Lecture 3, Part II
Models

Elizabeth Coppock

Introduction to Semantics · EGG 2019

Outline

Domain: ABBA

Some gossip

ABBA was composed of two married couples:

- Björn and Agneta
- Anni-Frid and Benny.

So these were facts:

Loves(ag,bj)
Loves(bj,ag)

Loves(be,an)
Loves(an,be)

Denotations for names

$[\![\text{ag}]\!]$ = Agneta

$[\![\text{bj}]\!]$ = Björn

$[\![\text{be}]\!]$ = Benny

$[\![\text{an}]\!]$ = Anni-Frid

Direct vs. indirect interpretation

Direct intepretation

$[\![\textit{Björn}]\!] = \text{Björn}$

Indirect interpretation

$\textit{Björn} \rightsquigarrow \textbf{bj}$

$[\![\textbf{bj}]\!] = \text{Björn}$

Semantics of binary predicates

$[\![\text{Loves}]\!] =$
$\{\langle \text{Agneta}, \text{Björn} \rangle, \langle \text{Björn}, \text{Agneta} \rangle, \langle \text{Anni-Frid}, \text{Benny} \rangle, \langle \text{Benny}, \text{Anni-Frid} \rangle\}$

Semantics of binary predicates

$[\![\text{Loves}]\!] =$
$\{\langle \text{Agneta}, \text{Björn}\rangle, \langle \text{Björn}, \text{Agneta}\rangle, \langle \text{Anni-Frid}, \text{Benny}\rangle, \langle \text{Benny}, \text{Anni-Frid}\rangle\}$

$[\![\text{Loves}(\text{ag}, \text{bj})]\!] = 1$
because $\langle \text{Agneta}, \text{Björn}\rangle \in [\![\text{Loves}]\!]$

Some more gossip

Both of the marriages ended. So we have to distinguish between two situations, which we will represent with **models**:

- M_{THEN}: how it was back in the day
- M_{NOW}: how it is now

$[\![\text{Loves}]\!]^{M_{\text{THEN}}} =$
$\{\langle \text{Agneta}, \text{Björn} \rangle, \langle \text{Björn}, \text{Agneta} \rangle, \langle \text{Anni-Frid}, \text{Benny} \rangle, \langle \text{Benny}, \text{Anni-Frid} \rangle\}$

$[\![\text{Loves}]\!]^{M_{\text{NOW}}} = \{\}$

Semantics of binary predicates

$[\![\text{Loves}]\!]^{M_{\text{THEN}}} =$
$\{\langle \text{Agneta}, \text{Björn} \rangle, \langle \text{Björn}, \text{Agneta} \rangle, \langle \text{Anni-Frid}, \text{Benny} \rangle, \langle \text{Benny}, \text{Anni-Frid} \rangle\}$

Semantics of binary predicates

$[\![\text{Loves}]\!]^{M_{\text{THEN}}} =$
$\{\langle \text{Agneta}, \text{Björn} \rangle, \langle \text{Björn}, \text{Agneta} \rangle, \langle \text{Anni-Frid}, \text{Benny} \rangle, \langle \text{Benny}, \text{Anni-Frid} \rangle\}$

$[\![\text{Loves(ag, bj)}]\!]^{M_{\text{THEN}}} = 1$
because $\langle \text{Agneta}, \text{Björn} \rangle \in [\![\text{Loves}]\!]^{M_{\text{THEN}}}$

Semantics of binary predicates

$$[\![\text{Loves}]\!]^{M_{\text{NOW}}} = \{\}$$

Semantics of binary predicates

$[\![\text{Loves}]\!]^{M_{\text{NOW}}} = \{\}$

$[\![\text{Loves}(\text{ag}, \text{bj})]\!]^{M_{\text{NOW}}} = 0$
because $\langle \text{Agneta}, \text{Björn} \rangle \notin [\![\text{Loves}]\!]^{M_{\text{NOW}}}$

Semantic rules for predication

Semantic rule: Unary predicates

If π is a unary predicate and α is a term, then:

$$[\![\pi(\alpha)]\!]^M = 1 \text{ if } [\![\alpha]\!]^M \in [\![\pi]\!]^M, \text{ and 0 otherwise.}$$

Semantic rule: Binary predicates

If π is a unary predicate and α and β are terms, then:

$$[\![\pi(\alpha, \beta)]\!]^M = 1 \text{ if } \langle [\![\alpha]\!]^M, [\![\beta]\!]^M \rangle \in [\![\pi]\!]^M, \text{ and 0 otherwise.}$$

What's in a model?

$M = \langle D, I \rangle$
where

- D is a set of individuals (the **domain**)
- I is a function that assigns a denotation to every non-logical constant (the **interpretation function**)

Logical vs. non-logical constants

Two types of constants:

- **Non-logical constants**: all names, predicates
- **Logical constants**: $\wedge$, $\vee$, $\neg$, $\rightarrow$ $\forall$, $\exists$, λ

Examples of interpretation functions

$M_{\text{THEN}} = \langle D_{\text{THEN}}, I_{\text{THEN}} \rangle$

$I_{\text{THEN}}(\text{Loves}) =$
$\{\langle \text{Agneta}, \text{Björn} \rangle, \langle \text{Björn}, \text{Agneta} \rangle, \langle \text{Anni-Frid}, \text{Benny} \rangle, \langle \text{Benny}, \text{Anni-Frid} \rangle\}$

$M_{\text{NOW}} = \langle D_{\text{NOW}}, I_{\text{NOW}} \rangle$

$I_{\text{NOW}}(\text{Loves}) = \{\}$

Semantic rule for non-logical constants

Semantic rule: Non-logical constants

If α is a non-logical constant, and $M = \langle D, I \rangle$,
then $[\![\alpha]\!]^M = I(\alpha)$.

Truth and entailment relative to a model

a ϕ is **true in** model M iff $[\![\phi]\!]^M = 1$

ϕ **logically entails** ψ iff:
In every model where ϕ is true, ψ is true too.

Models vs. possible worlds

- a **possible world** is a fully-specified way the world could be
- a **model** determines a denotation for all of the names and predicates (unary, binary, etc.) in the representation language (the **logical constants**)
  - Models can describe impossible situations, e.g. no requirement that bachelors are unmarried.

Two ways of defining entailment

- **Necessary consequence**

  ϕ is a necessary consequence of ψ iff:
  In every possible world where ϕ is true, ψ is true too.

- **Logical consequence**

  ϕ logically entails ψ iff:
  In every model where ϕ is true, ψ is true too
  (letting the universe of models have more or less total freedom to assign denotations the non-logical constants).

Necessary vs. logical consequence: Examples

- **Logical consequence**

  $[\mathrm{Loves}(ag, bj) \wedge \mathrm{Loves}(bj, ag)]$
  Therefore: $\mathrm{Loves}(ag, bj)$

- **Non-logical, but necessary consequence:**

  $\mathrm{Bachelor}(bj)$
  Therefore: $\mathrm{Male}(bj)$

Meaning postulates

Richard Montague suggested that we capture necessary consequence using **meaning postulates**.

These would constrain the space of models so that anything that satisfies Bachelor also satisfies Married.