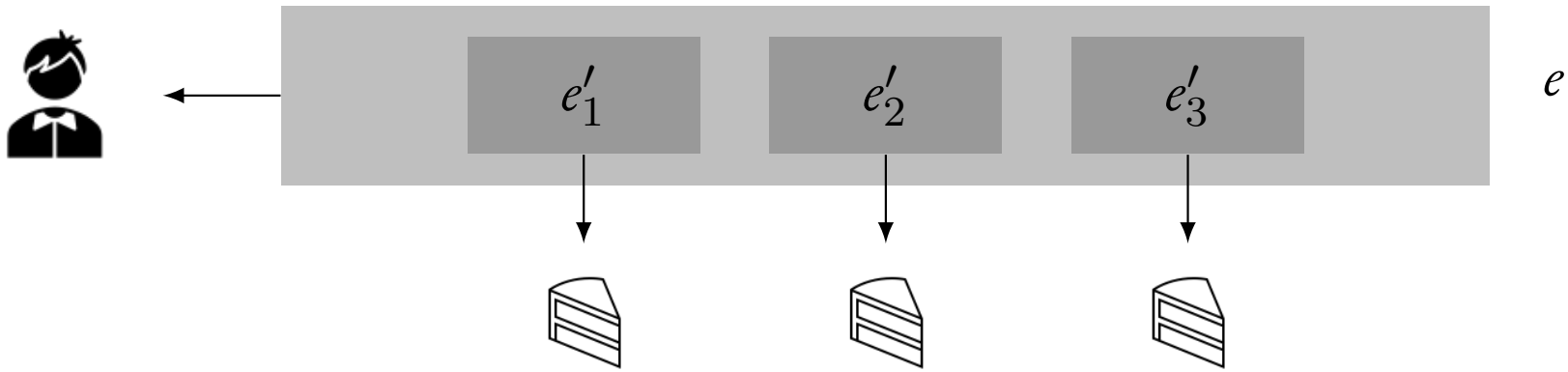


Cov is a free variable (hence bold) introduced by *Cov*.
$\textsc{part}(\mathbf{Cov}, e \oplus y)$ = '**Cov** is a partition over the sum of e and y'

Visualization: piece by piece (à la B&vS)

(32) John ate the cake piece by piece



Outline

My proposal

Unlike B&vS/H&B-F:

- Unified, or at least connected analysis of ratio and pluractional cases
- The complement of *-nként* has type d rather than $\langle e, t \rangle$
- What's expressed is a ratio, which turns into distributivity when enriched by a homogeneity assumption.

Default arguments ⇒ pluractional adverbial

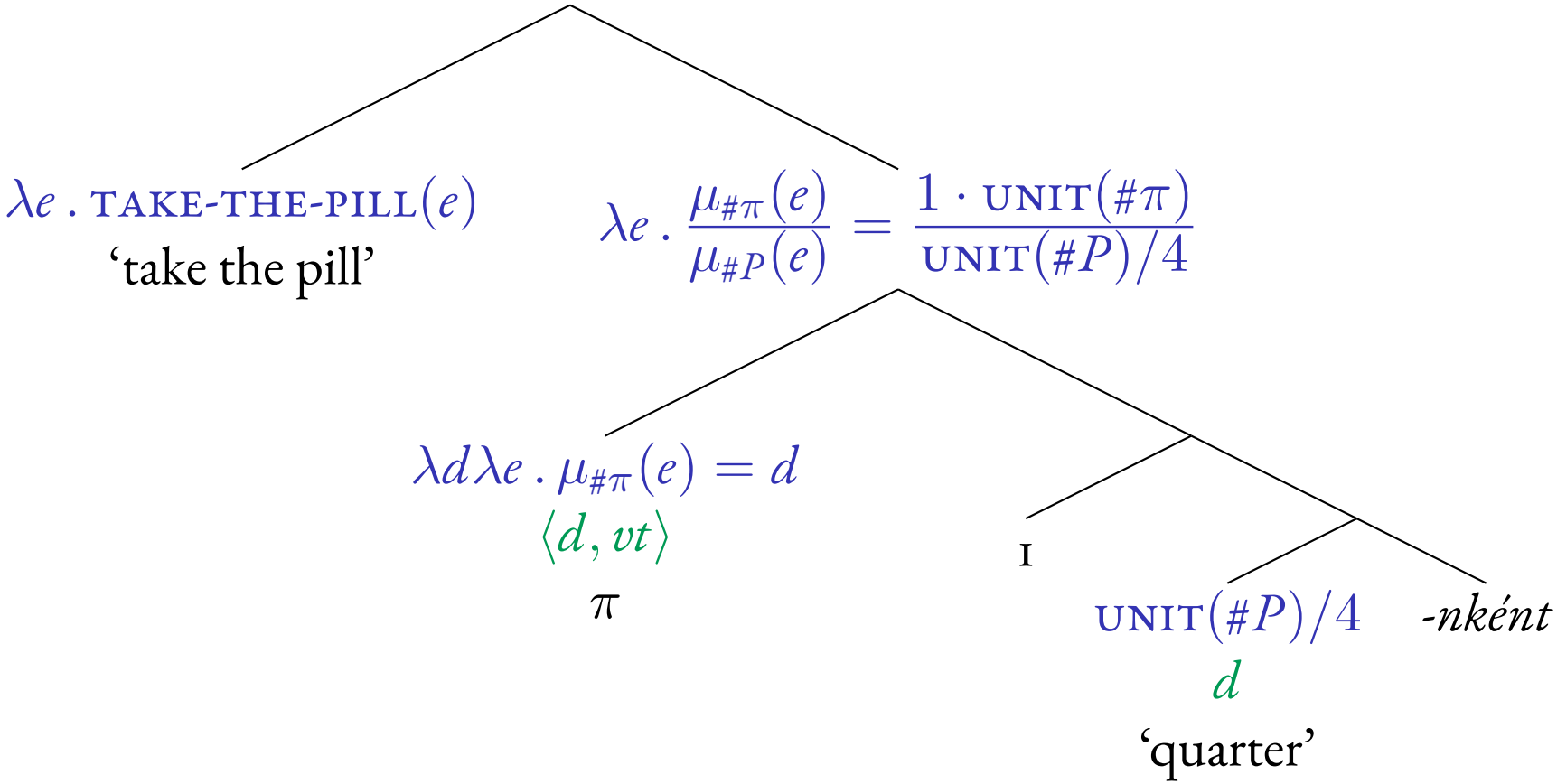
Quotient operator *per*:

$$per \rightsquigarrow \lambda d_d \,.\, \lambda q_d \,.\, \lambda G_{\langle d,\tau t\rangle} \,.\, \lambda \alpha_\tau \,.\, \frac{\textsc{max}(\lambda d' \,.\, G(d')(\alpha))}{\mu_{\dim(d)}(\alpha)} = \frac{q}{d}$$

Default settings given context-determined π, a salient partition of e:

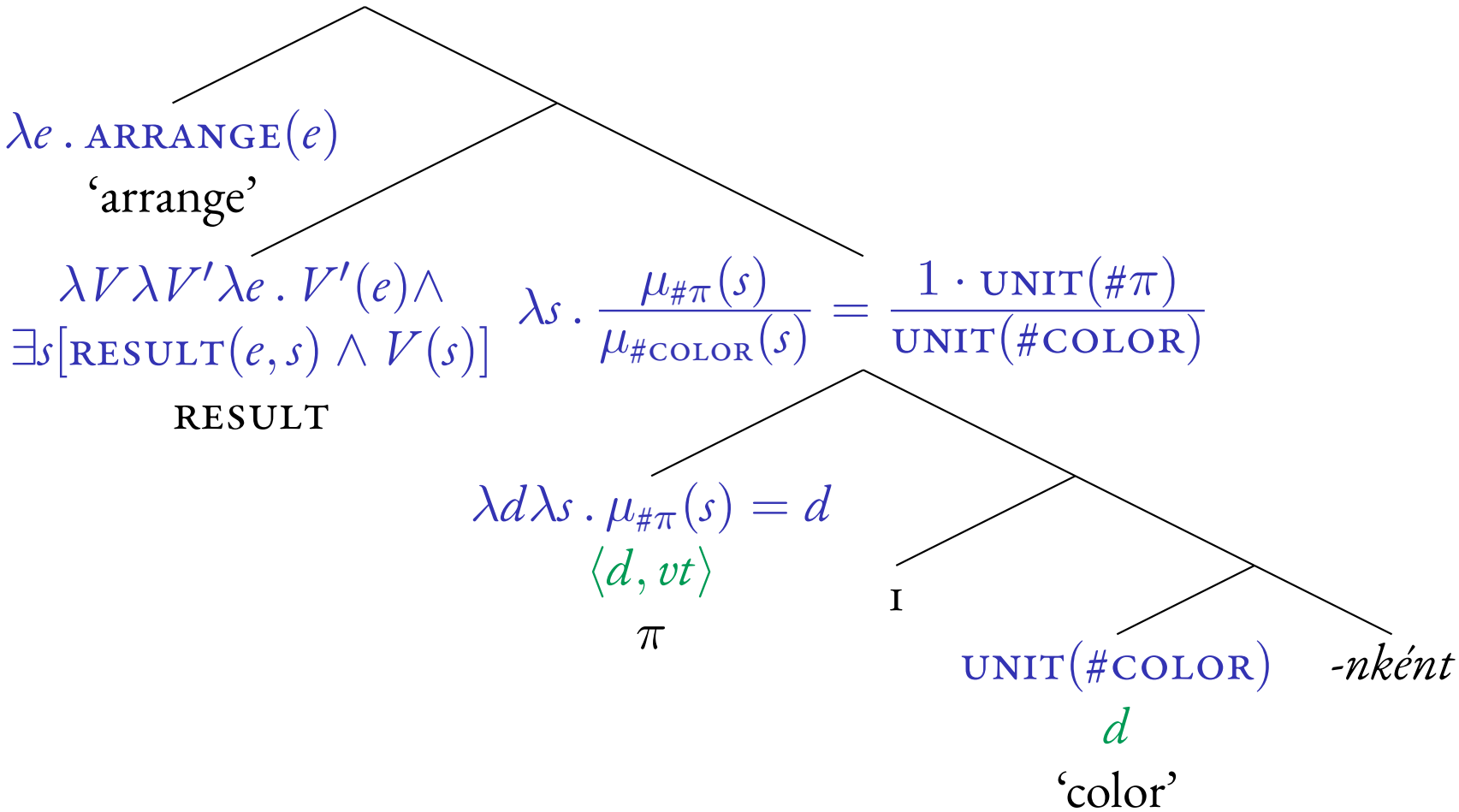
- $G \mapsto \lambda d \lambda e \,.\, \mu_{\#\pi}(e) = d$
  'how many cells of π does e instantiate'?
- $q \mapsto 1 \cdot \textsc{unit}(\#\pi)$

Taking the pill quarter by quarter



e is a taking-the-pill event with a 1 to 1/4 ratio between the number of cells in π that e instantiates and its count along a salient cardinality dimension $\#P$ (number of pills that the theme measures, by pragmatic reasoning).

Arranging by color



e is an arranging event whose result is a state s with a 1 to 1 ratio between the number of cells in π that e instantiates and its count in number of colors.

Outline

Summary

-nként has several uses:

- ‘two olives per martini’-type cases
  – non-decisive re: distributivity-marker vs. ratio-marker
- ‘cost per person’-type cases
  – support a ratio-marker analysis
- ‘visit Grandma weekly’-type cases
  – could be handled with a silent ‘once’
- ‘eat the pill by quarter’-type cases
  – can be handled by counting partition-cells

All of these cases can be obtained via a ratio-marker analysis, sometimes augmented by certain additional assumptions.

Conclusions

- Quantity calculus is useful in natural language semantics

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  - *-nként* picks out a concept covering both of them

Conclusions

- Quantity calculus is useful in natural language semantics
  - For example, English *per* and Hungarian *-nként* are ratio markers; they express arithmetic division
- Arithmetic and mereological division are conceptually adjacent
  - *-nként* picks out a concept covering both of them
  - Arithmetic division underlies a unified analysis
  - Distributivity can roughly be factored into arithmetic division and homogeneity
    $\Rightarrow$ Grammaticalization pathway:
    distributivity $\rightarrow$ loss of homogeneity $\rightarrow$ arithmetic division?

Yet to be explained: Internal readings

Sentence-internal 'differ' (Brasoveanu, 2011, i.a.):

(33) A tehetség mérték-e gyermek-enként különbözik
the talent measure-POSS child-DIV differs
'The degree of talent differs across children' (Istvan Kenesei, p.c.)

Pluractional comparisons (Beck, 2012, i.a.):

(34) és nap-onként számmal bővelkednek vala
and day-DIV number.with grow.3PL PAST
'And their number increased daily.' (Halm & Bende-Farkas, 2024)

Other asymmetric internal readings (Bumford, 2015, i.a.):

(35) naponként új jelentést kell írni
day-DIV new story-ACC must write
'A new story must be written every day.' (Balazs Suranyi, p.c.)

Thank you!

And thanks to members of the audiences at MIT, BU, Yale, NYU, and the Amsterdam Colloquium for discussions on earlier versions of some of the material. Special thanks to Lucas Champollion, Manfred Krifka, Hans Kamp, Ivano Ciardelli, Richard Luo, Paul Dekker, Fabrizio Cariani, Tom Roberts, Alexendre Cremers, and Flavia Nährlich.

Enormous thanks to Balazs Suranyi and the members of the audience at ELTE in Budapest for help and ideas on *-nként*.

Thanks to the research assistants in LiSLab who have been working with me to develop parallel corpora of ratio expressions, especially Nate Lambert, whose observations regarding the taxonomy of verbally-licensed uses of ratio markers in the EuroParl corpus helped helped me see the connection between arithmetic and mereological division.

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Outline

(36) **Még** gyerekenként kettő tablett **is** kevés volna.
even child-DIV two pill FOC little would.be
'Even two pills per child would be too little.'

Outline

Old Hungarian from-to case

(37) És reggel az fráterek ajtó-nkéd kenyeret kolulának.
and morning the friars door-DIV bread begged
'And in the morning, the friars went begging for bread from door to door.'
(Halm & Bende-Farkas, 2024)

Ungrammatical with *-nként* in Modern Hungarian.

(38) They went begging...
#per door.
#doorly.
#every door.
?door by door.
#by door
?by doors
#in doors
from door to door

Some comparable examples bad with *-nként*:

(39) #Ház-anként állt egy rendőr
house-DIV stand a police.officer
intended: 'A policeman was stationed at every house.'
Halm & Bende-Farkas (2024)

But *házanként* sometimes fine:

(40) Tiz ház-anként állt egy rendőr
ten house-DIV stand a police.officer
'A policeman was stationed every 10 houses.'

(41) Harom ház-anként kell ásni egy kutat
three house-DIV must build a well
'You'll have to dig a well for every 3 houses'

Outline

Distributive-internal 'differ'

(42) A matematikai tehetség mérték-e gyermek-enként
the maths talent measure-POSS child-DIV
különbözik
differs
'The degree of mathematical talent differs across children'
(Istvan Kenesei, p.c.)

(43) Az oxigénszint naponként/#naponta különbözik
the ogygen.level day-DIV/day-ITER differs
With *naponként*: 'The oxygen level differs day by day.'
With *naponta*: '...sometime during the day'
(Katalin É. Kiss, p.c.)

Outline

English adverbials and *differ(ent)*

The air quality is different every day.
??The air quality is different daily.
*The air quality is different day by day.

?The air quality differs every day.
??The air quality differs daily.
The air quality differs day by day.

Pluractional comparisons

(44) és nap-onként számmal bővelkednek vala
and day-DIV number.with grow.3PL PAST
'And their number increased daily.'

Compare:

(45) Their number increased...
#per day.
daily.
every day.
day by day.
#by day
#by days
#in days
from day to day

cf. Beck (2012) on pluractional comparisons

More asymmetric internal readings

(46) naponként új jelentést kell írni
day-DIV new story-ACC must write
'A new story must be written every day.'

Compare:

(47) a. Every day this calendar reveals a new Polaroid photo, each with its own little story.
b. Marriage is like a coffin, and each kid is another nail.
c. Every generation lives in a more Orwellian world.

(Brasoveanu, 2011; Bumford, 2015)

Rough idea

Pluractional *d-nként* describes an eventuality e composed of a sequence:

$$e_1, \ldots, e_n$$

such that for all e_i, e_{i+1} for $i < n$:

$$\Delta_{\dim(d)}(e_i, e_{i+1}) = d$$

so d is a measure of the distance among subevents/substates.