UNIFYING ARITHMETIC AND MEREOLOGICAL DIVISION

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Outline

Hungarian *-nként* and English *per*

Today's talk will center around Hungarian *-nként*:

(1) … hogy hajó-nként egy tudós-t alkalmazzunk,
… that boat-___DISTDIV one scientist-ACC employ.SBJ.1PL
'[It's not realistic] that we employ one scientist per vessel'
(EuroParl corpus)

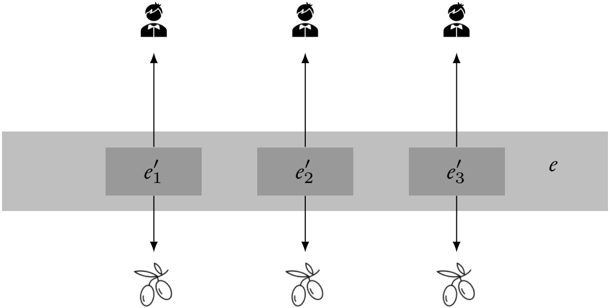
How to gloss *-nként*?

- DIST 'distributive'
  (Tompa, 1968; Kenesei et al., 1998; Csirmaz & Szabolcsi, 2012; Dékány & Hegedűs, 2021)
- I gloss it as DIV for 'division' (ambiguity intended).
  Division can be either arithmetic or mereological.

Event-mereology analysis of binominal *each*

They ate two olives each$_{\theta_{\text{agent}}}$ (Champollion 2017)

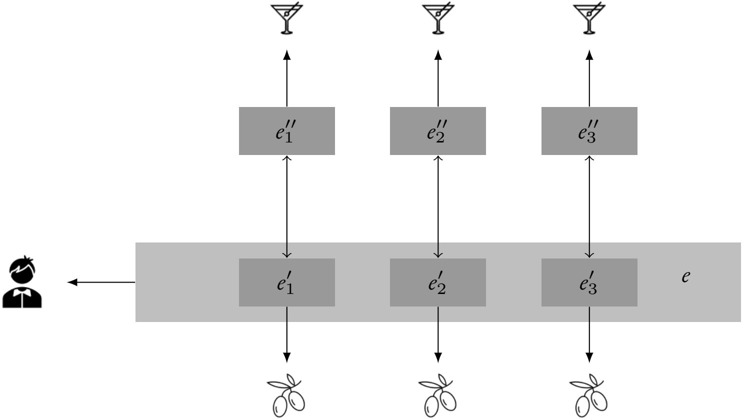
They = key; two olives = share



Distributivity marker analysis of *per* (or *-nként*)

James Bond ate two olives per martini (Panaitescu & Tovena 2019)

martini = key; two olives = share



'Match' function inspired by Boolos (1981), Rothstein (1995).

Ratio marker analysis of *per* (or *-nként*)

James Bond ate two olives per martini

$$= \frac{\text{olives eaten in } e}{\text{martinis drunk in } e}$$

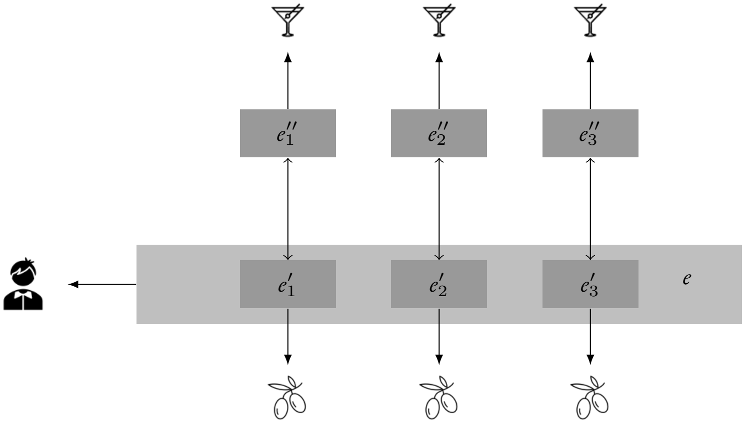
What I hope to convince you of

- A distributivity marker suffices neither for *per* nor for *-nként*; they (at least sometimes) express arithmetic division.
- But *-nként* also expresses mereological division of eventualities.
- Hence mereological division co-lexifies with arithmetic division.
- So they are adjacent in conceptual space and form a natural class.
- A unified analysis can be obtained via some tricks involving measurement and partitions.

Outline

Distributivity marker analysis (again)

James Bond ate two olives per martini (Panaitescu & Tovena 2019)



Outline

Predictions of distributivity marker analysis

- **Minimal size requirement**: The eventuality described by the clause should be divisible into one or more ‘key’-sized chunks.
- **Uniformity requirement**: The eventuality described by the clause should be divisible into subevents that uniformly manifest both the share and the key.
- **Indefinite share requirement**: A *per* phrase should only be able to modify (cardinal) indefinites.
- **Event predicates not terms**: A *per* phrase, together with its licensor, creates a predicate characterizing an event, and not a (degree-denoting) term.

No minimal size requirement for *per*

Unlike with *each*, the event is not always divisible into 'key'-sized chunks with *per*:

(2) James Bond drove 100 km per hour.
$\not\Rightarrow$??For each hour, James Bond drove 100 km.
(Event could last five minutes.)

(3) Do arm swing drills at 240 steps per minute for 20 seconds.

Call these 'sub-unit cases'.

Sub-unit cases with Hungarian *-nként*

Csirmaz & Szabolcsi (2012) mention *-nként* under 'rate expressions' and give the following example:

(4)

Az	a	vonat	órá-nként	400	kilométer-rel	halad
that	the	train	hour-DIV	400	km-INST	advances

'That train is travelling at 400 km/hour'

The event is not necessarily composed of hour-long subevents.

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Non-uniform scenarios

English:

(5) The Montreal Canadiens scored 2.82 goals per game in 2020-21.

Hungarian:

(6) a nitrát-irányelv 1,7 számosállat-egységről rendelkezik
the nitrate-directive 1.7 livestock-units provides.for
hektáronként.
hectare-DIV
'The Nitrates Directive provides for 1.7 livestock units per hectare.'

There are no subevents involving 1.7 livestock units.

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Gradable predicates

English *per* allows gradable predicate hosts, unlike adnominal *each*:

(7) a. It's \$2 {per person, each}.
    b. It's cheaper {per person, *each}.

Hungarian *-nként* is like *per* in this respect:

(8) Fej-enként olcs-óbb is, és környezetbarát-abb.
    head-DIV cheap-CMPR also, and environmentally.friendly-CMPR
    'It's cheaper per person, and more environmentally friendly.'

Dimension nouns

English *per* can be hosted by measure function-denoting nouns, unlike adnominal *each*:

(9) a. The price is \$2 {per person, each}.
b. The price {per person, *each} is \$2.

(10) a kilométer-enként kivetett díj csak növeked-het-ne
the km-DIV levied fee only increase-could-would
'The fee levied per km would only potentially increase.'

Predictions of distributivity marker analysis

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Term uses as differential argument of comparative

English:

(11) There are tables of various woods that put mahogany 200 kg per cubic meter **denser** than poplar.

Hungarian:

(12)

kilométer-enként	két	perc-cel	**gyors-abb**	tempó-t	ment.
kilometer-dist	2	minute-with	fast-er	tempo-acc	go.3sg

‘it went two minutes faster per kilometer’

(Cf. Rawlins 2013 on differential arguments)

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Representation language

$\mathcal{L}_{\mathscr{Q}}$: a λ-calculus with **dimension-centric quantity calculus**

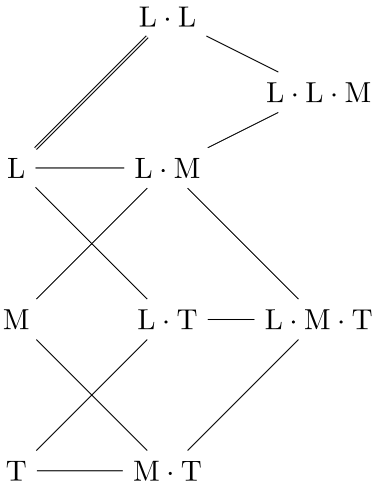
The semantic value of an expression ϕ in $\mathcal{L}_{\mathscr{Q}}$ is given by $[\![\phi]\!]^{\mathcal{M}}$, where:

$$\mathcal{M} = \langle [\langle \mathcal{D}_e, \oplus_e \rangle, \langle \mathcal{D}_v, \oplus_v \rangle, \langle \mathcal{D}_i, \oplus_i \rangle, \langle \mathcal{D}_d, +, * \rangle, \langle \mathcal{D}_m^{\mathscr{B}}, \cdot \rangle], \mathcal{I} \rangle$$

The history of quantity calculus goes back to Fourier 1822 (de Boer, 1994) and is studied in the field of metrology; see for example the International Vocabulary of Metrology (VIM). Here I borrow a dimension-centric approach from (Raposo, 2018).

International System of Units (SI)

Basic and derived dimensions



The dimensions form a group under multiplication

$\mathcal{D}_m = \mathscr{D}$ is a group under $\cdot$, so:

- if $\mathrm{A}, \mathrm{B} \in \mathscr{D}$, then $\mathrm{A} \cdot \mathrm{B} \in \mathscr{D}$
- $\mathscr{D}$ has an identity element $\mathbf{1}_{\mathscr{D}}$, such that for every $D \in \mathscr{D}$:

$$D \cdot \mathbf{1}_{\mathscr{D}} = \mathbf{1}_{\mathscr{D}} \cdot D = D$$

- There is a multiplicative inverse D^{-1} for every $D \in \mathscr{D}$: an element such that

$$D \cdot D^{-1} = \mathbf{1}_{\mathscr{D}}$$

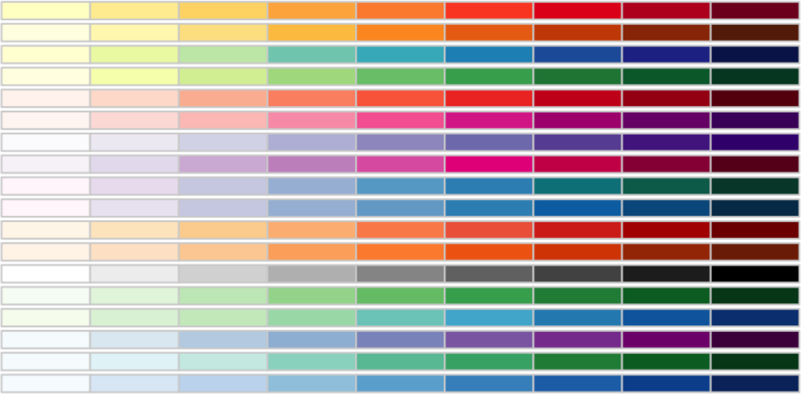
Example:

$\frac{\mathrm{M}}{\mathrm{L}^3}$ is the dimension 'weight per volume'.

Mapping from quantities to dimensions

$$\mathscr{Q} \xrightarrow{\text{dim}} \mathscr{D}$$

The space of quantities forms a fiber bundle



Each fiber is a vector space, with its own additive identity (zero) element.

Cross-dimensional multiplication

$\langle \mathcal{Q}, * \rangle$ is an **abelian monoid**, so:

- If $q_1, q_2 \in \mathcal{Q}$, then $q_1 * q_2 \in \mathcal{Q}$
- There is a **multiplicative identity** element $\mathbf{1}$ such that for all $q \in \mathcal{Q}$:

$$q * \mathbf{1} = \mathbf{1} * q = q$$

- If $q_1, q_2, q_3 \in \mathcal{Q}$ then
  $q_1 * (q_2 * q_3) = (q_1 * q_2) * q_3$ **(associativity)**
- $q_1 * q_2 = q_2 * q_1$ **(commutativity)**

Existence of inverses

$\langle \mathscr{Q}, * \rangle$ is an **abelian monoid**, not a group.

Not every quantity has an inverse; you can't divide by any $\mathbf{0}_D$ $(D \in \mathscr{D})$.

But for every *non-zero* quantity $q \in \mathscr{Q}$
there is an inverse q^{-1}:

$$q * q^{-1} = \mathbf{1}$$

Or: The set of non-zero quantities forms a group under multiplication.

Unit mapping

$$\mathscr{Q} \xleftarrow{\text{unit}} \mathscr{D}$$

where $\text{unit}(D)$ picks out a q such that $\dim(q) = D$

Restrictions:

- You can't pick the zero element (the additive identity).
- unit must be a group homomorphism:

$$\text{unit}(\text{A} \cdot \text{B}) = \text{unit}(\text{A}) * \text{unit}(\text{B})$$

Representation language

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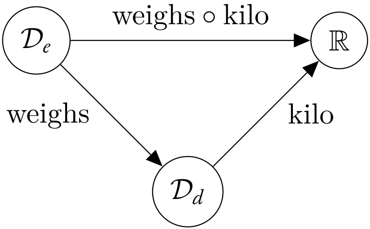
where:

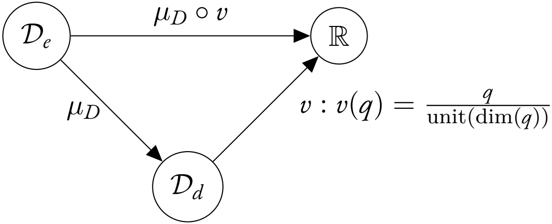
- $\mathscr{B}$ is a finite set of primitive dimensions
- $\langle \mathcal{D}_m^{\mathscr{B}}, \cdot \rangle$ is an abelian group with basis $\mathscr{B}$, a finite set of dimensions
- $\langle \mathcal{D}_d, * \rangle$ is an abelian monoid
- $\mathcal{I}(\textsc{unit})$ is a group homomorphism from $\mathscr{D}_m^{\mathscr{B}}$ to $\mathcal{D}_d$
- $\mathcal{I}(\textsc{dim})$ is a surjection map from $\mathcal{D}_d$ onto $\mathcal{D}_m^{\mathscr{B}}$
- For each $D \in \mathcal{D}_m^{\mathscr{B}}$, $\langle \mathcal{D}_d | D, +, * \rangle$ is a vector space over $\mathbb{R}$
- $\mathcal{I}$ maps each constant of type τ to an element of $\mathcal{D}_\tau$

The Lønning Triangle



(Lonning, 1987; Champollion, 2017)

The Lønning Triangle (à la metrologique)



μ_D: canonical measure function for dimension D

What dimensions does natural language make use of?

Complements of *per* in EuroParl:

distance: *kilometre (of intra-Community trade)*, *100 km*
area: *hectare/decare (of arable land)*, *square metre (live weight)*
time: *annum*, *calendar year*, *day*, *24 hours*, *season*
volume: *cubic centimetre*, *hectolitre (of pure alcohol)*, l*itre (of milk)*
weight: *kilo (of fertilizer)*, *reduced tonne of greenhouse gas*
power: *kilowatt (produced)*, *megajoule*
energy: *energy unit*, *kilowatt-hour (sold)*, *kW/hour*
extensive: *unit (of output/production/quantity/food)*
effort: *unit of effort*
information: *megabyte*
money: *euro (of subsidy)*, *mille of GNP*, *year of EU funding*

Cardinality denominator dimensions in EuroParl

card:human: *capita*, *head of population*, *child*, *farmer*, *taxpayer*, *pupil*
card:animate: *bird*, *fish*, *hen*, *ewe*, *million adult cattle*, *1000 animals*
card:organization: *household*, *farm*, *power station*, *country*, *NGO*
card:tangible: *car*, *cigarette*, *goods vehicle*, *olive tree*, *ship*, *dwelling*
card:intangible: *paragraph*, *policy area*, *category of cars*, *job created*
card:location: *continent*, *zone*, *region*, *port*, *lake*
card:event: *session*, *Presidency*, *accident*, *death*, *flight*, *money withdrawal*
card:human / distance: *passenger kilometer*

Cardinality dimensions (individuals)

Let us assume that for every subset of D_e $P \in D_{\langle e,t \rangle}$, there is a basic dimension $\#\textsc{dim}(P)$. I call these 'cardinality dimensions'.

$\textsc{unit}(\#\textsc{martini})$ denotes the quantity '1 martini'

Sortal nouns are ambiguous, e.g.:
martini $\rightsquigarrow \lambda x \,.\, \textsc{martini}(x)$ (type $\langle e, t \rangle$)
martini $\rightsquigarrow \textsc{unit}(\#\textsc{martini})$ (type d)

Outline

Lexical entries for *per*

Three ratio-marker senses of *per*:

- quotient function

  per $\rightsquigarrow \lambda d_d \lambda q_d \,.\, \frac{q}{d}$

- quotient operator

  $$per \rightsquigarrow \lambda d_d \,.\, \lambda q_d \,.\, \lambda G_{\langle d,\tau t\rangle} \,.\, \lambda \alpha_\tau \,.\, \frac{\textsc{max}(\lambda d' \,.\, G(d')(\alpha))}{\mu_{\dim(d)}(\alpha)} = \frac{q}{d}$$

- dimension quotient

  per $\rightsquigarrow \lambda g_{\langle e,d\rangle} \lambda f_{\langle e,d\rangle} \lambda x_e \,.\, \frac{f(x)}{g(x)}$

Challenge for the quotient function analysis

(13) It's estimated that 150 species per day go extinct.
150 species per day is a high rate.
#Therefore, a high rate is among those going extinct.

Target truth conditions:

$$\text{Gen}\, e\, .\, \frac{\text{the number of species that go extinct in } e}{\text{the duration of } e} = \frac{150}{\text{day}}$$

Compositional derivation for quotient operator use

vt

$$\lambda e \,.\, \frac{\textsc{max}(\lambda d \,.\, \textsc{ge}(e) \wedge \exists x[\textsc{th}(e) = x \wedge {}^{*}\textsc{sp}(x) \wedge \mu_D(x) = d])}{\mu_{\textsc{dim}(\textsc{day})}(e)} = \frac{150}{\textsc{day}}$$

$\langle\langle d, vt\rangle, vt\rangle$

$$\lambda G_{\langle d,vt\rangle} \,.\, \lambda e \,.\, \frac{\textsc{max}(\lambda d \,.\, G(d)(e))}{\mu_{\textsc{dim}(\textsc{day})}(e)} = \frac{150}{\textsc{day}}$$

d

150

|

150

$\langle d, \langle\langle d, vt\rangle, vt\rangle\rangle$

$$\lambda q_d \,.\, \lambda G_{\langle d,vt\rangle} \,.\, \lambda e \,.\, \frac{\textsc{max}(\lambda d \,.\, G(d)(e))}{\mu_{\textsc{dim}(\textsc{day})}(e)} = \frac{q}{\textsc{day}}$$

per$_1$ *day*

$\langle d, vt\rangle$

$$\lambda d \lambda e \,.\, \textsc{ge}(e) \wedge \exists x[\textsc{th}(e) = x \wedge {}^{*}\textsc{sp}(x) \wedge \mu_D(x) = d]$$

λd *d-many species go extinct*

$$per_1 \rightsquigarrow \lambda d_d \,.\, \lambda q_d \,.\, \lambda G_{\langle d,\tau t\rangle} \,.\, \lambda \alpha_\tau \,.\, \frac{\textsc{max}(\lambda d' \,.\, G(d')(\alpha))}{\mu_{\textsc{dim}(d)}(\alpha)} = \frac{q}{d}$$

(Coppock, 2022)

James Bond ate two olives per martini

vt

$$\lambda e\,.\,\frac{\textsc{max}(\lambda d\,.\,\textsc{eat}(e) \wedge \exists x[\textsc{th}(e) = x \wedge {}^*\textsc{olive}(x) \wedge \mu_D(x) = d])}{\mu_{\#\textsc{martini}}(e)} = \frac{2}{\textsc{unit}(\#\textsc{martini})}$$

$\langle\langle d, vt\rangle, vt\rangle$

$$\lambda G_{\langle d,vt\rangle}\,.\,\lambda e\,.\,\frac{\textsc{max}(\lambda d\,.\,G(d)(e))}{\mu_{\#\textsc{martini}}(e)} = \frac{2}{\textsc{unit}(\#\textsc{martini})}$$

$\langle d, vt\rangle$

$$\lambda d \lambda e\,.\,\textsc{eat}(e) \wedge \exists x[\textsc{th}(e) = x \wedge {}^*\textsc{olive}(x) \wedge \mu_D(x) = d]$$

λd eat d-many olives

d

2

2

$\langle d, \langle\langle d, vt\rangle, vt\rangle\rangle$

$$\lambda q_d\,.\,\lambda G_{\langle d,vt\rangle}\,.\,\lambda e\,.\,\frac{\textsc{max}(\lambda d\,.\,G(d)(e))}{\mu_{\#\textsc{martini}}(e)} = \frac{q}{\textsc{unit}(\#\textsc{martini})}$$

per$_1$ *martini*

'The ratio of how many olives are eaten in *e* to the measure of *e* along the number-of-martinis dimension is equal to 2 divided by one martini.'

Another challenge for the quotient function analysis

Measure function verbs like *weigh*:

(14) Water weighs 1 kg per liter.

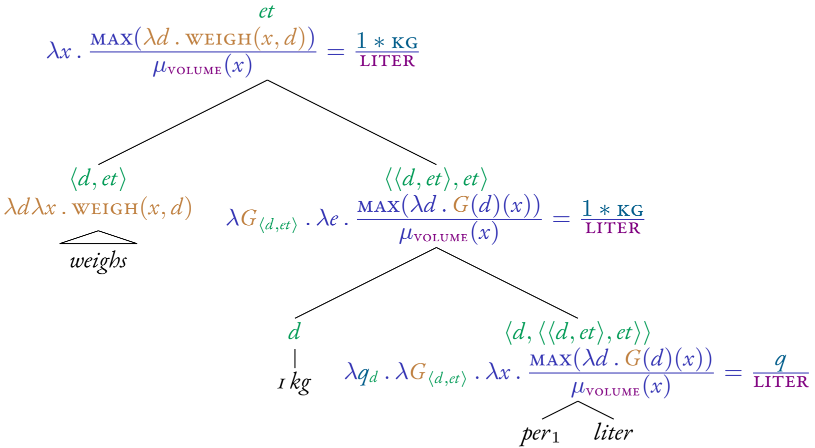
Schwarz & Bale (2022) point out that it does not suffice to treat '1 kg per liter' as degree-denoting term here; what water weighs is not a ratio of weight to volume.

$$\text{weight}(x) \neq \frac{1 * \text{kg}}{\text{liter}}$$

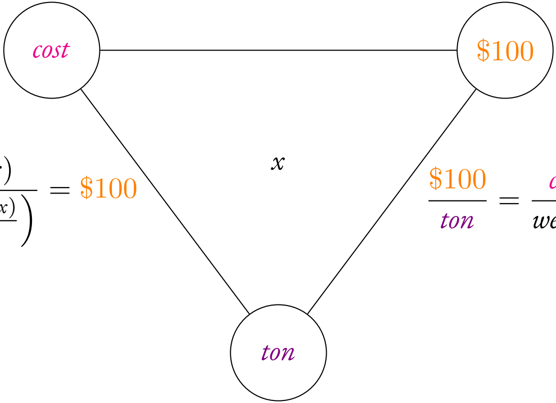
$$\dim(\text{weight}(x)) = \text{M} \qquad \dim\left(\frac{1 * \text{kg}}{\text{liter}}\right) = \frac{\text{M}}{\text{L}^3}$$

Compositional derivation for measure function verb case

Water weighs 1 kg per liter



The Quotient Triangle



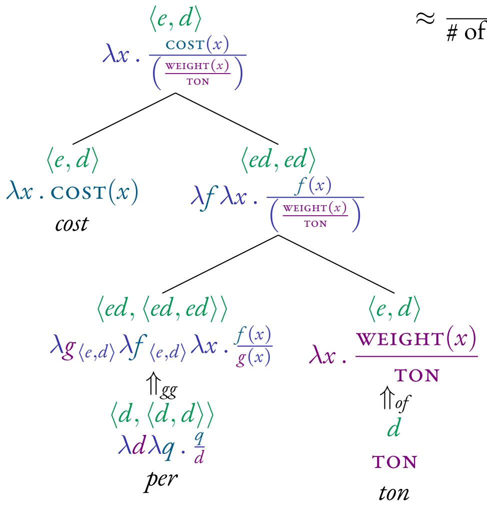
$$\frac{cost(x)}{\left(\frac{weight(x)}{ton}\right)} = \$100$$

$$\frac{\$100}{ton} = \frac{cost(x)}{weight(x)}$$

The cost per ton is $100

The cost is $100 per ton

Dimension quotient analysis

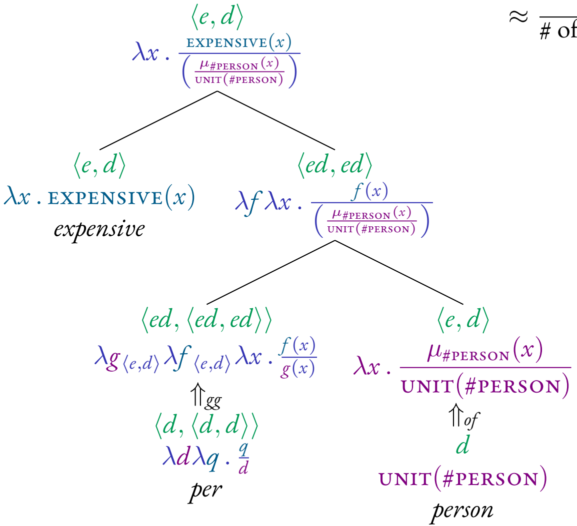


$$\approx \frac{\text{cost of x}}{\text{\# of tons x weighs}}$$

Gradable predicates

(how) expensive per person

$$\approx \frac{\text{cost of x}}{\text{\# of people x measures}}$$



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- dimension quotient
  per $\rightsquigarrow \lambda g_{\langle e,d\rangle} \lambda f_{\langle e,d\rangle} \lambda x_e \,.\, \frac{f(x)}{g(x)}$

I assume that *-nként* has all these uses, too.

Outline

Frequency uses

(15) Mari het-enként latogatja a nagymamát
Mary week-DIV visits the grandma.POSS.ACC
'Mary visits her grandma at least once weekly.'

Can this usage be captured using a ratio-marker analysis?

Maybe with a silent multiplicative *egyszer* 'once'?

(16) Mari legalább egyszer het-enként latogatja a nagymamát
Mary at.least once week-DIV visits the grandma.POSS.ACC
'Mary visits her grandma at least once weekly.'

Visiting gramma once

Lexical entry for the multiplicative (inspired by Wagiel 2023):
-szer $\rightsquigarrow \lambda n \lambda V_{\langle v,t \rangle} \lambda e \,.\, \mu_{\#V}(e) = n$

Assumption: For a given predicate of events V, $\mu_{\#V}(e) = n$ means that e contains as a (proper or improper) subpart exactly n instances of V.

Derivation tree:

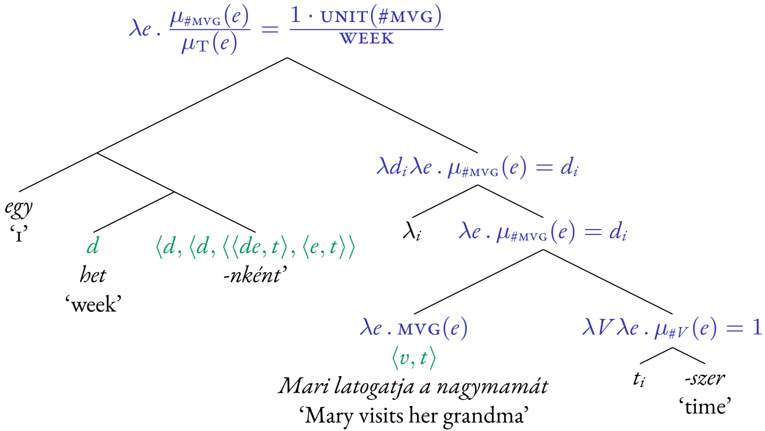
$\lambda e \,.\, \mu_{\#\textsc{mvg}}(e) = 1$

$\lambda e \,.\, \textsc{mvg}(e)$ $\langle v, t \rangle$
Mari latogatja a nagymamát
'Mary visits her grandma'

$\lambda V \lambda e \,.\, \mu_{\#V}(e) = 1$

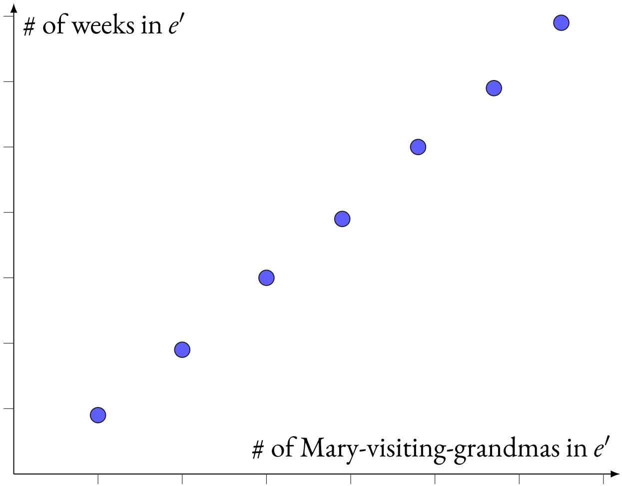
egy '1' *-szer* 'time'

Visiting grandma once per week



Getting even ratios

Let e' be a subevent of e.



Champollion's 'stratified reference'

Accoridng to Champollion, 'waltz for an hour' presupposes that *waltz* has stratified reference with respect to the dimension 'runtime' and the granularity 'one hour':

$$\forall e[\textsc{waltz}(e) \rightarrow e \in {}^*\lambda e'[\textsc{waltz}(e') \wedge \epsilon(1 \cdot \textsc{hour})(\tau(e'))]]$$

'Every waltzing event can be divided into one or more parts, each of which is a waltzing event whose runtime is very short compared with one hour.'

In general, V has **stratified reference** with respect to dimension θ and granularity $\epsilon(K)$, $SR_{\theta,\epsilon(K)}(V)$, iff:

$$\forall e[V(e) \rightarrow e \in {}^*\lambda e'[V(e') \wedge \epsilon(K)(\theta(e'))]]$$

Defining homogeneity

Let us say that e is **homogeneous** with respect to dimension D, granularity ϵ, and predicate V iff:

$$e \in *\lambda e'[V(e') \wedge \mu_D(e') = \epsilon]$$

Proposal: Quotient-operator *-nként* has can be strengthened with the inference that e is homogenous with respect to the dimension and granularity of its complement, and the ratio-predicate that it builds using its surrounding syntactic context.

Strengthening *per* with homogeneity

$per_1^{\mathcal{H}} \rightsquigarrow$

$$\lambda d_d \,.\, \lambda q_d \,.\, \lambda G_{\langle d,\tau t\rangle} \,.\, \lambda \alpha_\tau \,.\, \mathcal{H}(\alpha)(\lambda \alpha' \,.\, \frac{\textsc{max}(\lambda d' \,.\, G(d')(\alpha'))}{\mu_{\textsc{dim}(d)}(\alpha'))} = \frac{q}{d})(d)$$

where

$$\mathcal{H}(\alpha)(V)(d) \equiv V(\alpha) \wedge \alpha \in {}^{*}\lambda\alpha'[V(\alpha') \wedge \mu_{\text{dim}(d)}(\alpha') = d]$$

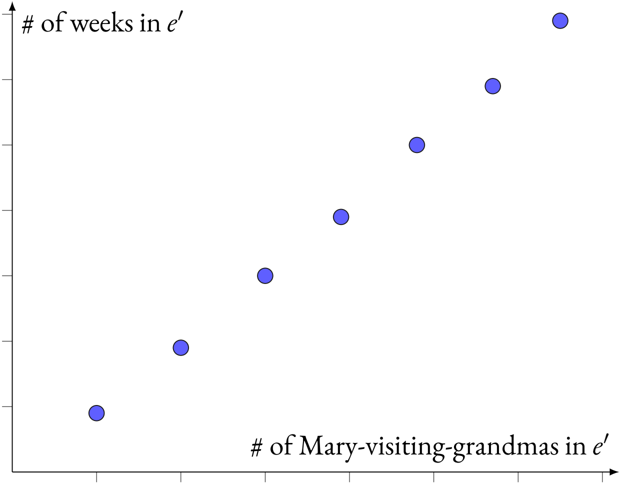
Example:

$$\lambda e \,.\, \mathcal{H}(e)(\lambda e' \,.\, \frac{\mu_{\#\textsc{mvg}}(e')}{\mu_{\text{T}}(e')} = \frac{1 \cdot \textsc{unit}(\#\textsc{mvg})}{\textsc{week}})(\textsc{week})$$

'The ratio of Mary-visiting-grandmas to weeks in *e* is 1:1, and *e* is composed of week-long subevents exhibiting the same ratio.'

Getting even ratios

Let e' be a subevent of e.



Begging from door to door

(17) És reggel az fráterek ajtó-nkéd kenyeret kolulának.
and morning the friars door-DIV bread begged
'And in the morning, the friars went begging for bread from door to door.'
(Bende-Farkas & Halm, 2024)

With a silent *egyszer* 'once', we can derive:

$$\lambda e \,.\, \frac{\mu_{\#\text{BEG.FOR.BREAD}}(e)}{\mu_{\#\text{DOOR}}(e)} = \frac{1 \cdot \text{UNIT}(\#\text{BEG.FOR.BREAD})}{\text{UNIT}(\#\text{DOOR})}$$

'The ratio of beggings for bread to doors in *e* is 1 (begging) to 1 (door).'
Then we can add a homogeneity assumption.

‘Top-down’ cases (Bende-Farkas & Halm, 2024)

Here we have only one event of taking the big pill:

(18) A nagymama negyed-enként vette be a nagy tablettát
the grandma quarter-DIV took the big pill.ACC
‘Grandma took the big pill quarter by quarter’

Here we have only one arrangement:

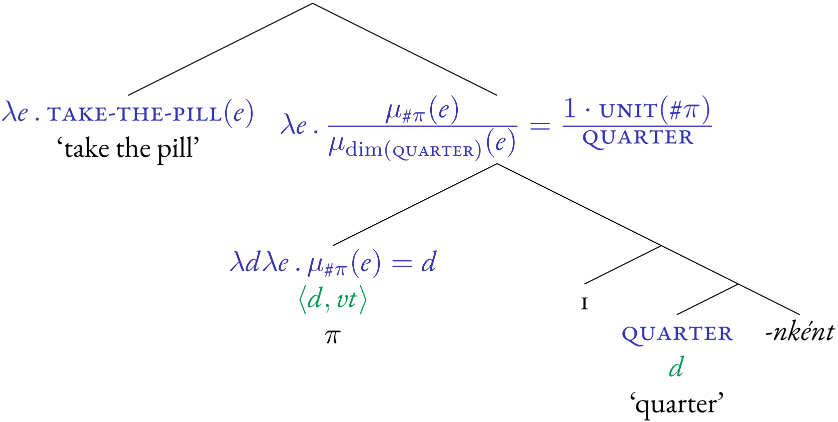
(19) Mari szín-enként rendezte el a ruhákat.
Mari color-DIV arranged the clothes.ACC
‘Mary arranged the clothes by color.’
(Balazs Suranyi, p.c.)

Similar to this type of use of *per* in EuroParl:

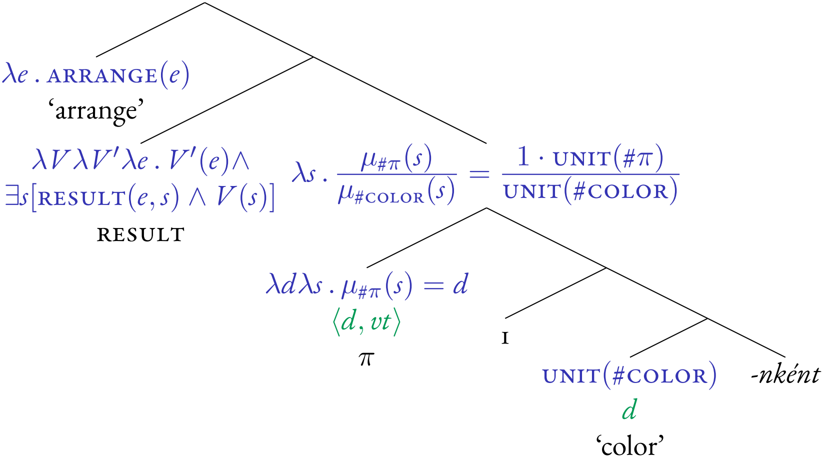
(20) The complete table with a breakdown of all applications per prior right and country of applicant can be found on the website.

Taking the pill quarter by quarter

Let π be a salient partition of e (possibly a state).
Let $\mu_{\#\pi}(e')$ count the number of cells in π that e' instantiates.



Arranging by color



Summary

-nként has several uses:

- 'two olives per martini'-type cases
  – non-decisive re: distributivity-marker vs. ratio-marker
- 'cost per person'-type cases
  – support a ratio-marker analysis
- 'visit Grandma weekly'-type cases
  – motivate a silent 'once', and a homogeneity assumption
- 'eat the pill by quarter'-type cases
  – motivate appeal to partitions

All of these cases can be obtained via a ratio-marker analysis, sometimes augmented by certain additional assumptions.

Outline

Conclusions

- Quantity calculus is useful in natural language semantics
  - For example, English *per* and Hungarian *-nként* are ratio markers; they express arithmetic division
- Arithmetic and mereological division are conceptually adjacent
  - *-nként* picks out a concept covering both of them
  - Arithmetic division underlies a unified analysis
  - Distributivity can roughly be factored into arithmetic division and homogeneity
    $\Rightarrow$ Grammaticalization pathway:
    distributivity $\rightarrow$ loss of homogeneity $\rightarrow$ arithmetic division?

Thank you!

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